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Energy in the Kantowski–Sachs space-time using teleparallel geometry

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Abstract. The purpose of this paper is to examine the energy content of the inflationary Universe described by Kantowski–Sachs space-time in quasilocal approach of teleparallel gravity and in the Hamiltonian structure of the teleparallel equivalent of general relativity. The teleparallel versions of field equations are also derived in such a space-time.

Keywords. Kantowski–Sachs metric; teleparallelism; gravitational energy.

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1. Introduction

Since the birth of general relativity, considered as a superb theory of space-time and gravitation, gravity is taken to be a manifestation of space-time geometry. The gravitational field of a massive body curves the space-time around it. The trajectories of particles in its gravitational field are described by the geodesics of curved space-time, not by force equation. Geometry here replaces the concept of force. But general relativity lacks the definition of energy and momentum. It is generally believed that the energy–momentum of a gravitational field cannot be localized because according to Einstein's equivalence principle, gravity cannot be detected at a point. This means energy–momentum is fundamentally non-local and hence non-tensorial [1]. If we take a finite region of space-time, localization of energy and momentum becomes a controversial problem. Different researchers show that they have adopted different types of gravitational field pseudotensors which depend on the reference frames. After Einstein proposed his gravitational energy–momentum complex [2], several other solutions were prescribed by different researchers [2–6]. Some of these were covariant, some were not. The works of Landau and Lifshitz [7] present energy–momentum pseudotensor dependent on the second derivative of the metric tensor. From these results obtained with Einstein's principle of equivalence, in the whole space-time of Minkowski, we can find a small region of space-time where the gravitational field is null. Problems naturally arise to define the energy of a gravitational field in the whole of the space-time region as well as in a small region. Consequently, pseudotensor approach has been largely questioned but not abandoned. The recent attempt to solve this problem is to use the so-called 'teleparallel equivalent of general relativity' (TEGR) where gravity is termed as a force like other fundamental forces of nature. The teleparallel theory has been considered long time ago with attempts to define the energy of gravitational field. The teleparallel gravity can be understood as a gauge theory of translational group. This theory predicts new alternative models to GR made out of torsion tensor. Torsion is here the field strength associated with the translation. Gravitational potential is a gauge potential. Thus, in this approach the gravitational interaction is described by a force similar to the Lorentz force in electrodynamics. It has been argued by some researchers that the energy–momentum can be localized in the teleparallel theory of gravity. Unfortunately, the localization of energy and momentum in this theory is also an open, unsolved disputed problem [8]. It is observed that the well-defined energy–momentum and angular momentum density at every point in a space-time cannot be associated with gravitational field. If we want to talk about gravitational energy and angular momentum then these quantities must be assigned to extended but finite space-time domains. Thus, since the gravitational interaction is local, some kinds of local – at least nearly local – description of these quantities are sought. These are observables associated with open subsets of space-time whose closure is compact, i.e. they are quasilocal (Szabados [9]). Many different definitions of this alternative concept of energy called quasilocal mass energy have been proposed by Penrose and many others. Their definitions are conceptually very important, yet these definitions have serious problems. The existence of total zero energy density has been established by different researchers for different Universes using Einstein energy–momentum complex, symmetric pseudotensor Landau–Lifshitz complexes etc. Recently, Cooperstock and Faraoni [9], Vargas [9], Salti *et al* [9] also led to the same conclusion.

Our present Universe appears to confirm to Robertson–Walker (RW) cosmology. However, the cosmology of the early Universe remains an open question. It might be that some other cosmological model developed in the early era and later changed over to RW model. In this context, one may think of some other model, for example, the Kantowski–Sachs model, as a possible candidate of the early Universe. Inflation is again considered to be one of the early phases of the Universe [10]. For testing gravitational energy–momentum tensor of the teleparallel theory in quasilocal approach and also in the Hamiltonian structure of TEGR, it is intended to calculate the total energy of inflationary Kantowski–Sachs-type Universe. Again, the field equations in TEGR are derived in such an inflationary early Universe. In literature no such work is still reported as far as the knowledge of the author goes.

In a recent paper, Gad and Fouad [11] and Megied and Gad [8] have found the energy and momentum distribution of Kantowski–Sachs space-time using different energy-complexes. Recently, Sousa *et al* [12] have shown the equivalence between GR and TEGR in FLRW Universe.

In the present work, we examine the energy for a Kantowski–Sachs Universe using quasilocal approach in the context of the general tetrad-teleparallel theory of gravity (Hayashi and Shirafuji [9]) and in Hamiltonian formulation implemented by Maluf *et al* [13]. In quasilocal approach, all these quantities vanish for our inflationary Universe in the context of teleparallel gravity independently of the three coupling constants. But, in Hamiltonian formalism, these quantities remain finite for a small region but become infinite for a large Universe. The equivalence between GR and TEGR by obtaining the teleparallel version of field equations is also verified. Lastly, we present our conclusion.

In §2, after presenting the fundamentals of teleparallel gravity in §2.1, we choose the tetrad fields in §2.2. Then the non-zero components of Weitzenböck connection, the torsion tensor are evaluated. Next, we calculate the total energy in our inflationary Universe defined by Kantowski–Sachs space-time in quasilocal approach.

In §3.1, we derive the field equations of our Universe using the field equations of TEGR. In §3.2, we calculate the energy of our Kantowski–Sachs Universe with the chosen set of tetrads. Finally, in §4, we present our conclusion.

2. Teleparallel theory of gravity

2.1 *The fundamentals of tetrad theory of gravitation: Teleparallel gravity*

We know that the teleparallel gravity is a theory of gravity which is based on Weitzenböck space-time. It is important to mention that Weitzenböck independently introduced a space-time that presents torsion with null curvature during the year 1923 [14]. To each point of space-time, a Minkowski tangent space is attached, on which the translational (gauge) group acts. The space-time indices are the Greek alphabets μ, ν, σ, λ, ... and the global $SO(3,1)$ indices are the Latin alphabets (a), (b), (c), ... = (0), (1), (2), (3). The Minkowski space-time has the metric $\eta_{(ab)} = (1, -1, -1, -1)$. The translational gauge potential $\wedge_\mu^{(a)}$ and the tetrad field $e_\mu^{(a)}$ are related by

$$e_\mu^{(a)} = \partial_\mu x^{(a)} + \wedge_\mu^{(a)}. \tag{1}$$

The tetrad field satisfies the orthogonality condition,

$$e_\mu^{(a)} e_{(a)}^\nu = \delta_\nu^\mu. \tag{2}$$

The basic tetrad field is related to the presence of gravitational field.

The tangent space indices are raised and lowered with the Minkowski metric coefficients $\eta_{(a)(b)}$ whereas the Riemannian space-time metric $g_{\mu\nu}$ is used to raise or lower the space-time indices μ, ν, ...

$$g_{\mu\nu} = \eta_{(a)(b)} e_\mu^{(a)}\, e_\nu^{(b)}. \tag{3}$$

The tetrad gives rise to the so-called Weitzenböck connection

$$\Gamma^\sigma_{\mu\nu} = e^\sigma_{(a)} \partial_\nu e_\mu^{(a)}. \tag{4}$$

It presents torsion but no curvature.

The non-vanishing torsion components of the Weitzenböck connection are given by

$$\mathrm{T}^\sigma_{\mu\nu} = \Gamma^\sigma_{\nu\mu} - \Gamma^\sigma_{\mu\nu}. \tag{5}$$

The action of teleparallel gravity in the presence of matter is given by [15].

$$S = \frac{1}{16\pi G} \int \mathrm{d}^4 x e S^{\lambda\tau\nu}\, T_{\lambda\tau\nu} + \int \mathrm{d}^4 x e L_{\mathrm{M}}, \tag{6}$$

where $e = \det(e_\mu^{(a)})$, L_{M} is the Lagrangian of the matter field and $S^{\lambda\tau\nu}$ is the tensor

$$S^{\lambda\tau\nu} = c_1 T^{\lambda\tau\nu} + \frac{c_2}{2}(T^{\tau\lambda\nu} - T^{\nu\lambda\tau}) + \frac{c_3}{2}(g^{\lambda\nu} T_{\sigma}^{\sigma\tau} - g^{\tau\lambda} T_{\sigma}^{\sigma\nu}). \tag{7}$$

It is stated by Maluf [15] that the teleparallel equivalent of general relativity (TEGR) is obtained with

$$c_1 = \frac{1}{4}, \quad c_2 = \frac{1}{4}, \quad c_3 = -1. \tag{8}$$

By varying action with respect to $e^{(a)}$, the teleparallel equivalent of Einstein's field equations in tetrad form is

$$\partial_\nu (e S_\lambda^{\tau\nu}) = \frac{1}{4k'} e (t_\lambda^\tau + \Theta_\lambda^\tau), \tag{9}$$

where

$$k' = \frac{1}{16\pi G} \tag{10}$$

$$t_\lambda^\tau = k'(e \Gamma^\nu_{\sigma\lambda} S_\nu^{\tau\sigma}) - \delta_\lambda^\tau L_G \tag{11}$$

is the gravitational energy–momentum pseudotensor [16] and Θ_λ^τ is the symmetric source energy–momentum tensor.

Interestingly, $S_\nu^{\tau\sigma}$ is antisymmetric in the last two indices.

∴ The conservation law can be obtained as

$$\partial_\mu [e(t_\lambda^\mu + T_\lambda^\mu)] = 0. \tag{12}$$

Again, in quasilocal approach using the Hamiltonian formalism, we get the Hamiltonian for a finite region as [17]

$$H(N) = \int_\Sigma N^\mu H_\mu \, \mathrm{d}\sigma + \oint_{\delta\Sigma} B(N). \tag{13}$$

The term $B(N)$ is known as the boundary term [15,16], Σ is a finite 3-hypersurface, N^μ is the vector field, $\partial\Sigma$ is the 2-surface boundary and H_μ is the covariant form of the Hamiltonian.

The quasilocal energy–momentum is obtained by the integral of $B(N)$ over $\partial\Sigma$ of Σ as

$$P_\nu = \frac{1}{4k'} \oint_{\partial\Sigma} e S_\nu^{\alpha\beta} (\mathrm{d}\sigma)_{\alpha\beta}. \tag{14}$$

2.2 *Quasilocal energy*

In this section, we calculate the quasilocal energy in teleparallel gravity given by [15].

The inflationary Universe described by Kantowski–Sachs space-time has the line element in spherical coordinates as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \wedge^2 \mathrm{d}r^2 - k^2\left[\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right], \tag{15}$$

where $\wedge$, k are functions of time only.

We take the non-zero vierbeins or the tetrad components as

$$e_0^{(0)} = 1, \quad e_2^{(1)} = k, \quad e_3^{(2)} = k\sin\theta, \quad e_1^{(3)} = \wedge. \tag{16}$$

Using the inverse metric tensor $g^{\mu\nu}$, we can write the inverse tetrads

$$e_{(a)}^{\mu} = g^{\mu\nu} e_{(a)\nu} \tag{17}$$

as

$$e_{(a)}^{\mu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{k} & 0 \\ 0 & 0 & 0 & \frac{1}{k\sin\theta} \\ 0 & \frac{1}{\wedge} & 0 & 0 \end{pmatrix}, \tag{18}$$

where the determinant is

$$e = \wedge k^2 \sin\theta. \tag{19}$$

The corresponding non-vanishing Weitzenböck connections are

$$\Gamma^1_{10} = \frac{\dot{\wedge}}{\wedge}, \tag{20}$$

$$\Gamma^2_{20} = \Gamma^3_{30} = \frac{\dot{k}}{k}, \tag{21}$$

where dot denotes differentiation with respect to t.

The non-vanishing torsion components are

$$T^1_{01} = -T^1_{10} = \frac{\dot{\wedge}}{\wedge}, \tag{22}$$

$$T^2_{02} = T^3_{03} = -T^2_{20} = -T^3_{30} = \frac{\dot{k}}{k} \tag{23}$$

while the other components vanish.

The non-zero components of the tensor $S_{\nu}^{\delta\tau}$ are obtained as

$$S^{110} = -S^{101} = \frac{\dot{\wedge}}{\wedge^3}\left(c_1 + \frac{c_2}{2}\right) + \frac{c_3}{2\wedge^2}\left(\frac{\dot{\wedge}}{\wedge} + \frac{2\dot{k}}{k}\right) \tag{24}$$

$$S^{220} = -S^{202} = \frac{\dot{k}}{k^3}\left(c_1 + \frac{c_2}{2}\right) + \frac{c_3}{2k^2}\left(\frac{\dot{\wedge}}{\wedge} + \frac{2\dot{k}}{k}\right) \tag{25}$$

$$S^{330} = -S^{303} = \frac{\dot{k}}{k^3 \sin^2\theta}\left(c_1 + \frac{c_2}{2}\right) + \frac{c_3}{2k^2 \sin^2\theta}\left(\frac{\dot{\wedge}}{\wedge} + \frac{2\dot{k}}{k}\right). \tag{26}$$

Here the condition

$$S_{\chi\beta\gamma} + S_{\beta\gamma\chi} + S_{\gamma\chi\beta} = 0 \tag{27}$$

is found to be fulfilled.

Interestingly there is no component like $S_0^{\alpha\beta}$, for any α, β.

$$\therefore S_0^{\alpha\beta} = 0. \tag{28}$$

The quasilocal energy within any region is therefore

$$P_0 = \frac{1}{4\kappa'} \oint_{\partial\Sigma} e S_0^{\alpha\beta} (\mathrm{d}\sigma)_{\alpha\beta} = 0. \tag{29}$$

Our calculation agrees with Megied and Gad, Banerjee and Sen [8].

3. Equivalence between GR and TEGR

3.1 *Field equations in TEGR*

In the work [12,18], it is shown that TEGR is the only viable, consistent, teleparallel gravity theory. Here also we try to establish the same. In this model, equivalence between GR and TEGR is tested. We follow the method presented by Sousa *et al* [12] for FLRW Universe.

With the same choice of tetrads in §1, for Kantowski–Sachs model which is a good candidate for our early Universe, the non-vanishing torsion components are

$$T_{(1)20} = -T_{(1)02} = \dot{k} \tag{30}$$

$$T_{(2)30} = -T_{(2)03} = \dot{k} \sin\theta \tag{31}$$

$$T_{(3)10} = -T_{(3)01} = \dot{\wedge} \tag{32}$$

$$T_{(2)32} = -T_{(2)23} = k \cos\theta \tag{33}$$

which are obtained using

$$T_{(a)\mu\nu} = \partial_\mu e_{(a)\nu} - \partial_\nu e_{(a)\mu}. \tag{34}$$

It is observed that the torsion components are antisymmetrical under the exchange of the last two indices. The energy–momentum tensor of a perfect fluid is given by

$$T^\mu_\nu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}, \tag{35}$$

where ρ is the density and p is the pressure.

The non-vanishing components of the tensor $\Sigma^{(a)(b)(c)}$ appearing in the Lagrangian density for gravitational field defined by [18]

$$\Sigma^{(a)(b)(c)} = \frac{1}{4}[T^{(a)(b)(c)} + T^{(b)(a)(c)} - T^{(c)(a)(b)}] + \frac{1}{2}[\eta^{(a)(c)}T^{(b)} - \eta^{(a)(b)}T^{(c)}] \tag{36}$$

with

$$T^{(a)(b)(c)} = T_{(d)\mu\nu}e^{(b)\mu}e^{(c)\nu}\eta^{(a)(d)} \tag{37}$$

$$T^{(b)} = T_{(d)\mu\nu}e^{(b)\nu}e^{(d)\mu} \tag{38}$$

are found to be

$$\Sigma^{(1)(0)(1)} = -\Sigma^{(1)(1)(0)} = \Sigma^{(2)(0)(2)} = -\Sigma^{(2)(2)(0)} = \frac{1}{2}\left(\frac{\dot{\wedge}}{\wedge} + \frac{\dot{\kappa}}{\kappa}\right) \tag{39}$$

$$\Sigma^{(3)(0)(3)} = -\Sigma^{(3)(3)(0)} = \frac{\dot{\kappa}}{\kappa} \tag{40}$$

$$\Sigma^{(0)(0)(1)} = -\Sigma^{(0)(1)(0)} = -\frac{\cot\theta}{2\kappa} \tag{41}$$

$$\Sigma^{(3)(1)(3)} = -\Sigma^{(3)(3)(1)} = -\frac{\cot\theta}{2\kappa} \tag{42}$$

$$\Sigma^{(0)(1)(1)} = \Sigma^{(0)(2)(2)} = \frac{\dot{\kappa}}{2\kappa} \tag{43}$$

$$\Sigma^{(0)(3)(3)} = \frac{\dot{\wedge}}{2\wedge}. \tag{44}$$

Next, to obtain the components of field equations, we use $\{(a) = (0), \mu = 0\}$, $\{(a) = (3), \mu = 1\}$, $\{(a) = (1), \mu = 2\}$ and $\{(a) = (2), \mu = 3\}$. These are used to obtain field equations involving T_{00}, T_{11}, T_{22} and T_{33} respectively.

Using $(a) = (0)$, $\mu = 0$ we get

$$\begin{aligned} 4\pi G e T_{00} = {} & e_{(0)0}e_{(0)0}\partial_2\left[ee^0_{(0)}e^2_{(1)}\Sigma^{(0)(0)(1)}\right] \\ & - e\left[e^2_{(1)}T_{(1)20}\Sigma^{(1)(1)(0)} + e^3_{(2)}T_{(2)30}\Sigma^{(2)(2)(0)}\right. \\ & + e^1_{(3)}T_{(3)10}\Sigma^{(3)(3)(0)} - \frac{1}{4}e_{(0)0} \\ & \times\left\{e^0_{(0)}e^2_{(1)}T_{(1)02}\Sigma^{(1)(0)(1)} + e^2_{(1)}e^0_{(0)}T_{(1)20}\right. \\ & \times\Sigma^{(1)(1)(0)} + e^3_{(2)}e^0_{(0)}T_{(2)03}\Sigma^{(2)(0)(2)} + e^3_{(2)}e^0_{(0)}T_{(2)30} \\ & \times\Sigma^{(2)(2)(0)} + e^1_{(3)}e^0_{(0)}T_{(3)01}\Sigma^{(3)(0)(3)} + e^1_{(3)}e^0_{(0)}T_{(3)10} \\ & \times\Sigma^{(3)(3)(0)} + e^1_{(3)}e^2_{(1)}T_{(3)12} \\ & \left.\left.\times\Sigma^{(3)(3)(1)} + e^2_{(1)}e^1_{(3)}T_{(3)21}\Sigma^{(3)(1)(3)}\right\}\right] \\ & -\frac{1}{2}ee^{(0)}_0\Delta \end{aligned} \tag{45}$$

from where we arrive at the field equation

$$\frac{2\dot{\wedge}\dot{\kappa}}{\wedge\kappa} + \frac{1+\dot{\kappa}^2}{\kappa^2} = 8\pi G\rho + \Delta \tag{46}$$

using equations (30)–(33), (35), (39)–(44).

The second equation is obtained using $a = (3)$, $\mu = 1$ as

$$\begin{aligned} 4\pi G e e^{1}_{(3)} T_{11} = {} & e_{(3)1} e_{(3)1} \partial_0 \left[e e^{1}_{(3)} e^{0}_{(0)} \Sigma^{(3)(3)(0)} \right] \\ & - e \Big[\eta_{(3)(3)} e^{0}_{(0)} \Sigma^{(3)(0)(3)} T_{(3)01} \\ & \quad - \frac{1}{4} e_{(3)1} \left\{ e^{0}_{(0)} e^{2}_{(1)} T_{(1)02} \Sigma^{(1)(0)(1)} \right. \\ & \quad + e^{2}_{(1)} e^{0}_{(0)} T_{(1)20} \Sigma^{(1)(1)(0)} + e^{3}_{(2)} e^{0}_{(0)} T_{(2)03} \Sigma^{(2)(0)(2)} \\ & \quad + e^{3}_{(2)} e^{0}_{(0)} T_{(2)30} \Sigma^{(2)(2)(0)} + e^{1}_{(3)} e^{0}_{(0)} T_{(3)01} \Sigma^{(3)(0)(3)} \\ & \quad + e^{1}_{(3)} e^{0}_{(0)} T_{(3)10} \Sigma^{(3)(3)(0)} + e^{1}_{(3)} e^{2}_{(1)} T_{(3)12} \Sigma^{(3)(3)(1)} \\ & \quad \left. + e^{2}_{(1)} e^{1}_{(3)} T_{(3)21} \Sigma^{(3)(1)(3)} \right\} \Big] \\ & - \frac{1}{2} e e^{(0)}_{0} \Delta \end{aligned} \tag{47}$$

which gives, on simplification using (30)–(33), (35), (39)–(44),

$$2\left(\frac{\ddot{\kappa}}{\kappa}\right) + \frac{1+\dot{\kappa}^2}{\kappa^2} = -8\pi G p + \Delta. \tag{48}$$

Again, using the other two sets of components $\{(a) = (1), \mu = 2\}$ and $\{(a) = (2), \mu = 3\}$ we get

$$\begin{aligned} 4\pi G e\, e^{2}_{(1)} T_{22} = {} & e_{(1)2} e_{(1)2} \partial_0 \left[e e^{2}_{(1)} e^{0}_{(0)} \Sigma^{(1)(1)(0)} \right] \\ & - e \Big[\eta_{(1)(1)} e^{0}_{(0)} T_{(1)02} \Sigma^{(1)(0)(1)} \\ & \quad - \frac{1}{4} e_{(1)2} \left\{ e^{0}_{(0)} e^{2}_{(1)} T_{(1)02} \Sigma^{(1)(0)(1)} \right. \\ & \quad + e^{2}_{(1)} e^{0}_{(0)} T_{(1)20} \Sigma^{(1)(1)(0)} + e^{0}_{(0)} e^{3}_{(2)} T_{(2)03} \Sigma^{(2)(0)(2)} \\ & \quad + e^{3}_{(2)} e^{0}_{(0)} T_{(2)30} \Sigma^{(2)(2)(0)} + e^{1}_{(3)} e^{0}_{(0)} T_{(3)01} \Sigma^{(3)(0)(3)} \\ & \quad + e^{1}_{(3)} e^{0}_{(0)} T_{(3)10} \Sigma^{(3)(3)(0)} + e^{1}_{(3)} e^{2}_{(1)} T_{(3)12} \Sigma^{(3)(3)(1)} \\ & \quad \left. + e^{2}_{(1)} e^{1}_{(3)} T_{(3)21} \Sigma^{(3)(1)(3)} \right\} \Big] \\ & + \frac{1}{2} e e_{(1)2} \Delta. \end{aligned} \tag{49}$$

Again, using eqs (30)–(33), (35), (39)–(44) we get

$$\frac{\ddot{\kappa}}{\kappa} + \frac{\ddot{\wedge}}{\wedge} + \frac{\dot{\wedge}\dot{\kappa}}{\wedge\kappa} = -8\pi G p + \Delta. \tag{50}$$

The third set of equations is deduced using another set of components $\{(a) = (2), \mu = 3\}$. Here we get

$$
\begin{aligned}
4\pi G e e^{3}_{(2)} T_{33} = 0 - e \Big[& \eta_{(2)(2)} e^{0}_{(0)} \Sigma^{(2)(0)(2)} T_{(2)03} \\
& + \eta_{(2)(2)} e^{2}_{(1)} T_{(2)\,23} \Sigma^{(2)(1)(2)} \\
& - \frac{1}{4} e_{(2)\,3} \{ e^{0}_{(0)} e^{2}_{(1)} T_{(1)\,02} \Sigma^{(1)(0)(1)} \\
& + e^{2}_{(1)} e^{0}_{(0)} T_{(1)20} \Sigma^{(1)(1)(0)} + e^{0}_{(0)} e^{3}_{(2)} T_{(2)03} \Sigma^{(2)(0)(2)} \\
& + e^{3}_{(2)} e^{0}_{(0)} T_{(2)30} \Sigma^{(2)(2)(0)} + e^{1}_{(3)} e^{0}_{(0)} T_{(3)01} \Sigma^{(3)(0)(3)} \\
& + e^{1}_{(3)} e^{0}_{(0)} T_{(3)10} \Sigma^{(3)(3)(0)} + e^{1}_{(3)} e^{2}_{(1)} T_{(3)12} \Sigma^{(3)(3)(1)} \\
& + e^{2}_{(1)} e^{1}_{(3)} T_{(3)21} \Sigma^{(3)(1)(3)} \} \Big] \\
+ \frac{1}{2} e e_{(2)3} \Delta. &
\end{aligned}
\tag{51}
$$

Again, using eqs (30)–(33), (35), (39)–(44), we get

$$
2\frac{\dot{\kappa}^2}{\kappa} + 2\frac{\dot{\kappa}\,\dot{\wedge}}{\kappa\wedge} = 8\pi G p - \Delta. \tag{52}
$$

Equations (46), (48), (50), (52) are equivalent to the field equations for Kantowski–Sachs cosmology from general relativity.

3.2 *Total energy in TEGR*

The total energy comprising gravitational and matter fields can be obtained using

$$
P^{(a)} = \int_{\nu} \mathrm{d}^3 x 4\kappa' \partial_\nu \left(e \Sigma^{(a)(0)(c)} e^{0}_{(0)} e^{\nu}_{(c)} \right), \tag{53}
$$

where $a = 0$ corresponds to the system energy.

$\therefore$ Total energy will be given by

$$
\begin{aligned}
P^{(0)} &= \int \mathrm{d}^3 x 4\kappa' \left[\partial_2 \left\{ e\, \Sigma^{(0)(0)(c)}\, e^{0}_{(0)}\, e^{2}_{(c)} \right\} \right] \\
&= \int \mathrm{d}^3 x\, 4\kappa' \left(+\frac{\wedge}{2} \right) \sin\theta \\
&= \left(\frac{\wedge}{2} \right) 4\kappa' \int_0^r \int_0^\pi \int_0^{2\pi} r^2\ \sin^2\theta\, \mathrm{d}r\, \mathrm{d}\theta\, \mathrm{d}\phi \\
&= \text{finite, if } r \text{ is finite,} \\
&= \text{infinite for large } r.
\end{aligned}
\tag{54}
$$

4. Conclusion

To find the gravitational energy, we have considered the teleparallel quasilocal approach and also the Hamiltonian structure of teleparallel equivalent of general relativity in case inflationary Kantowski–Sachs-type cosmology.

The quasilocal energy of the inflationary Kantowski–Sachs Universe within a finite 3-hypersurface region is found to be zero agreeing with the results obtained by Megied, Gad [8], for Kantowski–Sachs space-time using energy–momentum definitions of Møller's energy in the tetrad theory of gravitation. In Loi So and Vargas [9] work, zero value was found for quasilocal energy of Bianchi type I–type II Universes in teleparallel theory of gravity. Like their result, in our paper also zero value for total energy has been found for Kantowski–Sachs Universe. Like their result, our result is also independent of the three teleparallel dimensionless coupling constants because energy contributions from the matter and gravitational fields inside our 3-hypersurface region cancel each other.

We also calculated the total energy of inflationary Kantowski–Sachs cosmology in Hamiltonian structure of TEGR following the method of Sousa *et al* [12] for FLRW Universe. The total energy, in our case, is finite for a finite region irrespective of the equations of the state of the cosmic matter. But, interestingly, the total energy becomes infinite for a very large Universe which seems to be very unphysical. So, it is hereby concluded that perhaps the method adopted by Sousa, Moura, Pereira for a spherical FLRW Universe is not applicable to inflationary Kantowski–Sachs Universe of very large size.

Finally, we have analysed the equivalence between general relativity and TEGR. We have considered Lagrangian density containing quadratic combination of tetrads and used the constraint that the null curvature is present. With all these assumptions, the field equations obtained are the same as those in GR. Thus, tetrad formalism brings the equivalence between TEGR and GR for Kantowski–Sachs space-time agreeing with results in [12] obtained for spherical FLRW Universe.

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