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Fission dynamics of the compound nucleus ^{213}Fr formed in heavy-ion-induced reactions

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Abstract. A stochastic approach based on one-dimensional Langevin equations was used to calculate the average pre-fission multiplicities of neutrons, light charged particles and the fission probabilities for the compound nucleus ^{213}Fr and the results are compared with the experimental data. In these calculations, a modified wall and window dissipation with a reduction coefficient, k_s, has been used in the Langevin equations. It was shown that the results of the calculations are in good agreement with the experimental data by using values of k_s in the range $0.3 \leq k_s \leq 0.5$.

Keywords. Fission; fission probability; pre-scission particle multiplicity.

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1. Introduction

The problem of the origin and nature of nuclear dissipation is one of the most interesting questions in nuclear physics at low and intermediate excitation energies. At present, there are several models for dissipation but they give dependences which are very different from each other. For example, linear response theory [1,2] predicts that dissipation increases with temperature, whereas the two-body dissipation model [3] predicts the decrease of dissipation with temperature as T^{-2}. On the other hand, there are certain indications that the nuclear dissipation is deformation-dependent. In the paper [4] authors made a detailed study of the fission dynamics and pre-scission particle emission using Langevin equations. A comparison of the calculated pre-scission neutron multiplicity and fission probability with the experimental data for a number of nuclei led to a phenomenological shape-dependent nuclear friction [4]. The phenomenological friction turned out to be smaller than the standard wall formula value for nuclear friction up to the saddle point, and it would sharply increase between saddle and scission points. Further, Nix and Sierk suggested [5,6] in their analysis of mean fragment kinetic energy data that the dissipation is about four times weaker than that predicted by the wall plus window formula of one-body dissipation.

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The wall formula for nuclear dissipation was developed by Blocki *et al* [7] in a simple classical picture of one-body dissipation. One crucial assumption of the wall formula concerns the randomization of the nucleon motion due to the successive collisions it suffers at the nuclear surface. The derivation of the wall formula assumes that the nucleon motion due to the successive collisions is fully randomized. It was earlier understood that any deviation from this full randomization assumption would give rise to a reduction in the strength of the wall formula friction [7,8]. Furthermore, many authors, while analysing different aspects of the nuclear fission, assumed a constant nuclear dissipation [9–12].

In this paper, we use a modified wall and window dissipation with a reduction coefficient [6,13] in one-dimensional Langevin equations to simulate the dynamics of the nuclear fission of ^{213}Fr formed in ^{16}O+^{197}Au reactions and reproduce experimental data on the average pre-fission multiplicities of neutrons, light charged particles and the fission probabilities. It should be stressed that in our calculation we want to consider the magnitude of the reduction coefficient as a free parameter.

The present paper has been arranged as follows. In §2 the model and basic equations are described. The results of the calculations are presented in §3. Finally concluding remarks are given in §4.

2. Description of the model and basic equations

In the present study, a stochastic approach based on one-dimensional Langevin equations is used to describe the fission dynamics of ^{213}Fr. In the stochastic approach the evolution of the collective coordinates can be considered as the motion of Brownian particle in a viscous heat bath [14,15]. The heat bath in this picture represents the rest of all other nuclear degrees of freedom which are assumed to be in thermal equilibrium. In order to specify the collective coordinates for a dynamical description of nuclear fission, we use the ‘funny hills’ shape parameters $\{c, h, \alpha\}$ as suggested by Brack *et al* [16]. However, we simplify the calculation by considering only symmetric fission ($\alpha = 0$) and further neglecting the neck degree of freedom ($h = 0$). Consequently, in terms of the one-dimensional potential, $V(c)$, the coupled Langevin equations in one dimension take the form [17]

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\frac{p^2}{2}\frac{\partial}{\partial c}\left(\frac{1}{m}\right) - \frac{\partial F}{\partial c} - \eta\dot{c} + R(t),$$
$$\frac{\mathrm{d}c}{\mathrm{d}t} = \frac{p}{m(c)}, \tag{1}$$

where $R(t)$ is a random force with the properties $\langle R(t)\rangle = 0$, $\langle R(t)R(t')\rangle = 2\eta T\delta(t-t')$ and F is the free energy of the system. In the Fermi gas model, F is related to the level density parameter

$$F(c, T) = V(c) - a(c)T^2, \tag{2}$$

where T is the temperature of the system. The coordinate-dependent level density parameter is of the form

$$a(c) = a_v A + a_s A^{2/3} B_s(c), \tag{3}$$

where A is the mass number of the compound nucleus and B_s is the dimensionless functional of the surface energy in the liquid drop model. The values of the parameters $a_v = 0.073$ MeV^{-1} and $a_s = 0.095$ MeV^{-1} in eq. (3) are taken from the work of Ignatyuk *et al* [18].

The Langevin trajectories are simulated starting from the ground state of the compound nucleus with the excitation energy $E*$. The initial conditions for Langevin equations can be chosen by the Neumann method with the generating function

$$\Phi\left(c_0, p_0, l_0, t=0\right) \propto \exp\left[-\frac{V(c_0)+E_{\mathrm{coll}}(c_0, p_0)}{T}\right]\delta(c_0 - c_{gs})\frac{\mathrm{d}\sigma(l)}{\mathrm{d}l}. \quad (4)$$

The initial state is assumed to be characterized by the thermal equilibrium momentum distribution and by the spin distribution of the compound nuclei $\mathrm{d}\sigma(l)/\mathrm{d}l$ according to scaled prescription [19], which reproduces to a certain extent the dynamical results of the surface friction model [20] for the fusion of two heavy ions.

The collective inertia, m, is calculated in the frame of the Werner–Wheeler approach and the nuclear temperature is defined as $T = \sqrt{E_{\mathrm{int}}/a(c)}$ with

$$E_{\mathrm{int}} = E^* - p^2/(2m) - V(c) - E_{\mathrm{rot}} - E_{\mathrm{evap}}(t), \quad (5)$$

where E_{rot} and E_{evap} are the rotational energy and the nucleus excitation energy that light particles have carried away by the instant t, respectively. The potential energy $V(c)$ is obtained from the liquid drop model [21].

As was shown in refs [6,13] the modified wall and window dissipation formula friction can be given as

$$\eta = \frac{1}{2}\rho_{\mathrm{m}}\bar{v}\left\{\left(\frac{\partial r}{\partial c}\right)^2\Delta\sigma + k_s\pi\left[\int_{z_{\min}}^{z_N}\left(\frac{\partial\rho^2}{\partial c}+\frac{\partial\rho^2}{\partial z}\frac{\partial D_1}{\partial c}\right)^2\left(\rho^2+\left(\frac{1}{2}\frac{\partial\rho^2}{\partial z}\right)^2\right)^{-1/2}\mathrm{d}z + \int_{z_N}^{z_{\max}}\left(\frac{\partial\rho^2}{\partial c}+\frac{\partial\rho^2}{\partial z}\frac{\partial D_2}{\partial c}\right)^2\left(\rho^2+\left(\frac{1}{2}\frac{\partial\rho^2}{\partial z}\right)^2\right)^{-1/2}\mathrm{d}z\right]\right\}, \quad (6)$$

where ρ_{m} is the mass density of the nucleus, $\bar{v}$ is the average nucleon speed inside the nucleus, r is the distance between the centres of masses of the future fission fragments, $\Delta\sigma$ is the area of the window between the two parts of the system, ρ^2 is the surface of the nucleus, D_1, D_2 are the positions of the centres of mass of the two parts of the fissioning system relative to the centre of mass of the whole system, $z_{\min}$ and $z_{\max}$ are the two extreme ends of the nuclear shape along the z-axis and z_N is the position of the neck plane.

The surface of a nucleus of mass number A with elongation c can be defined as

$$\rho^2(z) = \left(1-\frac{z^2}{c^2}\right)\left(Ac^2 + Bz^2 + \alpha zc\right), \quad (7)$$

where the coefficients A and B are expressed as

$$A = \frac{1}{c^3} - \frac{B}{5}$$
$$B = \frac{c-1}{2}. \quad (8)$$

The quantity α is a parameter which depends upon the asymmetry parameter (α_{asy}). Asymmetry parameter is defined as

$$\alpha_{\text{asy}} = \frac{(A_1 - A_2)}{A_{\text{CN}}}, \quad (9)$$

where A_{CN} is the mass of the compound nucleus and A_1, A_2 are the masses of the right and left lobes, respectively. The quantity α and the value of c at which scission occurs are denoted as follows [22]:

$$\alpha = 0.11937\alpha_{\text{asy}}^2 + 0.24720\alpha_{\text{asy}}$$
$$c_{\text{sc}} = -2.0\alpha^2 + 0.032\alpha + 2.0917. \quad (10)$$

The decay widths for n, p, α, γ emissions are calculated at each Langevin time step Δt. The emission of a particle is allowed by asking at each time step along the trajectory whether the ratio of the Langevin time step Δt to the particle decay time τ_{part} is larger than a random number ξ

$$\Delta t/\tau_{\text{part}} > \xi, \quad 0 \le \xi \le 1, \quad (11)$$

where

$$\tau_{\text{part}} = \hbar/\Gamma_{\text{tot}} \quad \text{and} \quad \Gamma_{\text{tot}} = \sum_{\nu} \Gamma_{\nu}.$$

The probabilities of decay via different channels can be calculated by using a standard Monte Carlo cascade procedure where the kind of decay selected with the weights $\Gamma_{\nu}/\Gamma_{\text{tot}}$ with $\nu = n, p, \alpha, \gamma$. After the emission particle of type ν, the kinetic energy ε_{ν} of the emitted particle is calculated by hit and miss Monte Carlo procedure. Then the intrinsic excitation energy of the residual mass and spin of the compound nucleus are recalculated and the dynamics is continued. The loss of angular momentum is taken into account by assuming that each neutron, proton, or a γ quanta carries away $1\hbar$ angular momentum while the α particle carries away $2\hbar$ angular momentum.

Figure 1 shows several typical Langevin trajectories reaching the scission point calculated by Langevin equations. The particle emission width of a particle of type ν is given by ref. [23]

$$\Gamma_{\nu} = (2s_{\nu} + 1)\frac{m_{\nu}}{\pi^2\hbar^2\rho_{\text{c}}(E_{\text{int}})}\int_0^{E_{\text{int}}-B_{\nu}} \text{d}\varepsilon_{\nu}\rho_{\text{R}}(E_{\text{int}} - \varepsilon_{\nu})\varepsilon_{\nu}\sigma_{\text{inv}}(\varepsilon_{\nu}), \quad (12)$$

where s_{ν} is the spin of the emitted particle ν, m_{ν} is its reduced mass with respect to the residual nucleus. $\rho_{\text{c}}(E_{\text{int}})$ and $\rho_{\text{R}}(E_{\text{int}} - \varepsilon_{\nu})$ are the level densities of the compound and residual nuclei.

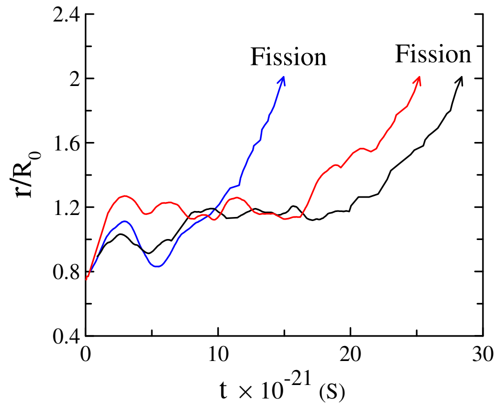

Figure 1. Typical Langevin trajectories reach the scission point. R_0 is the radius of the spherical nucleus.

The variable ε_ν is the kinetic energy of the evaporated particle ν. The intrinsic energy and the separation energy of the particle ν are denoted by $E_{\rm int}$ and B_ν.

The inverse cross-sections can be written as [23]

$$\sigma_{\rm inv}(\varepsilon_\nu) = \begin{cases} \pi R_\nu^2 \left(1 - V_\nu/\varepsilon_\nu\right), & \text{for} \quad \varepsilon_\nu > V_\nu \\ 0, & \text{for} \quad \varepsilon_\nu < V_\nu \end{cases}, \tag{13}$$

with

$$R_\nu = 1.21\left[(A - A_\nu)^{1/3} + A_\nu^{1/3}\right] + (3.4/\varepsilon_\nu^{1/2})\delta_{\nu,n}, \tag{14}$$

where A_ν is the mass number of the emitted particle $\nu = n, p, \alpha$. The barriers for the charged particles are

$$V_\nu = \frac{[(Z - Z_\nu)Z_\nu K_\nu]}{(R_\nu + 1.6)}, \tag{15}$$

with $K_\nu = 1.32$ for α and 1.15 for proton.

The width of the gamma emission is calculated by the following formula [24]:

$$\Gamma_\gamma \cong \frac{3}{\rho_{\rm c}(E_{\rm int})} \int_0^{E_{\rm int}} {\rm d}\varepsilon\, \rho_{\rm c}(E_{\rm int} - \varepsilon) f(\varepsilon), \tag{16}$$

where ε is the energy of the emitted γ quanta and $f(\varepsilon)$ is defined by

$$f(\varepsilon) = \frac{4}{3\pi}\frac{e^2}{\hbar c}\frac{1+k}{m_n c^2}\frac{NZ}{A}\frac{\Gamma_{\rm G}\varepsilon^4}{(\Gamma_{\rm G}\varepsilon)^2 + (\varepsilon^2 - E_{\rm G}^2)^2}, \tag{17}$$

with $E_{\rm G} = 80\ A^{-1/3}$, $\Gamma_{\rm G} = 5$ MeV and $k = 0.75$ [25], $E_{\rm G}$ and $\Gamma_{\rm G}$ are the position and width of the giant dipole resonance, respectively.

In calculations, a Langevin trajectory either reaches the scission point or counts as an evaporation residue event if the intrinsic excitation energy becomes smaller than either the fission barrier or the binding energy of a neutron.

If the Langevin trajectory has not fissioned and has not been counted as an evaporation residue event after a delay time, when stationary flux over the saddle point is reached, we stop the dynamical calculation and switch over to the statistical description with a Kramers-type fission decay [26].

3. Results of the calculations and discussion

In this paper we carried out calculations of the average pre-fission multiplicities of neutrons, light charged particles and fission probabilities for ^{213}Fr formed in ^{16}O+^{197}Au reactions.

Figures 2, 3 and 4 show the results of the average pre-fission multiplicities of neutrons and light charged particles for ^{213}Fr.

It can be seen from figure 2 that at lower excitation energies the values of the average pre-fission multiplicities of neutrons calculated with different values of reduction coefficient are very close together and also to the experimental data, but at higher excitation energies the experimental data can be reproduced by considering values of k_s in the range $0.3 \leq k_s \leq 0.5$. It can be explained as follows: at lower excitation energy a compound nucleus is formed with a lower value of spin and then the height of the fission barrier is large (see figure 5), and so the neutron widths are much larger than the fission width. Consequently, if we use different values of reduction coefficient, the neutrons have enough time to be emitted before fission. On the other hand, at higher excitation energy, a compound nucleus is formed with a larger value of spin. Thus the fission barrier height will be reduced and therefore the neutron widths are comparable to the fission width.

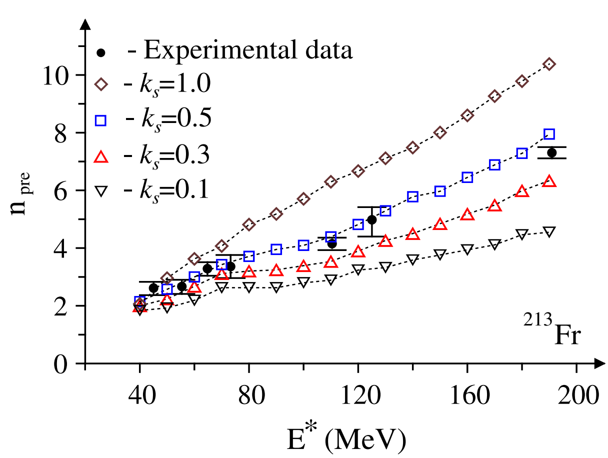

Figure 2. Pre-scission neutron multiplicity as a function of excitation energy for ^{213}Fr. The calculated values are connected by dotted lines to guide the eye. The experimental data (filled circles) are taken from refs [27–29].

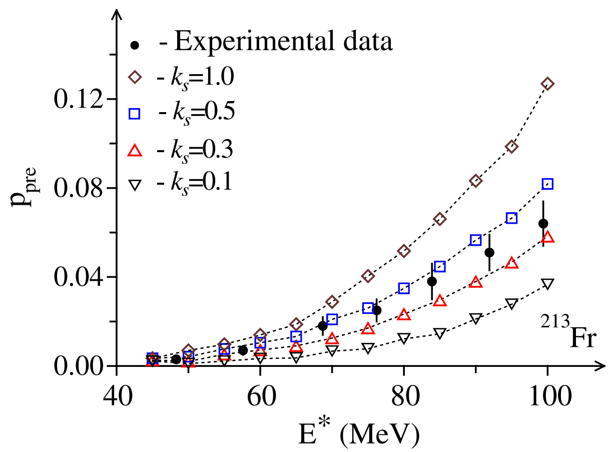

Figure 3. Pre-scission proton multiplicity as a function of excitation energy for ^{213}Fr. The calculated values are connected by dotted lines to guide the eye. The experimental data (filled circles) are taken from ref. [30].

Consequently, in these cases the value of reduction coefficient is a very important parameter to reproduce pre-scission neutron multiplicities.

Similar arguments can be considered for interpreting figures 3 and 4.

The calculated and experimental values of the fission probability are shown in figure 6 for ^{213}Fr.

It can be seen from figure 6 that at higher excitation energies the fission probability reaches a stationary value. This is because with increasing excitation energy, pre-fission multiplicities of neutrons and light charged particles increase and emission of each light particle carries away angular momentum and excitation energy. Therefore fission barrier

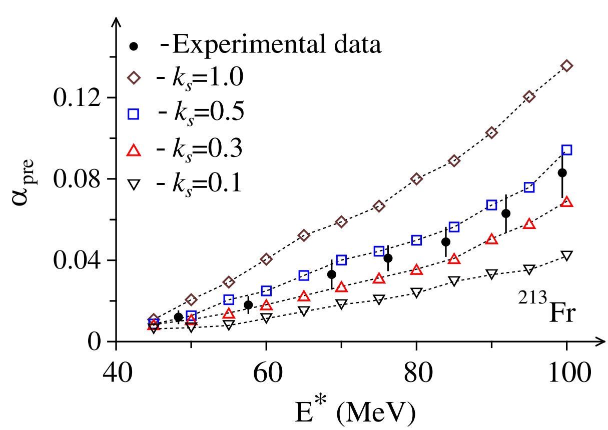

Figure 4. Pre-scission alpha multiplicity as a function of excitation energy for ^{213}Fr. The calculated values are connected by dotted lines to guide the eye. The experimental data (filled circles) are taken from ref. [30].

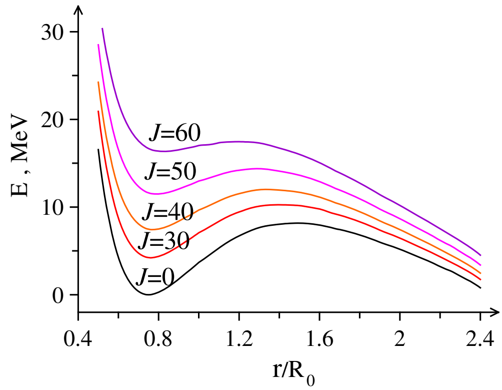

Figure 5. Potential energy surfaces at $J = 0, 30, 40, 50, 60\hbar$. R_0 is the radius of the spherical nucleus.

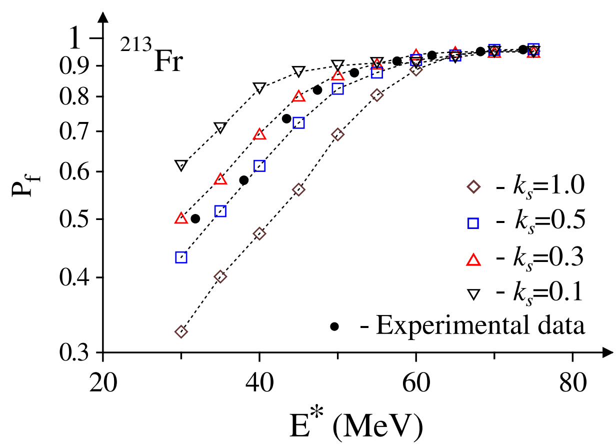

Figure 6. Fission probability calculated with different values of k_s. The calculated values are connected by dotted lines to guide the eye. The experimental data (filled circles) are taken from ref. [31].

height of the residual nucleus increases and consequently the fission event will be less and less probable.

Moreover, it can be seen that at higher excitation energies the fission probability calculated with different values of k_s are very close and in agreement with the experimental data but at lower excitation energies the experimental data can be reproduced by considering k_s in the range $0.3 \leq k_s \leq 0.5$. It can be explained as follows: at higher excitation energy a compound nucleus will be formed with a larger value of spin and then the fission barrier height and the fission time are decreased. Consequently, the value of k_s is not very important for calculating fission probability.

It should be stressed that Chaudhuri and Pal [32] used both the usual wall formula friction and its chaos-weighted version in the Langevin equations to calculate the fission probability and pre-scission neutron multiplicity for ^{213}Fr and some other compound nuclei.

4. Conclusions

A stochastic approach to fission dynamics based on one-dimensional Langevin equations was applied to calculate the experimental data of the pre-scission particle multiplicities and fission probability for the compound nucleus ^{213}Fr formed in ^{16}O+^{197}Au reactions.

We used a modified wall and window dissipation with a reduction coefficient in the Langevin equations and assumed the magnitude of reduction coefficient as a free parameter. It was shown that the fission probability and pre-scission particle multiplicities can be reproduced with k_s in the range $0.3 \leq k_s \leq 0.5$ for the compound nucleus ^{213}Fr. It should be stressed that our result for k_s is consistent with other researches [33,34]. Authors in refs [33,34] have performed a systematic study of many different systems and showed that for reproducing the measured neutron multiplicities, the variance of the fission fragment mass–energy distribution, dependence of the pre-scission neutron multiplicity on the fragment mass asymmetry and total kinetic energy, the reduced coefficient of the contribution from a wall formula has to be decreased at least by half of the one-body dissipation strength ($0.25 \leq k_s \leq 0.5$).

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