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String cosmology in Bianchi type-VI_0 dusty Universe with electromagnetic field

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Abstract. In this paper, the effect of electromagnetic field in the string Bianchi type-VI_0 Universe is investigated. Einstein's field equations have been solved exactly with suitable physical assumptions for two types of strings: (i) massive strings and (ii) Nambu strings. It is found that when the Universe is dominated by massive strings, the existence of electromagnetic field is necessary as it accelerates the expansion of the Universe. But when our Universe is dominated by Nambu strings, the electromagnetic field does not have significant effect on the evolution of the Universe. We have also shown that the early massive string-dominated Universe got converted to Nambu string-dominated Universe later. Our models are derived from an early deceleration phase to an accelerating phase which is consistent with the recent observations of supernovae type-Ia. The physical and geometrical behaviour of these models are also discussed.

Keywords. Bianchi type-VI_0 Universe; Nambu string; massive string.

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1. Introduction

In recent years, there has been considerable interest in string cosmology. Cosmic strings are linear defects, analogous to flux tubes in type-II superconductors, or to vortex filaments in superfluid helium. They can be formed at a phase transition anywhere between the grand unified theories (GUT) and electroweak scales [1–6]. The best developed of the many cosmic-string scenarios concerns GUT-scale strings, which generate both density perturbations and perturbations in the microwave background of roughly the observed order of magnitude. In a CDM Universe, they do not seem very attractive, but the combination of strings and HDM is a very promising alternative to the popular inflationary models and definitely does improve the agreement with observation [7]. The vacuum strings may generate sufficient density fluctuations to explain the galaxy formation [8]. The strings that form the cloud are massive strings instead of geometrical strings. Each massive string is formed by a geometrical string with particles attached along its

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extension. Hence, the strings that form the cloud are the generalization of Takabayasi's relativistic model of strings (called p-strings). This is the simplest model wherein we have particles and strings together. In principle, we can eliminate the strings and end up with a cloud of particles. This desirable property of a model of a string cloud can be used in cosmology since strings are not observed at the present time of the evolution of the Universe (see [9–12]). These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general relativistic treatment of strings was initiated by Letelier [13,14].

The discovery of a cosmological magnetic field of the order of 10^{-8} G [15] increased the interest in the study of cosmological models that admit an electromagnetic field. The presence of such a field demands the use of a class of space-times more general than the Friedmann–Robertson–Walker ones, since isotropy is broken. The most natural extension is the study of spatially homogeneous models, which were classified as the Bianchi type models [16,17]. Zel'dovich [18], Harrison [19], Misner *et al* [20], Asseo and Sol [21], Pudritz and Silk [22], Kim *et al* [23], Perley and Taylor [24], Kronberg *et al* [25], Wolfe *et al* [26], Kulsrud *et al* [27] and Barrow [28] have pointed out the importance of magnetic field in different contexts. It is reasonable to consider magnetic fields in the energy–momentum tensor of the early Universe. The string cosmological models with a magnetic field are also discussed by Banerjee *et al* [29], Chakraborty [30], Tikekar and Patel [31,32], Patel and Maharaj [33], Singh and Singh [34]. Singh [35,36] has studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times. Lidsey *et al* [37] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Bali *et al* [38–43], Yadav *et al* [44–46] have investigated Bianchi type-I, -II, -III, -V, -IX and -VI_0 magnetized string cosmological models in the presence of bulk viscosity. Recently, Pradhan and Lata [47] have investigated the magnetized Bianchi type-VI_0 bulk viscous barotropic massive string Universe with decaying vacuum energy density Λ in general relativity.

Motivated by the situations discussed above, in this paper, we shall focus upon the problem of establishing a formalism for studying the massive string in Bianchi type-VI_0 space-time. In the present study, we have obtained some Bianchi type-VI_0 string cosmological models in the presence and absence of magnetic field. The paper is organized as follows. The metric and the field equations are presented in §2. In §3, we deal with solution of the field equations with cloud of strings and electromagnetic field. In §3.1 the field equations have been solved for massive string. We describe some physical and geometric properties of the model in §3.1.1. In §3.2 the field equations have been solved for Nambu string. We describe some physical and geometric properties of the model in §3.2.1. Finally, in §4, concluding remarks are given.

2. The metric and field equations

We consider the Bianchi type-VI_0 metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2\mathrm{e}^{2x}\mathrm{d}y^2 + C^2\mathrm{e}^{-2x}\mathrm{d}z^2, \tag{1}$$

where A, B and C are functions of t only. The energy–momentum tensor for a cloud of strings in the presence of electromagnetic field is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j + \left[g^{lm} F_{il} F_{km} - \frac{1}{4} g_{ik} F_{lm} F^{lm} \right], \tag{2}$$

where u_i and x_i satisfy the condition

$$u_i u^i = -x^i x_i = -1, \qquad u^i x_i = 0, \tag{3}$$

where ρ is the proper energy density for a cloud string with particles attached to them, λ is the string tension density, u^i is the four-velocity of the particles and x^i is a unit space-like vector representing the direction of string. The string tension density λ can take positive or negative values. Positive value of λ represents a Universe filled with no strings but only an anisotropic fluid whereas its negative value represents strings loaded with particles forming the surface of world sheet [13,14]. In a co-moving coordinate system, we have

$$u^i = (0, 0, 0, 1), \qquad x^i = \left(\frac{1}{A}, 0, 0, 0 \right), \tag{4}$$

i.e., the direction of strings are assumed to be along the x-axis.

The Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0, \tag{5}$$

are satisfied by

$$F_{23} = K\,(\text{say}) = \text{constant}, \tag{6}$$

where a semicolon (;) stands for covariant differentiation. The particle density of the configuration is given by

$$\rho = \rho_{\rm p} + \lambda. \tag{7}$$

The Einstein's field equations (with $8\pi G = 1$ and $C = 1$)

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -T_{ij}, \tag{8}$$

for the metric (1) leads to the following system of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = \lambda + \frac{K^2}{2B^2C^2}, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\frac{K^2}{2B^2C^2}, \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\frac{K^2}{2B^2C^2}, \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = \rho + \frac{K^2}{2B^2C^2}, \tag{12}$$

$$\frac{1}{A}\left[\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right] = 0, \tag{13}$$

where an overdot stands for the first and double overdot for second derivative with respect to t.

The average scalar factor a for Bianchi type-VI_0 is given by

$$a = (ABC)^{1/3}. \tag{14}$$

A volume scale factor V is defined as

$$V = a^3 = ABC. \tag{15}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{16}$$

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$ and $H_3 = \dot{C}/C$ are the directional Hubble's parameters in x, y and z directions respectively.

From eqs (14) and (15), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \tag{17}$$

An important observational quantity is the deceleration parameter q, which is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \tag{18}$$

The scalar expansion θ, the shear scalar σ^2 and the average anisotropy parameter A_m are defined as

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{19}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right), \tag{20}$$

$$A_m = \frac{1}{3}\sum_{i}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{21}$$

where $\Delta H_i = H_i - H, i = 1, 2, 3$.

3. Solution of the field equations

Fist of all we observe that eq. (13) leads to

$$C = mB, \tag{22}$$

where m is an integrating constant.

Therefore, the field equations (9)–(12) are a system of three equations with five unknown parameters A, B, C, ρ, λ as eqs (10) and (11) are the same under the condition given by eq. (22). Two additional constraints relating these parameters are required to obtain explicit solutions of the system. First, we assume that the expansion θ in the model is proportional to the shear σ. This condition, by using eq. (22), leads to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = b\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{23}$$

which yields

$$\frac{\dot{A}}{A} = n\frac{\dot{B}}{B}, \tag{24}$$

where $n = (2b\sqrt{3}+1)/(1-b\sqrt{3})$ and b are constants. Equation (24), after integration, reduces to

$$A = \beta B^n, \tag{25}$$

where β is a constant of integration.

From eqs (9) and (12) we get

$$\lambda = \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} - \frac{K^2}{2B^2C^2}, \tag{26}$$

and

$$\rho = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} - \frac{K^2}{2B^2C^2}. \tag{27}$$

The highly non-linear field equations can be solved for the following two types of strings.

3.1 *Case I: Massive string*

In this case we assume that the sum of the rest energy density and tension density for cloud of strings vanish [48–50], i.e.,

$$\rho + \lambda = 0. \tag{28}$$

From eqs (26)–(28) we obtain

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + 2\frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \frac{K^2}{B^2C^2}. \tag{29}$$

From eqs (22) and (25), eq. (29) can be written as

$$2\ddot{B} + 2(n+1)\frac{\dot{B}^2}{B} = \frac{K^2}{m^2 B^3}. \tag{30}$$

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = ff'$, where $f' = \mathrm{d}f/\mathrm{d}B$. Hence eq. (30) takes the form

$$\frac{\mathrm{d}}{\mathrm{d}B}(f^2) + \frac{2(n+1)}{B}f^2 = \frac{K^2}{m^2 B^3}, \tag{31}$$

which by integration reduces to

$$\mathrm{d}t = \frac{\mathrm{d}B}{\sqrt{(K^2/2m^2 n)B^{-2} + \ell_0 B^{-2(n+1)}}}, \tag{32}$$

where ℓ_0 is an integration constant. Therefore the metric (1) reduces to

$$\mathrm{d}s^2 = -\frac{\mathrm{d}B^2}{(K^2/2m^2 n)B^{-2} + \ell_0 B^{-2(n+1)}} + \beta^2 B^{2n}\mathrm{d}x^2 + B^2 \mathrm{e}^{2x}\mathrm{d}y^2 + m^2 B^2 \mathrm{e}^{-2x}\mathrm{d}z^2. \tag{33}$$

Now by using a suitable transformation of coordinates, i.e.

$$B = T, \qquad \beta x = X, \qquad y = Y, \qquad mz = Z, \tag{34}$$

the model (33) takes the form

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{(K^2/2m^2 n)T^{-2} + \ell_0 T^{-2(n+1)}} + T^{2n}\mathrm{d}X^2 + T^2 \mathrm{e}^{2X}\mathrm{d}Y^2 + T^2 \mathrm{e}^{-2X}\mathrm{d}Z^2. \tag{35}$$

3.1.1 *The geometric and physical significance of the model.* The energy density (ρ), the string tension (λ) and the particle density (ρ_{p}) for the model (35) are given by

$$\rho = \frac{(n+1)K^2}{2m^2 n}T^{-4} + \left[(2n+1)\ell_0 T^{-4} - \frac{1}{\beta^2}\right]T^{-2n}, \tag{36}$$

$$\lambda = -\frac{(n+1)K^2}{2m^2 n}T^{-4} - \left[(2n+1)\ell_0 T^{-4} - \frac{1}{\beta^2}\right]T^{-2n}, \tag{37}$$

$$\rho_{\mathrm{p}} = 2\frac{(n+1)K^2}{2m^2 n}T^{-4} + 2\left[(2n+1)\ell_0 T^{-4} - \frac{1}{\beta^2}\right]T^{-2n}. \tag{38}$$

From eqs (36) and (38), we see that energy conditions, $\rho \geq 0$ and $\rho_{\mathrm{p}} \geq 0$ are satisfied under the condition

$$T^{2(n-2)} \geq \frac{2m^2 n}{(n+1)K^2}\left[\frac{1}{\beta^2} - (2n+1)\ell_0 T^{-4}\right]. \tag{39}$$

From eqs (36) and (38), it is noted that the proper energy density $\rho(t)$ and particle density $\rho_{\mathrm{p}}(t)$ are decreasing functions of time and tend to zero at a later time. These behaviours of $\rho(t)$ and $\rho_{\mathrm{p}}(t)$ are clearly depicted in figure 1 as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well-known situations.

From eq. (37) we observe that the tension density λ is an increasing function of time and it is always negative and it approaches zero at a later time. Letelier [14] pointed out that λ may be positive or negative. When $\lambda < 0$, the string phase of the Universe disappears, i.e. we have an anisotropic fluid of particles. This behaviour of λ is also shown in figure 1.

The expressions for the scalar of expansion θ, magnitude of shear σ^2, proper volume V, deceleration parameter q and the average anisotropy parameter A_m for the model (35) are given by

$$\theta = 3H = (n+2)\left[\frac{K^2}{2m^2 n}T^{-4} + \ell_0 T^{-2(n+2)}\right]^{1/2}, \tag{40}$$

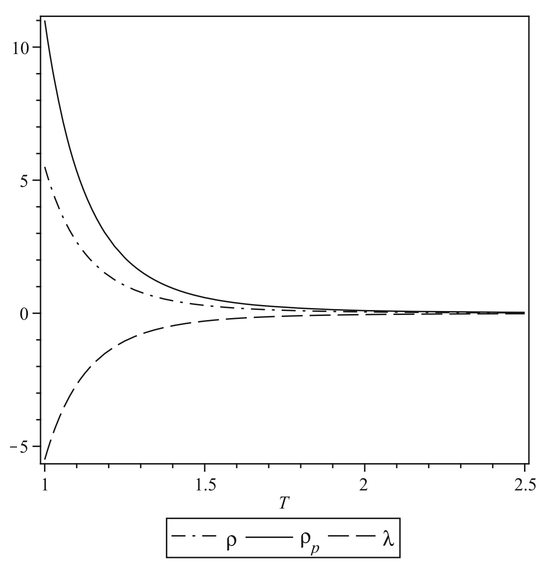

Figure 1. The plot of energy density ρ, particle density ρ_{p} and tension density λ vs. T for $n = 2$ and $K = m = \ell_0 = \beta = 1$.

$$\sigma^2 = \frac{(n-1)^2}{3}\left[\frac{K^2}{2m^2 n}T^{-4} + \ell_0 T^{-2(n+2)}\right], \tag{41}$$

$$V = m\beta T^{n+2}, \tag{42}$$

$$q = -\left[\frac{\big((n-4)/6m^2 n\big)K^2 T^{-4} - 2((n+2)/3)\ell_0 T^{-2(n+2)}}{((n+2)/3)\left((K^2/2m^2 n)T^{-4} + \ell_0 T^{-2(n+2)}\right)}\right], \tag{43}$$

$$A_m = 2\left(\frac{m-1}{2m+1}\right)^2. \tag{44}$$

From eq. (43), we observe that

$$q < 0, \qquad \text{if} \quad T > \left[\frac{2m}{K}\sqrt{\frac{n\ell_0(n+2)}{n-4}}\right]^{1/n}, \tag{45}$$

and

$$q > 0, \qquad \text{if} \quad T < \left[\frac{2m}{K}\sqrt{\frac{n\ell_0(n+2)}{n-4}}\right]^{1/n}. \tag{46}$$

To indicate whether a model inflates or not one can see the sign of q. A negative sign $-1 \le q < 0$, indicates inflations whereas a positive sign corresponds to the standard decelerating model. Recent observations [51,52] show that the deceleration parameter of the Universe is in the range $-1 \le q < 0$ and the present day Universe is undergoing an accelerated expansion. From figure 2 we observe that the model (35) successfully describes the expansion of our Universe from a decelerating phase to an accelerating one.

In this case, from eqs (35) and (36), we obtain

$$\frac{\rho_{\mathrm{p}}}{|\lambda|} = 2 > 1. \tag{47}$$

Thus, in our model, the Universe is dominated by massive strings throughout the whole process of evolution [1,53].

3.1.2 *Solution in the absence of electromagnetic field.* In the absence of magnetic field, i.e. when $K \to 0$, eq. (32) has the following exact solution:

$$B^2 = (n+2)^{2/(n+2)}\left(\sqrt{\ell_0}t + c_0\right)^{2/(n+2)}. \tag{48}$$

Therefore

$$C^2 = m^2(n+2)^{2/(n+2)}\left(\sqrt{\ell_0}t + c_0\right)^{2/(n+2)}, \tag{49}$$

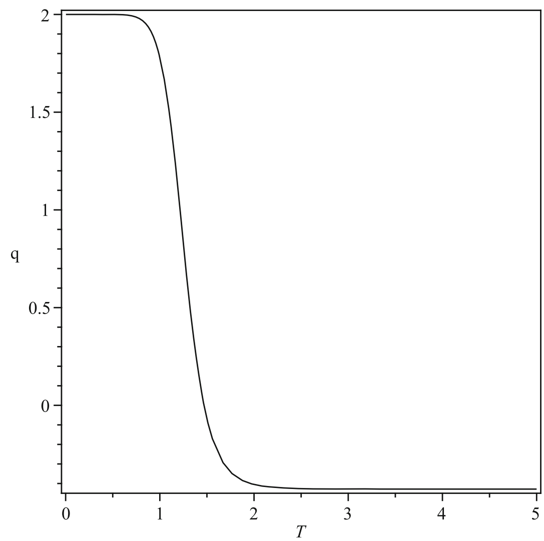

Figure 2. The plot of deceleration parameter q vs. T for $n = 5$ and $K = m = \ell_0 = \beta = 1$.

and

$$A^2 = \beta^2(n+2)^{2n/(n+2)}\left(\sqrt{\ell_0}t + c_0\right)^{2n/(n+2)}, \tag{50}$$

where c_0 is an integration constant.

Thus the metric (1) reduces to

$$\begin{aligned} ds^2 =& -dt^2 + \beta^2(n+2)^{2n/(n+2)}\left(\sqrt{\ell_0}t + c_0\right)^{2n/(n+2)}dx^2 \\ &+ (n+2)^{2/(n+2)}\left(\sqrt{\ell_0}t + c_0\right)^{2/(n+2)}e^{2x}dy^2 \\ &+ m^2(n+2)^{2/(n+2)}\left(\sqrt{\ell_0}t + c_0\right)^{2/(n+2)}e^{-2x}dz^2. \end{aligned} \tag{51}$$

Now, by using a suitable transformation of coordinates, i.e.

$$\begin{aligned} &\sqrt{\ell_0}t + c_0 = T, \quad \beta(n+2)^{n/(n+2)}x = X, \\ &(n+2)^{1/(n+2)}y = Y, \quad m(n+2)^{1/(n+2)}z = Z, \end{aligned} \tag{52}$$

the model (51) takes the form

$$ds^2 = -\frac{dT^2}{\ell_0} + T^{2n/(n+2)}dX^2 + T^{2/(n+2)}e^{2X}dY^2 + T^{2/(n+2)}e^{-2X}dZ^2. \tag{53}$$

The energy density (ρ), the string tension (λ) and the particle density (ρ_p) for the model (53) are given by

$$\rho = \left(\frac{\sqrt{(2n+1)\ell_0}}{n+2}\right)^2 \frac{1}{T^2} - \frac{L^{-2n}}{\beta^2} T^{-2n/(n+2)}, \tag{54}$$

$$\lambda = -\left(\frac{\sqrt{(2n+1)\ell_0}}{n+2}\right)^2 \frac{1}{T^2} + \frac{L^{-2n}}{\beta^2} T^{-2n/(n+2)}, \tag{55}$$

$$\rho_p = 2\left(\frac{\sqrt{(2n+1)\ell_0}}{n+2}\right)^2 \frac{1}{T^2} - 2\frac{L^{-2n}}{\beta^2} T^{-2n/(n+2)}, \tag{56}$$

where $L = (n+2)^{1/(n+2)}$. From (54) and (56), we see that energy conditions, $\rho \geq 0$ and $\rho_p \geq 0$, are satisfied under the conditions

$$T \leq \left[\frac{\beta\sqrt{(2n+1)\ell_0}L^n}{(n+2)}\right]^{(n+2)/2}. \tag{57}$$

From eqs (54) and (56) we observe that the energy density $\rho(t)$ and the particle density $\rho_p(t)$ are decreasing functions of time whereas the tension density λ is an increasing function of time and always negative. The behaviours of these physical quantities are depicted in figure 3.

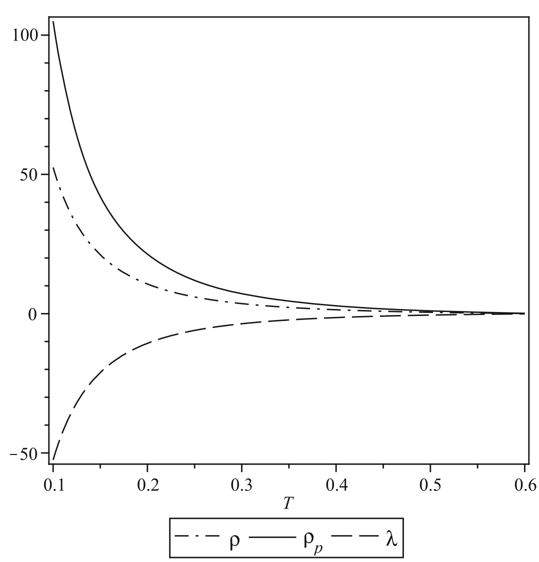

Figure 3. The plot of energy density ρ, particle density ρ_p and tension density λ vs. T for $n = 2$, $\ell_0 = 32$ and $L = \beta = 1$.

The expressions for the scalar of expansion θ, magnitude of shear σ^2, proper volume V, deceleration parameter q and the average anisotropy parameter A_m for the model (35) are given by

$$\theta = 3H = \sqrt{\ell_0}\frac{1}{T}, \tag{58}$$

$$\sigma^2 = \left(\frac{n-1}{n+2}\right)^2 \frac{\ell_0}{3}\frac{1}{T^2}, \tag{59}$$

$$V = m\beta L^{n+2} T^2, \tag{60}$$

$$q = \frac{1}{2}. \tag{61}$$

From (61), it is interesting to note that in the absence of electromagnetic field, the expansion of the Universe is always decelerating which is a contradictory result and hence we conclude that electromagnetic field plays an important role on the evolution of the Universe.

3.2 *Case II: Nambu string*

In this case we assume

$$\rho - \lambda = 0. \tag{62}$$

This corresponds to the state equation for a cloud of massless geometric (Nambu) strings, i.e. $\rho_{\mathrm{p}} = 0$. From eqs (26), (27) and (62) we obtain

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{2}{A^2} = 0. \tag{63}$$

From eqs (22) and (25), eq. (63) can be written as

$$2\ddot{B} - 2n\frac{\dot{B}^2}{B} = -\frac{2}{\beta^2}B^{1-2n}. \tag{64}$$

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = ff'$, where $f' = \mathrm{d}f/\mathrm{d}B$. Hence eq. (64) takes the form

$$\frac{\mathrm{d}}{\mathrm{d}B}(f^2) - \frac{2n}{B}f^2 = -\frac{2}{\beta^2}B^{1-2n}, \tag{65}$$

which by integrating reduces to

$$\mathrm{d}t = \frac{\mathrm{d}B}{\sqrt{\left(B^{2(1-n)}/\beta^2(2n-1)\right) + \ell B^{2n}}}, \tag{66}$$

where ℓ is an integration constant. Therefore, the metric (1) reduces to

$$\mathrm{d}s^2 = -\frac{\mathrm{d}B^2}{(B^{2(1-n)}/\beta^2(2n-1)) + \ell B^{2n}} + \beta^2 B^{2n}\mathrm{d}x^2 + B^2\mathrm{e}^{2x}\mathrm{d}y^2 + m^2B^2\mathrm{e}^{-2x}\mathrm{d}z^2, \tag{67}$$

which after suitable transformation of coordinates takes the form

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{(T^{2(1-n)}/\beta^2(2n-1)) + \ell T^{2n}} + \beta^2 T^{2n}\mathrm{d}x^2 + T^2\mathrm{e}^{2x}\mathrm{d}y^2 + m^2T^2\mathrm{e}^{-2x}\mathrm{d}z^2. \tag{68}$$

3.2.1 *The geometric and physical significance of the model.* The energy density (ρ), the string tension (λ) and the particle density (ρ_{p}) for the model (35) are given by

$$\rho = \lambda = \left[\frac{2T^{-2n}}{\beta^2(2n-1)} + (2n-1)\ell T^{2(n-1)}\right] - \frac{K^2}{2m^2}T^{-4}, \tag{69}$$

$$\rho_{\mathrm{p}} = 0. \tag{70}$$

From eq. (69) we see that the energy condition, $\rho \geq 0$, is satisfied under the condition

$$T^{-2(n-2)}\left[\frac{2}{\beta^2(2n-1)} + (2n+1)\ell T^{2(2n-1)}\right] \geq \frac{K^2}{2m^2}. \tag{71}$$

From eq. (69) we observe that the energy density is a decreasing function of time and it is always positive under the condition given by eq. (71). This behaviour of $\rho(t)$ can be seen in figure 4.

The expressions for the scalar of expansion θ, magnitude of shear σ^2, proper volume V, deceleration parameter q and the average anisotropy parameter A_m for the model (35) are given by

$$\theta = 3H = (n+2)\left[\frac{T^{-2n}}{\beta^2(2n-1)} + \ell T^{2(n-1)}\right]^{1/2}, \tag{72}$$

$$\sigma^2 = \frac{(n-1)^2}{3}\left[\frac{T^{-2n}}{\beta^2(2n-1)} + \ell T^{2(n-1)}\right], \tag{73}$$

$$V = m\beta T^{n+2}, \tag{74}$$

$$q = -\frac{(2/3\beta^2)\,((1-n)/(2n-1))\,T^{-2n} + [(4n-1)/3]\ell T^{2(n-1)}}{((n+2)/3)\left[[1/(\beta^2(2n-1)T^{-2n})] + \ell T^{2(n-1)}\right]}. \tag{75}$$

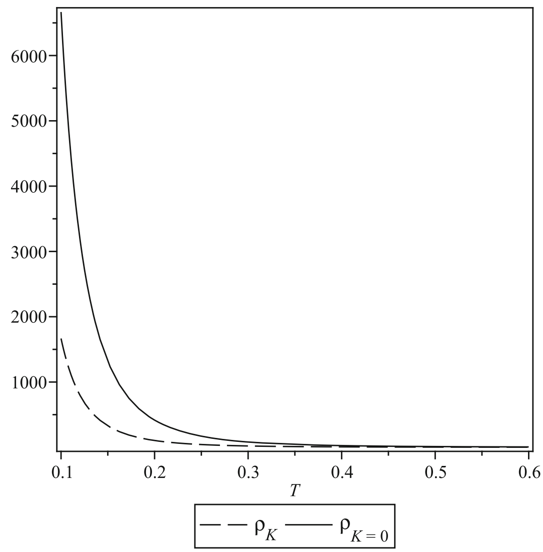

Figure 4. The plot of energy density ρ in the presence of magnetic field (dashed line) and without magnetic field (solid line) vs. T for $n = 2$, $K = \beta = m = \ell = 1$.

From eq. (75), we observe that

$$q < 0, \qquad \text{if} \quad T > \left[\frac{2}{\beta^2\ell\left((n-1)/(8n^2-6n+1)\right)}\right]^{1/(4n-1)} \tag{76}$$

and

$$q > 0, \qquad \text{if} \quad T < \left[\frac{2}{\beta^2\ell\left((n-1)/(8n^2-6n+1)\right)}\right]^{1/(4n-1)}. \tag{77}$$

From eq. (75), we observe that $q < 0$ for the condition given by (76) and $q > 0$ for the condition given by (77). It is observed that our model is evolving from a deceleration phase to an acceleration phase. Figure 5 plots the deceleration parameter (q) vs. time which gives the behaviour of q from decelerating to accelerating phase.

Since in this case the metric components are not dependent on K, the electromagnetic field, in the absence of magnetic field the model (68) remains the same as well as all geometrical properties of the model. But, of course, the physical properties of the model will change in the absence of magnetic field as follows:

$$\rho = \lambda = \left[\frac{2T^{-2n}}{\beta^2(2n-1)} + (2n-1)\ell T^{2(n-1)}\right], \tag{78}$$

$$\rho_{\mathrm{p}} = 0. \tag{79}$$

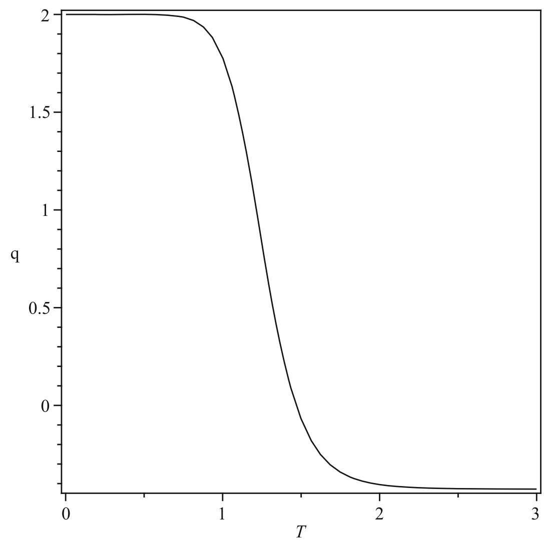

Figure 5. The plot of deceleration parameter q vs. T for $n = 2$ and $\beta = 1,\ \ell = 0.5$.

From eq. (78) we observe that ρ is a decreasing function of time and it approaches a small positive value at a later time. This behaviour of the energy density is clearly depicted in figure 4 (solid line). Also the energy condition, $\rho \geq 0$, is satisfied under the condition

$$T \geq \left[-\frac{2}{\beta^2(2n-1)^2}\right]^{1/(2(2n-1))}. \tag{80}$$

It is worth mentioning that the energy density ρ decreases faster, with time, due to the presence of electromagnetic field.

In this case, from eqs (78) and (79), we obtain

$$\frac{\rho_{\mathrm{p}}}{|\lambda|} = 0. \tag{81}$$

Hence, in this case the strings dominate over the particles.

4. Conclusion

In the present study, we have investigated the effect of electromagnetic filed in the string Bianchi type-VI$_0$ Universe. Einstein's field equations have been solved exactly with suitable physical assumptions for two types of geometric strings: (i) massive strings and (ii) Nambu strings. It is found that when the Universe is dominated by massive strings, the existence of electromagnetic field is necessary as it accelerates the expansion of the Universe. But when our Universe is dominated by Nambu strings, the electromagnetic field does not have significant effect on the evolution of the Universe. Therefore, we concluded that early massive string-dominated Universe might have converted to Nambu string-dominated Universe at a later time. There is a point type singularity in our models at $T = 0$ [54]. Since $(\sigma/\theta) =$ constant, the models do not approach isotropy except for $n = 1$. It is observed that our models (except the model of massive string without electromagnetic field) are evolving from a deceleration phase to an acceleration phase. Also, recent observations of SNe Ia expose that the present Universe is accelerating and the value of deceleration parameter lies in the range $-1 < q < 0$. It follows that in our derived models, one can choose the value of deceleration parameter consistent with the observation. Thus the derived models are realistic.

We also note that the horizon distance in both massive and Nambu string-dominated Universe is obtained as

$$d_{\mathrm{H}}(\tau) = a(\tau)\int_0^{\tau'} \frac{\mathrm{d}\tau'}{a(\tau')}, \tag{82}$$

where $\tau = (m\beta)^{1/(n+2)}T$ and $a(\tau) = \tau^{(n+2)/3}$. Therefore, from eq. (82) we get

$$d_{\mathrm{H}} = \left(\frac{3}{n+5}\right)\tau^{(2(n+2)+3)/3}. \tag{83}$$

From eq. (83) we obtain

$$d_{\mathrm{H}}(\tau) > a(\tau), \qquad \text{for} \quad \tau > \left(\frac{n+5}{3}\right)^{3/(n+5)}. \tag{84}$$

From eq. (84) we observe that the horizon grows more rapidly than the scale factor implying a colder and darker Universe. It is like a flat or open Universe with a dominance of dark energy.

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