PRAMANA — journal of physics

© Indian Academy of Sciences

Vol. 80, No. 3
March 2013
pp. 519–531

Head-on collision of dust-ion-acoustic solitons in electron-dust-ion quantum plasmas

PRASANTA CHATTERJEE[1], MALAY KUMAR GHORUI[1,2,*] and RAJKUMAR ROYCHOUDHURY[3]

[1]Department of Mathematics, Siksha Bhavana, Visva Bharati University, Santiniketan 731 235, India
[2]Department of Mathematics, B.B. College, Asansol 713 303, India
[3]Physics and Applied Mathematics, Indian Statistical Institute, Kolkata 700 009, India
*Corresponding author. E-mail: malaykr_ghorui@rediffmail.com

MS received 25 November 2011; revised 21 August 2012; accepted 18 September 2012

Abstract. In this paper, we study the head-on collision between two dust-ion-acoustic (DIA) solitons in quantum electron-dust-ion plasma. Using the extended Poincaré–Lighthill–Kuo (PLK) method, we obtain the Korteweg–de Vries (KdV) equations, the phase shifts and the trajectories after the head-on collision of the two DIA solitons. We investigate the effect of quantum diffraction parameters for electrons and ions (H_{e}, H_{i}), the Fermi temperature ratio (σ) and the dust charged number density (n_{d0}) on the phase shifts. Different values of $\mu = z_{\mathrm{d0}}(n_{\mathrm{d0}}/n_{\mathrm{i0}})$ and $\mu_{\mathrm{d}} = z_{\mathrm{d0}}(m_{\mathrm{i}}/m_{\mathrm{d}})$ are taken to discuss the effects on phase shifts, where z_{d0} denotes the dust charge number, n_{j0} represents the equilibrium number density and m_j is the mass of the jth species ($j = e$, i, d for electrons, ions and dust particles, respectively). It is observed that the phase shifts are significantly affected by the plasma parameters.

Keywords. Dust-ion-acoustic solitons; electron-dust-ion plasma; quantum plasma; phase shift; Pioncaré–Lighthill–Kuo (PLK) method; KdV equation; head-on collision.

PACS Nos 52.35.Sb; 52.27.Lw; 52.20.Hv

1. Introduction

The physics of dusty plasma has attracted a great deal of interest during the last few decades mainly owing to its applications in asteroid zones, cometary tails, planetary ring, interstellar medium, lower part of Earth's ionosphere and magnetosphere [1–5] in low-temperature physics, radiofrequency plasma discharge [6], coating and etching of thin films [7], plasma crystal [8], etc. Several authors [9–11] have theoretically investigated various aspects of non-linear acoustic wave propagation in three-component dusty plasmas consisting of electrons, ions and dust grains. Depending on time scales, there can be different types of acoustic modes in dusty plasmas in the form of solitons, shocks and periodic wave trains. One such mode, an ultra low-frequency electrostatic mode, called the dust-acoustic (DA) wave has been predicted theoretically by Rao *et al* [12] by including the dust collective dynamics. Moreover, at higher frequencies, Shukla and Silin [13] studied the dust-ion acoustic (DIA) waves involving only ion and electron-dynamics. Several authors [14,15] have also paid attention to different aspects of these waves.

Dusty degenerate plasmas can be found in the laboratory as well as astrophysical bodies. It is now known that dust clouds can pollute the surfaces of white dwarfs [16]. Dust may also be added to white dwarfs when a star bulges out into a red giant and the planets moving around it are engulfed by it. These objects are converted to dust grains due to the frictional forces and after the supernova phase, a white dwarf may appear with the dust grains. In microelectromechanical systems, ultrasmall electronic devices, etc., impurities play the role of dust particles [17]. Thus, dust plays an important role in modifying plasma dynamics. In these examples, densities are high enough to make the plasma degenerate. Generally, dust particles are heavier than both ions and electrons, and electrons are considered to be degenerate owing to their small mass, but to make the system more accurate, we consider the degenerate ions also, and the charged dust is treated classically. Several authors [18–20] carried out investigations on the quantum mechanical behaviour of both ions and electrons.

The electron-dust-ion plasma behaves like Fermi gas and the quantum mechanical effect plays a significant role in the behaviour of charge particles [21–28]. When the plasma is cooled to extremely low temperatures, the de Broglie wavelength of the charge carriers becomes comparable to the scale lengths, such as Debye length or Larmor radius in the system. In such systems, the ultracold dense plasma behaves as a Fermi gas and quantum mechanical effects might play a crucial role in the behaviour of charged particles of these plasmas under extreme conditions. Quantum effects in such plasmas become important when the thermal de Broglie wavelength ($\lambda_{\mathrm{B}j}$) is comparable to or larger than the average interparticle distance $n_j^{-1/3}$, i.e., when $n_j\lambda_{\mathrm{B}j}^3 \geq 1$, where $\lambda_{\mathrm{B}j} = (h/2\pi m_j V_{Tj}) = (a_j/\lambda_{\mathrm{D}j})$ is the thermal de Broglie wavelength for the jth species and $a_j = h/4q_j\sqrt{\pi m_j n_{j0}}$ characterizes the Bohr radius per unit number density n_{j0}, while q_j is the charge and $\lambda_{\mathrm{D}j}$ is the Debye length, and $j = e, i, d$ for electrons, ions and dusts. Here, we assume $\lambda_{\mathrm{B}j} \gg \lambda_{\mathrm{D}j}$, whence the quantum effects can no longer be ignored. Hass *et al* [18] have developed the quantum hydrodynamic model (QHD) to study quantum ion-acoustic wave in electron-ion plasmas. The QHD model gives rise to a set of equations describing the transport of charge, momentum and energy in a charge particle system interacting through a self-electrostatic potential. Nonlinear structures such as solitons, shocks, vertices, etc. and solitary waves in dusty quantum plasmas have also been analysed by a number of authors. Masood *et al* [20] have studied linear and nonlinear DIA waves propagating in an electron-dust-ion plasma from both analytical and numerical perspectives by employing the two-fluid QHD model. Chatterjee *et al* [29] have described the DIA KdV solitons and second-order correction to KdV solitons in a quantum pair-ion plasmas. Chatterjee *et al* [30] also have investigated nonlinear propagation of quantum ion-acoustic waves in dense quantum plasma containing electrons, positrons and positive ions using a QHD model. DIA soliton in electron-dust-ion plasma, by employing a two-fluid QHD model, has also been explained by Chatterjee *et al* [19].

One of the seminal properties of solitons is their asymptotic preservation of form when they undergo collision, as first noted by Zabusky and Kruskal [31]. In a one-dimensional system, there are two distinct soliton interactions. One is the overtaking collision and the other is the head-on collision [32]. The overtaking collision of solitary wave can be studied by the inverse scattering transformation method [33]. For head-on collision between two solitary waves, $\theta = \pi$, where θ is the angle between two propagation directions of two solitons. To search for the evolution of waves travelling to both sides, we need to employ a suitable asymptotic expansion to solve the original fluid dynamic equations. Xue [34] investigated the head-on collision between dust-ion solitary waves in unmagnetized dusty plasma. He found that the dust charge variation significantly modifies the phase shift. Han *et al* [35,36] analysed the head-on collision of two ion-acoustic solitary waves in weakly relativistic electron–positron–ion plasma and in unmagnetized electron–positron–ion plasma, respectively. El-Shamy [37,38] described the head-on collision of ion thermal solitary waves in pair-ion plasmas containing charged dust impurities [37] and that of ion-acoustic solitary waves in a Thomas–Fermi plasma containing degenerate electrons and positrons [38]. Chatterjee *et al* [39] studied the head-on collision of ion-acoustic solitary waves in a three-component unmagnetized plasma with cold ions, Boltzmann distributed positrons and superthermal electrons. The effects of the ratio of the electron temperature to the positron temperature, together with the spectral index of the electron kappa distribution and fractional concentration of positron component on the phase shifts are studied. El-Shamy [40,41] investigated the head-on collision of thermal waves in magnetized pair-ion plasma containing charged dust impurities [40] and that of two ion-acoustic solitary waves in unmagnetized multicomponent plasma consisting of hot ions, hot positrons and two-hot-electrons [41]. Akbari-Moghanjoughi [42] found the small-amplitude propagation and quasielastic head-on collision of ion-acoustic solitary waves in degenerate Thomas–Fermi electron–positron–ion magnetized plasma. Head-on collision of solitons are also studied in quantum plasma. El-Labany *et al* [43] determined the characteristics of the head-on collision between two quantum ion-acoustic solitary waves in dense electron–positron–ion plasma. It was the first attempt to illustrate the effects of both the quantum diffraction corrections and the Fermi temperature ratio of positrons-to-electrons on the phase shifts. Xu *et al* [44] investigated the interaction of two solitary waves in quantum electron–positron–ion plasma. The effects of the quantum parameter, the ratio of Fermi positron temperature to Fermi electron temperature, and the ratio of Fermi positron number density to Fermi electron number density and the ratio of Fermi ion temperature to Fermi electron temperature on the phase shifts are studied.

The head-on collision of dust-ion-acoustic soliton in electron-dust-ion quantum plasma has not yet been investigated. To fill up this lacuna, we study here the dynamics of ion-acoustic dusty solitons in a quantum plasma system consisting of electron-dust-ion, based on a one-dimensional model. The head-on collision between two DIAWs is studied by the extended Poincaré–Lighthill–Kuo (PLK) method. It is shown that the bi-directional DIA solitary waves are propagated and head-on collision of these two solitons occurs. The phase shifts and the trajectories of the two solitons after collision are deduced. Our aim is to find the effects of H_e, H_i, σ and (n_{d0}) for different values of μ or μ_d on the phase shifts.

2. Basic equations

Consider the propagation of DIAWs in an unmagnetized collisionless quantum plasma comprising electrons, ions and dust. We consider inertial dusts and ions, while assuming electrons to be inertialess under the condition that the phase velocity of the wave is much less than the Fermi velocity of electrons (i.e., $V_{\mathrm{Fd}} < V_{\mathrm{Fi}} < \omega/k < V_{\mathrm{Fe}}$). The dynamics of DIAWs in quantum dusty plasma is governed by the following set of hydrodynamic equations:

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{i}} v_{\mathrm{i}}) = 0 \tag{1}$$

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{d}} v_{\mathrm{d}}) = 0 \tag{2}$$

$$\frac{\partial v_{\mathrm{i}}}{\partial t} + v_{\mathrm{i}}\frac{\partial v_{\mathrm{i}}}{\partial x} = -\frac{e}{m_{\mathrm{i}}}\frac{\partial \phi}{\partial x} - \frac{1}{m_{\mathrm{i}} n_{\mathrm{i}}}\frac{\partial p_{\mathrm{i}}}{\partial x} + \frac{\hbar^2}{2m_{\mathrm{i}}^2}\frac{\partial}{\partial x}\left(\frac{(\partial^2/\partial x^2)(\sqrt{n_{\mathrm{i}}})}{\sqrt{n_i}}\right) \tag{3}$$

$$\frac{\partial v_{\mathrm{d}}}{\partial t} + v_{\mathrm{d}}\frac{\partial v_{\mathrm{d}}}{\partial x} = \frac{z_{\mathrm{d}0} e}{m_{\mathrm{d}}}\frac{\partial \phi}{\partial x} \tag{4}$$

$$\frac{e}{m_{\mathrm{e}}}\frac{\partial \phi}{\partial x} - \frac{1}{m_{\mathrm{e}} n_{\mathrm{e}}}\frac{\partial p_{\mathrm{e}}}{\partial x} + \frac{\hbar^2}{2m_{\mathrm{e}}^2}\frac{\partial}{\partial x}\left(\frac{(\partial^2/\partial x^2)(\sqrt{n_{\mathrm{e}}})}{\sqrt{n_{\mathrm{e}}}}\right) = 0 \tag{5}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_{\mathrm{e}} - n_{\mathrm{i}} + z_{\mathrm{d}0} n_{\mathrm{d}}). \tag{6}$$

At equilibrium, the charge neutrality condition $n_{\mathrm{i}0} = n_{\mathrm{e}0} + z_{\mathrm{d}0} n_{\mathrm{d}0}$ is applied, where $z_{\mathrm{d}0}$ denotes the dust charge and n_{j0} is the equilibrium number density of the jth species. As the charge on the dust is sufficiently large ($z_{\mathrm{d}} \gg 1$), the dust charge fluctuations are very small and the charging effects can be neglected. Now we use the normalized variables as $x \to x/\lambda_{\mathrm{D}}$, $t \to \omega_{pi} t$, $n_j \to n_j/n_{j0}$, $v_j \to v_j/c_{\mathrm{si}}$ and $\phi \to e\phi/K_{\mathrm{B}} T_{\mathrm{Fe}}$, where $j = e, i, d$ for electrons, ions and dusts, respectively, $c_{\mathrm{si}} = (2K_{\mathrm{B}} T_{\mathrm{Fe}}/m_{\mathrm{i}})^{1/2}$ is the quantum ion-acoustic speed, $\lambda_{\mathrm{D}} = (2K_{\mathrm{B}} T_{\mathrm{Fe}}/4\pi n_{\mathrm{i}0} e^2)$ is the quantum Debye length and $\omega_{\mathrm{pi}}^{-1} = \sqrt{m_{\mathrm{i}}/4\pi n_{\mathrm{i}0} e^2}$ is the ion plasma period. We assume that the plasma particles for one-dimensional zero-temperature Fermi gas obey the pressure law

$$p_j = \frac{m_j V_{\mathrm{F}j}^2}{3n_{j0}^2} n_j^3. \tag{7}$$

Here, m_j is the mass of the jth species and $V_{\mathrm{F}j} = \sqrt{2K_{\mathrm{B}} T_{\mathrm{F}j}/m_j}$ is the Fermi speed. Then the normalized basic sets of equations for DIAWs in quantum dusty plasma are [20]:

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{i}} v_{\mathrm{i}}) = 0, \tag{8}$$

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \frac{\partial}{\partial x}(n_{\mathrm{d}} v_{\mathrm{d}}) = 0, \tag{9}$$

$$\frac{\mathrm{d}v_{\mathrm{i}}}{\mathrm{d}t} = -\frac{\partial \phi}{\partial x} - \sigma n_{\mathrm{i}}\frac{\partial n_{\mathrm{i}}}{\partial x} + \frac{H_{\mathrm{i}}^2}{2}\frac{\partial}{\partial x}\left(\frac{(\partial^2/\partial x^2)(\sqrt{n_{\mathrm{i}}})}{\sqrt{n_{\mathrm{i}}}}\right), \tag{10}$$

$$\frac{\mathrm{d}v_\mathrm{d}}{\mathrm{d}t} = \mu_\mathrm{d}\frac{\partial\phi}{\partial x}, \tag{11}$$

$$\frac{\partial\phi}{\partial x} - n_\mathrm{e}\frac{\partial n_\mathrm{e}}{\partial x} + \frac{H_\mathrm{e}^2}{2}\frac{\partial}{\partial x}\left(\frac{(\partial^2/\partial x^2)(\sqrt{n_\mathrm{e}})}{\sqrt{n_\mathrm{e}}}\right) = 0, \tag{12}$$

$$\frac{\partial^2\phi}{\partial x^2} = \mu n_\mathrm{d} + (1-\mu)n_\mathrm{e} - n_\mathrm{i}. \tag{13}$$

Here, $\mu = z_{\mathrm{d}0}n_{\mathrm{d}0}/n_{\mathrm{i}0}$, $\mu_\mathrm{d} = z_{\mathrm{d}0}m_\mathrm{i}/m_\mathrm{d}$ and the ratio of ion to electron Fermi temperature $\sigma = T_{\mathrm{Fi}}/T_{\mathrm{Fe}}$. The non-dimensional quantum diffraction parameters for ions and electrons are defined as

$$H_\mathrm{i} = \frac{\hbar\omega_{\mathrm{pi}}}{2K_\mathrm{B}T_{\mathrm{Fe}}} \quad \text{and} \quad H_\mathrm{e} = \frac{\hbar\omega_{\mathrm{pi}}}{2K_\mathrm{B}T_{\mathrm{Fe}}}\sqrt{\frac{m_\mathrm{i}}{m_\mathrm{e}}},$$

$\hbar$ is the reduced Planck constant. Linearization of eqs (8)–(13) gives the dispersion relation for the DIA wave in quantum dusty plasma in terms of normalized frequency ω and wave number k as (for details, see ref [20]).

$$\left\{k^2\left(1+\frac{H_\mathrm{e}^2k^2}{4}\right)+(1-\mu)\right\}\omega^4 - \left[\left\{k^2\left(1+\frac{H_\mathrm{e}^2k^2}{4}\right)+(1-\mu)\right\}\right.$$
$$\left.\times\left(k^2\sigma+\frac{H_\mathrm{i}^2k^4}{4}\right)+k^2(\mu\mu_\mathrm{d}+1)\left(1+\frac{H_\mathrm{e}^2k^2}{4}\right)\right]\omega^2$$
$$+\mu\mu_\mathrm{d}k^2\left(k^2\sigma+\frac{H_\mathrm{i}^2k^4}{4}\right)\left(1+\frac{H_\mathrm{e}^2k^2}{4}\right) = 0.$$

Integrating eq. (12) with the boundary condition, viz. $n_\mathrm{e} = 1$ and $\phi = 0$ at $\pm\infty$, we have

$$\phi = -\frac{1}{2} + \frac{1}{2}n_\mathrm{e}^2 - \frac{H_\mathrm{e}^2}{2}\frac{(\partial^2\sqrt{n_\mathrm{e}}/\partial x^2)}{\sqrt{n_\mathrm{e}}}. \tag{14}$$

We consider two solitons S_1 and S_2 asymptotically far apart in the initial state and travel towards each other in the plasma. After some time they interact, collide and then depart. We consider that the solitons have small amplitudes ϵ (a small formal perturbation parameter characterizing the strength of nonlinearity) and the interactions between solitons are weak. We use the extended PLK perturbation method. Accordingly, the dependent variables are expanded as

$$\psi = \psi^{(0)} + \sum_{1}^{\infty}\epsilon^{n+1}\psi^{(n)}, \tag{15}$$

where $\psi = (n_\mathrm{e}, n_\mathrm{i}, n_\mathrm{d}, v_\mathrm{i}, v_\mathrm{d}, \phi)$ with $\psi^{(0)} = (1,1,1,0,0,0)$. We now introduce the stretched coordinates as

$$\xi = \epsilon(x-\lambda t) + \epsilon^2 P^{(0)}(\eta,\tau) + \epsilon^3 P^{(1)}(\xi,\eta,\tau) + \cdots, \tag{16}$$

$$\eta = \epsilon(x+\lambda t) + \epsilon^2 Q^{(0)}(\xi,\tau) + \epsilon^3 Q^{(1)}(\xi,\eta,\tau) + \cdots, \tag{17}$$

$$\tau = \epsilon^3 t, \tag{18}$$

where ξ and η denote the trajectories of two solitons travelling towards each other (i.e., to the right and left, respectively), λ is the unknown phase velocity of DIAWs (to be determined later). We have to determine the variables in $P^{(0)}(\eta, \tau)$ and $Q^{(0)}(\xi, \tau)$. Substituting eqs (15)–(18) in eqs (8)–(11), (13), (14) and equating the coefficients of equal powers of ϵ and we have, to the leading order

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{\rm i}^{(1)}+\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)v_{\rm i}^{(1)}=0, \tag{19}$$

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{\rm d}^{(1)}+\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)v_{\rm d}^{(1)}=0, \tag{20}$$

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)v_{\rm i}^{(1)}+\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\phi^{(1)}+\sigma\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)n_{\rm i}^{(1)}=0, \tag{21}$$

$$\lambda\left(-\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)v_{\rm d}^{(1)}-\mu_{\rm d}\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}\right)\phi^{(1)}=0, \tag{22}$$

$$\mu n_{\rm d}^{(1)}+(1-\mu)n_{\rm e}^{(1)}-n_{\rm i}^{(1)}=0, \tag{23}$$

$$\phi^{(1)}=n_{\rm e}^{(1)}. \tag{24}$$

Solving eqs (19)–(24), finally, we obtain the following relations among different physical quantities:

$$\phi^{(1)}=\phi_\xi^{(1)}(\xi,\tau)+\phi_\eta^{(1)}(\eta,\tau), \tag{25}$$

$$n_{\rm e}^{(1)}=\phi^{(1)}=\phi_\xi^{(1)}(\xi,\tau)+\phi_\eta^{(1)}(\eta,\tau), \tag{26}$$

$$n_{\rm d}^{(1)}=\frac{1+(\mu-1)(\lambda^2-\sigma)}{\lambda^2-\sigma}(\phi_\xi^{(1)}(\xi,\tau)+\phi_\eta^{(1)}(\eta,\tau)), \tag{27}$$

$$n_{\rm i}^{(1)}=\frac{1}{\lambda^2-\sigma}(\phi_\xi^{(1)}(\xi,\tau)+\phi_\eta^{(1)}(\eta,\tau)), \tag{28}$$

$$v_{\rm d}^{(1)}=\frac{1+(\mu-1)(\lambda^2-\sigma)}{\lambda^2-\sigma}\lambda(\phi_\xi^{(1)}(\xi,\tau)-\phi_\eta^{(1)}(\eta,\tau)), \tag{29}$$

$$v_{\rm i}^{(1)}=\frac{\lambda}{\lambda^2-\sigma}(\phi_\xi^{(1)}(\xi,\tau)-\phi_\eta^{(1)}(\eta,\tau)). \tag{30}$$

To satisfy the solvability condition (the condition to find a unique solution from eqs (19)–(24) when $\phi^{(1)}$ is given by eq. (25), the wave velocity λ has to obey the relation

$$\frac{\lambda^2}{\lambda^2-\sigma}+\mu\mu_{\rm d}=\lambda^2(1-\mu).$$

The unknown functions $\phi_\xi^{(1)}$ and $\phi_\eta^{(1)}$ will be determined from the next orders. It is inferred from eqs (25)–(30) that at the leading order, there are two waves, namely $\phi_\xi^{(1)}$ and $\phi_\eta^{(1)}$, travelling to right and left, respectively. At the next order, the solutions have the following forms:

$$\phi^{(2)}=\phi_\xi^{(2)}(\xi,\tau)+\phi_\eta^{(2)}(\eta,\tau), \tag{31}$$

$$n_{\rm e}^{(2)}=\phi^{(2)}=\phi_\xi^{(2)}(\xi,\tau)+\phi_\eta^{(2)}(\eta,\tau), \tag{32}$$

$$n_{\rm d}^{(2)} = \frac{1 + (\mu - 1)(\lambda^2 - \sigma)}{\lambda^2 - \sigma} \left(\phi_\xi^{(2)}(\xi, \tau) + \phi_\eta^{(2)}(\eta, \tau) \right), \tag{33}$$

$$n_{\rm i}^{(2)} = \frac{1}{\lambda^2 - \sigma} \left(\phi_\xi^{(2)}(\xi, \tau) + \phi_\eta^{(2)}(\eta, \tau) \right), \tag{34}$$

$$v_{\rm d}^{(2)} = \frac{1 + (\mu - 1)(\lambda^2 - \sigma)}{\lambda^2 - \sigma} \lambda \left(\phi_\xi^{(2)}(\xi, \tau) - \phi_\eta^{(2)}(\eta, \tau) \right), \tag{35}$$

$$v_{\rm i}^{(2)} = \frac{\lambda}{\lambda^2 - \sigma} \left(\phi_\xi^{(2)}(\xi, \tau) - \phi_\eta^{(2)}(\eta, \tau) \right). \tag{36}$$

Furthermore, the next higher-order perturbation leads to

$$\begin{aligned} &- \frac{2}{[\sigma/(\lambda^2 - \sigma)^2] + 1 - \mu} \frac{\partial^2}{\partial \xi \partial \eta} \left(\frac{v_{\rm i}^{(3)}}{\lambda^2 - \sigma} - \mu v_{\rm d}^{(3)} \right) \\ &= \frac{\partial}{\partial \xi} \left[\frac{\partial \phi_\xi^{(1)}}{\partial \tau} + A \phi_\xi^{(1)} \frac{\partial \phi_\xi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi_\xi^{(1)}}{\partial \xi^3} \right] \\ &\quad + \frac{\partial}{\partial \eta} \left[\frac{\partial \phi_\eta^{(1)}}{\partial \tau} - A \phi_\eta^{(1)} \frac{\partial \phi_\eta^{(1)}}{\partial \eta} - B \frac{\partial^3 \phi_\eta^{(1)}}{\partial \eta^3} \right] \\ &\quad + \left[C \frac{\partial P^{(0)}}{\partial \eta} + D \phi_\eta^{(1)} \right] \frac{\partial^2 \phi_\xi^{(1)}}{\partial \xi^2} \\ &\quad - \left[C \frac{\partial Q^{(0)}}{\partial \xi} + D \phi_\xi^{(1)} \right] \frac{\partial^2 \phi_\eta^{(1)}}{\partial \eta^2}. \end{aligned} \tag{37}$$

Integrating eq. (37) with respect to the variables ξ and η, we have

$$\begin{aligned} &- \frac{2}{[\sigma/(\lambda^2 - \sigma)^2] + 1 - \mu} \left(\frac{v_{\rm i}^{(3)}}{\lambda^2 - \sigma} - \mu v_{\rm d}^{(3)} \right) \\ &= \int \left[\frac{\partial \phi_\xi^{(1)}}{\partial \tau} + A \phi_\xi^{(1)} \frac{\partial \phi_\xi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi_\xi^{(1)}}{\partial \xi^3} \right] {\rm d}\eta \\ &\quad + \int \left[\frac{\partial \phi_\eta^{(1)}}{\partial \tau} - A \phi_\eta^{(1)} \frac{\partial \phi_\eta^{(1)}}{\partial \eta} - B \frac{\partial^3 \phi_\eta^{(1)}}{\partial \eta^3} \right] {\rm d}\xi \\ &\quad + \int \int \left[C \frac{\partial P^{(0)}}{\partial \eta} + D \phi_\eta^{(1)} \right] \frac{\partial^2 \phi_\xi^{(1)}}{\partial \xi^2} {\rm d}\xi {\rm d}\eta \\ &\quad - \int \int \left[C \frac{\partial Q^{(0)}}{\partial \xi} + D \phi_\xi^{(1)} \right] \frac{\partial^2 \phi_\eta^{(1)}}{\partial \eta^2} {\rm d}\xi {\rm d}\eta, \end{aligned} \tag{38}$$

where

$$A = \frac{\frac{1}{2}\left[-\frac{2\mu\mu_{\mathrm{d}}^2}{\lambda^3} + \frac{2\lambda}{(\lambda^2-\sigma)^2} + \frac{\lambda(\lambda^2+3\sigma)}{(\lambda^2-\sigma)^3} + \lambda(1-\mu)\right]}{1-\mu+\frac{\sigma}{(\lambda^2-\sigma)^2}},$$

$$B = \frac{\frac{\lambda}{2}\left[1 - \frac{H_{\mathrm{i}}^2}{4(\lambda^2-\sigma)^2} - \frac{(1-\mu)H_{\mathrm{e}}^2}{4}\right]}{1-\mu+\frac{\sigma}{(\lambda^2-\sigma)^2}}, \quad C = 2\lambda,$$

$$D = \frac{\frac{1}{2}\left[\frac{\mu\mu_{\mathrm{d}}^2}{\lambda^3} - \frac{\lambda}{(\lambda^2-\sigma)^2} + \lambda(1-\mu)\right]}{1-\mu+\frac{\sigma}{(\lambda^2-\sigma)^2}}. \tag{39}$$

The first (second) term in the right-hand side of eq. (38) will be proportional to $\eta(\xi)$ because the integrand is independent of $\eta(\xi)$. Thus, these two terms are secular, which must be eliminated in order to avoid spurious resonances. Hence, we have

$$\frac{\partial\phi_\xi^{(1)}}{\partial\tau} + A\phi_\xi^{(1)}\frac{\partial\phi_\xi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi_\xi^{(1)}}{\partial\xi^3} = 0, \tag{40}$$

$$\frac{\partial\phi_\eta^{(1)}}{\partial\tau} - A\phi_\eta^{(1)}\frac{\partial\phi_\eta^{(1)}}{\partial\eta} - B\frac{\partial^3\phi_\eta^{(1)}}{\partial\eta^3} = 0. \tag{41}$$

The third and fourth terms in eq. (38) on its right-hand side are not secular in the next order, and so we have

$$C\frac{\partial P^{(0)}}{\partial\eta} = -D\phi_\eta^{(1)}. \tag{42}$$

$$C\frac{\partial Q^{(0)}}{\partial\xi} = -D\phi_\xi^{(1)}. \tag{43}$$

Equations (40) and (41) are the two-side travelling wave KdV equations in the reference frames of ξ and η, respectively, the solutions of which are as follows:

$$\phi_\xi^{(1)} = \phi_A^2 \,\mathrm{sech}\left[\left(\frac{A\phi_A}{12B}\right)^{1/2}\left(\xi - \frac{1}{3}A\phi_A\tau\right)\right], \tag{44}$$

$$\phi_\eta^{(1)} = \phi_B^2 \,\mathrm{sech}\left[\left(\frac{A\phi_B}{12B}\right)^{1/2}\left(\eta + \frac{1}{3}A\phi_B\tau\right)\right], \tag{45}$$

where ϕ_A and ϕ_B are the amplitudes of the two solitons in their initial positions. If $A = 0$, then the nonlinear terms in eqs (40) and (41) vanish. On the other hand if $B = 0$, then the dispersion terms vanish. Hence, the condition for existence of solitons are: (i) A and B are of the same sign if ϕ_A and ϕ_B are both positive or A and B are of the opposite sign if

ϕ_A and ϕ_B are both negative, (ii) $A \neq 0$ and (iii) $B \neq 0$. The leading phase changes due to the collision can be calculated from eqs (42) and (43) and are given by

$$P^{(0)}(\eta,\tau) = -\frac{D}{C}\left(\frac{12B\phi_B}{A}\right)^{1/2} \times \left[\tanh\left(\frac{A\phi_B}{12B}\right)^{1/2}\left(\eta+\frac{1}{3}A\phi_B\tau\right)+1\right], \quad (46)$$

$$Q^{(0)}(\xi,\tau) = -\frac{D}{C}\left(\frac{12B\phi_A}{A}\right)^{1/2} \times \left[\tanh\left(\frac{A\phi_A}{12B}\right)^{1/2}\left(\xi-\frac{1}{3}A\phi_A\tau\right)-1\right]. \quad (47)$$

Therefore up to $O(\epsilon^2)$, the trajectories of the two solitary waves for weak head-on interactions are

$$\xi = \epsilon(x-\lambda t) - \epsilon^2\frac{D}{C}\left(\frac{12B\phi_B}{A}\right)^{1/2} \times \left[\tanh\left(\frac{A\phi_B}{12B}\right)^{1/2}\left(\eta+\frac{1}{3}A\phi_B\tau\right)+1\right]+\cdots, \quad (48)$$

$$\eta = \epsilon(x+\lambda t) - \epsilon^2\frac{D}{C}\left(\frac{12B\phi_A}{A}\right)^{1/2} \times \left[\tanh\left(\frac{A\phi_A}{12B}\right)^{1/2}\left(\xi-\frac{1}{3}A\phi_A\tau\right)-1\right]+\cdots. \quad (49)$$

To obtain the phase shifts after a head-on collision of the two solitons, we assume that the solitary pulses S_1 and S_2 are asymptotically far from each other at the initial time when $t=-\infty$. Soliton S_1 is at $\xi=0$, $\eta=-\infty$ and soliton S_2 is at $\xi=\infty$, $\eta=0$. After the collision, when $t=\infty$, the soliton S_1 is far to the right of soliton S_2, i.e., soliton S_1 is at $\xi=0$, $\eta=\infty$ and soliton S_2 is at $\xi=-\infty$, $\eta=0$. Using eqs (48) and (49), we obtain the corresponding phase shifts $\Delta P^{(0)}$ and $\Delta Q^{(0)}$ as follows:

$$\Delta P^{(0)} = \epsilon(x-\lambda t)|_{\xi=0,\eta=\infty} - \epsilon(x-\lambda t)|_{\xi=0,\eta=-\infty} = 2\epsilon^2\frac{D}{C}\left(\frac{12B\phi_B}{A}\right)^{1/2}. \quad (50)$$

$$\Delta Q^{(0)} = \epsilon(x+\lambda t)|_{\xi=-\infty,\eta=0} - \epsilon(x+\lambda t)|_{\xi=\infty,\eta=0} = -2\epsilon^2\frac{D}{C}\left(\frac{12B\phi_A}{A}\right)^{1/2}. \quad (51)$$

3. Numerical results and discussion

Here we have investigated the head-on collision of two DIAWs in an unmagnetized collisionless quantum plasma composed of electrons, dusts and ions. For simplicity, we assumed that the size, mass and charge of the dust particles remain constant throughout, and we consider a special situation of tiny dust particles that are heavily charged such that the ratio m_d/m_i goes up to 10^2–10^7 and the dust charge number z_{d0} reaches the values of the order 10^2–10^7. Under these circumstances, it is appropriate to include the dust dynamics with the ion inertia [9]. We used the extended PLK method. The effects of the parameters on phase shifts are as follows: soliton S_1 is travelling from the right and S_2 is travelling from the left. Equations (44) and (45) help us to say that each soliton has a phase shift in its own travelling direction. The magnitudes of the phase shifts depend upon the parameters ϵ, H_i, H_e, σ, λ, μ, μ_d and $\phi_A(\phi_B)$. From eqs (44) and (45), it is clear that the coefficients A and B are to be always of the same sign. The phase shifts are positive or negative depending on the coefficient D. The positive phase shifts after collision between two solitary waves are, on an average slower than the solitary waves without interactions; on the other hand, the negative phase shifts after collision between two solitary waves are, on an average faster than the solitary waves without interactions. Let us now consider the dimensionless physical parameters from high-density astrophysical situations (such as white dwarfs, megnetars, etc.), where the plasma densities are nearly 10^{25}–10^{35}cm^{-3} and the temperature range is 8000 K–40000 K.

Figures 1a and 1b show the variations of phase shifts against the quantum diffraction parameter H_e for electron. Here, we consider $\mu = 0.2$ (solid line), 0.65 (dashed line), 0.7 (dotted line), $\mu_d = 10^{-3}$ for figure 1a and $\mu_d = 10^{-3}$ (solid line), 10^{-1} (dotted line), 0.2 (dashed line), $\mu = 0.7$, for figure 1b. Also we consider $\sigma = 0.00203276$, $H_i = 0.00658181$, $\epsilon = 0.01$ and $\phi_B = 1$ for both the figures. In all the cases we have $n_{e0}\lambda_{Fe}^3 \gg 1$, and so the quantum effects become important. From figures 1a and 1b it is clear that the phase shifts $\Delta P^{(0)}$ decrease with increasing H_e and ultimately vanish where the coefficient $B = 0$. Further, it is to be noted that when $B = 0$ the soliton solution will collapse. Here, the rate of decrease of phase shifts decreases with increasing μ, but increases with increasing μ_d.

Figures 2a and 2b show the variations of phase shifts against the quantum diffraction parameter H_i for ion. Here, we consider all the physical parameters except H_i as in figures 1a and 1b, respectively and $H_e = 3.95737$, but $\phi_B = -1$ because of the fact

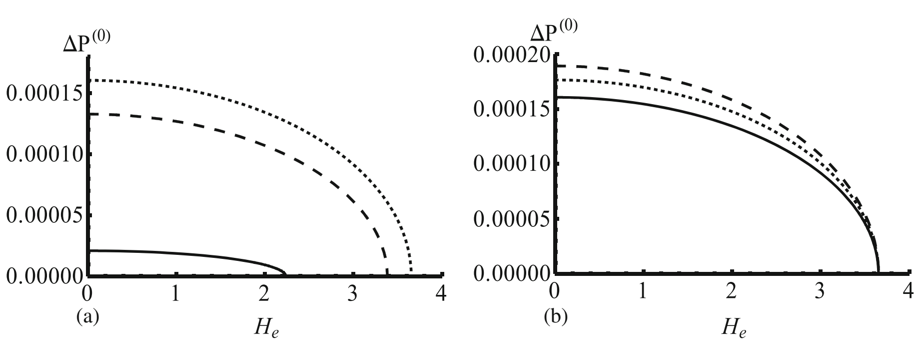

Figure 1. Variation of phase shift $\Delta P^{(0)}$ with H_e, where $\sigma = 0.00203276$, $H_i = 0.00658181$, $\epsilon = 0.01$ and $\phi_B = 1$. (**a**) for $\mu = 0.2$ (solid line); 0.65 (dashed line); 0.7 (dotted line), $\mu_d = 10^{-3}$ and (**b**) for $\mu_d = 10^{-3}$ (solid line), 10^{-1} (dotted line), 0.2 (dashed line), $\mu = 0.7$.

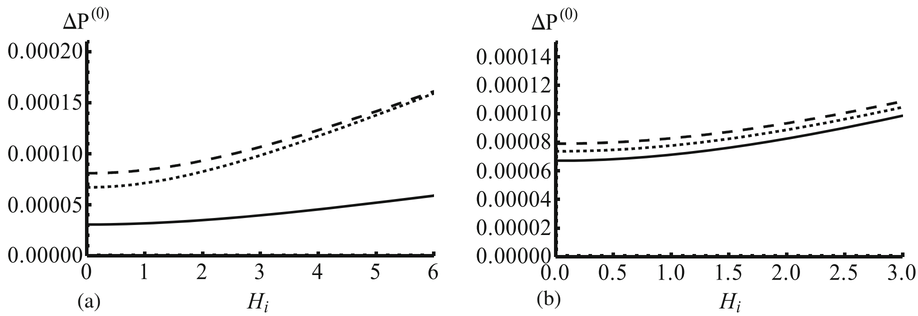

Figure 2. Variation of phase shift $\Delta P^{(0)}$ with $H_{\rm i}$, where $\sigma = 0.00203276$, $H_{\rm e} = 3.95737$ $\epsilon = 0.01$ and $\phi_B = -1$. (**a**) for $\mu = 0.2$ (solid lines); 0.65 (dashed line); 0.7 (dotted line), $\mu_{\rm d} = 10^{-3}$ and (**b**) for $\mu_{\rm d} = 10^{-3}$ (solid line), 10^{-1} (dotted line), 0.2 (dashed line), $\mu = 0.7$.

that the coefficients A and B are of opposite sign in the considered domain of $H_{\rm i}$. From figures 2a and 2b, it is clear that the phase shifts $\Delta P^{(0)}$ increase with increasing $H_{\rm i}$. The increasing rate increases with increasing μ, but the increasing rate remain constant for increasing or decreasing $\mu_{\rm d}$.

Figures 3a and 3b show the variations of phase shifts against the Fermi temperature ratio σ. Here, we consider all the physical parameters except σ as in figures 2a and 2b, respectively and $H_{\rm i} = 0.00658181$ for both the figures. From figures 3a and 3b, it is clear that the phase shifts $\Delta P^{(0)}$ decrease with increasing σ and the decreasing rate increases with decreasing μ, but it is constant for increasing or decreasing $\mu_{\rm d}$.

Figure 4 shows the variations of the phase shifts against the dust number density $n_{\rm d0} \geq 10^{29}{\rm cm}^{-3}$ for different values of the ratio $\mu_{\rm d} = 7\times10^{-4}$ (solid line), 8×10^{-4} (dotted line) and 9×10^{-4} (dashed line). Here, we consider $n_{\rm i0} = 10^{33}{\rm cm}^{-3}$, $\epsilon = 0.01$ and $\phi_B = 1$. From figure 4, it is clear that initially the phase shift increases slowly and then it increases rapidly as the unperturbed dust number density $n_{\rm d0}$ increases. For a given value of $n_{\rm d0}$, the effect on the phase shifts $\Delta P^{(0)}$ decreases as $\mu_{\rm d}$ decreases.

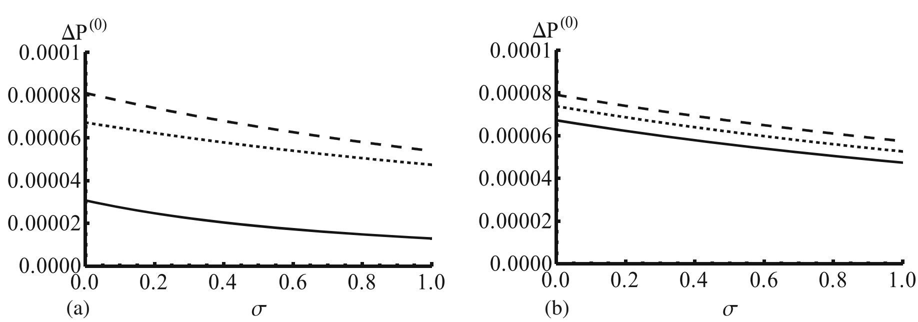

Figure 3. Variation of phase shift $\Delta P^{(0)}$ with σ, where $H_{\rm e} = 3.95737$, $H_{\rm i} = 0.00658181$, $\epsilon = 0.01$ and $\phi_B = -1$. (**a**) for $\mu = 0.2$ (solid lines); 0.65 (dashed line); 0.7 (dotted line), $\mu_{\rm d} = 10^{-3}$ and (**b**) for $\mu_{\rm d} = 10^{-3}$ (solid line), 10^{-1} (dotted line), 0.2 (dashed line), $\mu = 0.7$.

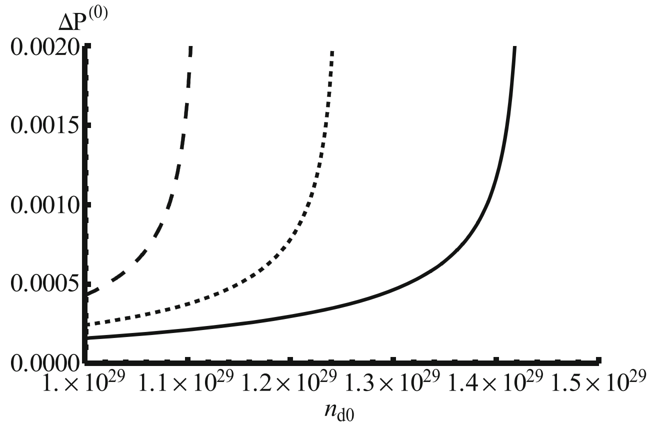

Figure 4. Variation of phase shift $\Delta P^{(0)}$ with n_{d0}, where $n_{\mathrm{i0}} = 10^{33}\mathrm{cm}^{-3}$, $m_{\mathrm{i}} = 10^{-24}\mathrm{g}$, $m_{\mathrm{d}} = 10^{-7}\mathrm{g}$, $\epsilon = 0.01$ and $\phi_B = 1$ for $\mu_{\mathrm{d}} = 7 \times 10^{-4}$ (solid line), 8×10^{-4} (dotted line) and 9×10^{-4} (dashed line).

4. Conclusion

In this paper, we study the effect of H_{e}, H_{i}, σ and n_{d0} for different values of μ and μ_{d} on the phase shifts. Usually the dust grain size ranges from nanometres to millimetres, unless they are man-made. Although the dust size distribution affects the main characteristics of the plasma system, we consider all the dust grains to have the same size for simplicity. The results obtained here indicate that the effects of the parameters H_{e}, H_{i}, σ, μ_{d}, μ and n_{d0} on phase shifts play important roles not only on the formations and the dynamical behaviour of the solitary waves but also on the soliton collision. In short, the phase shifts and the trajectories describing the collision of two DIAWs in an unmagnetized collisionless quantum plasma composed of electrons, positive and negative ions, and immobile negatively charged dust grains are investigated by the extended PLK method. Finally, the investigated result should be useful to investigate the collective phenomena related to DIAWs collisions that are of considerable interest in space and laboratory plasmas as well as in plasma applications.

Acknowledgement

The authors thank the referee for useful suggestions towards improving the manuscript.

References

[1] M Horanyi and D A Mendis, *Astrophys. J.* **294**, 357 (1985)
[2] C K Goertz, *Rev. Geophys.* **27**, 271, DOI: 10.1029/RG027i002p00271 (1989)
[3] F Verheest, *Space Sci. Rev.* **77**, 267 (1996)
[4] P K Shukla, D A Mendis and T Desai, *Advances in dusty plasmas* (World Scientific, Singapore, 1997)

[5] P K Shukla and A A Mamun, *Introduction to dusty plasma physics* (Institute of Physics Publishing, Bristol, 2002)
[6] J H Chu, J B Du and I Lin, *J. Phys.* **D27**, 296 (1994)
[7] G S Selwyn, *J. Appl. Phys.* Part 1 **32**, 2068 (1993)
[8] H Thomas, G E Mortill and V Dammel, *Phys. Rev. Lett.* **73**, 652 (1994)
[9] S Ghosh, S Sarkar, M Khan and M R Gupta, *Phys. Plasmas* **7**, 3594 (2006)
[10] J X Ma and J Y Liu, *Phys. Plasmas* **4**, 253 (1997)
[11] D L Xiao, J X Ma and Y F Li, *Phys. Plasmas* **12**, 052314 (2005)
[12] N N Rao, P K Shukla and M Y Yu, *Planet. Space Sci.* **38**, 543 (1990)
[13] P K Shukla and V P Silin, *Phys. Scr.* **45**, 508 (1992)
[14] S I Popel, A P Golub, T V Losseva, A V Ivlev, S A Khrapak and G Morfill, *Phys. Rev.* **E67**, 056402 (2003)
[15] A Barkan, R L Merlino and N D Angelo, *Planet. Space Sci.* **44**, 239 (1996)
[16] W T Reach, M J Kuchner, T V Hippel, A Burrow, F Mullally, M Kilic and D E Winger, *Astrophys. J.* **635**, L161 (2005)
[17] D Kremp, T Bornath, M Bonitz and M Schlanges, *Phys. Rev.* **E60**, 4725 (1999)
[18] F Hass, L G Garcia, J Goedert and G Manfredi, *Phys. Plasmas* **10**, 3858 (2003)
[19] P Chatterjee, B Das, G Mondal, S V Muniandy and C S Wong, *Phys. Plasmas* **17**, 1 (2010)
[20] W Masood, A Mushtaq and R Khan, *Phys. Plasmas* **14**, 123702 (2007)
[21] Y Nakamura, H Bailung and P K Shukla, *Phys. Rev. Lett.* **83**, 1602 (1999)
[22] P K Shukla and S Ali, *Phys. Plasmas* **12**, 114502 (2005)
[23] P K Shukla and L Stenflo, *Phys. Lett* **A355**, 378 (2006)
[24] S Ali and P K Shukla, *Eur. Phys. J.* **D41**, 319 (2007)
[25] S Ali and P K Shukla, *Phys. Plasmas* **13**, 022313 (2006)
[26] P K Shukla, *Phys. Lett.* **A352**, 242 (2006)
[27] W F El-Taibany and M Wadati, *Phys. Plasmas* **14**, 042302 (2007)
[28] W M Moslem, P K Shukla, S Ali and R Schlickeiser, *Phys. Plasmas* **14**, 042107 (2007)
[29] P Chatterjee, K Roy, S V Muniandy and C S Wong, *Phys. Plasmas* **16**, 112106 (2009)
[30] P Chatterjee, K Roy, G Mondal, S V Muniandy and S L Yap, *Phys. Plasmas* **16**, 122112 (2009)
[31] N J Zabusky and M D Kruskal, *Phys. Rev. Lett.* **15**, 240 (1965)
[32] C H Su and R M Mirie, *J. Fluid Mech.* **98**, 509 (1980)
[33] C S Gardner, J M Greener, M D Kruskal and R M Miura, *Phys. Rev. Lett.* **19**, 1095 (1967)
[34] J K Xue, *Phys. Rev.* **F69**, 016403 (2004)
[35] J N Han, X X Yang, D X Tian and W S Duan, *Phys. Lett.* **A372**, 4817 (2008)
[36] J N Han, X X Yang, S C Li and W S Duan, *Eur. Phys. J.* **D47**, 197 (2008)
[37] E F EL-Shamy, *Eur. Phys. J.* **D56**, 73 (2010)
[38] E F EL-Shamy, W M Moslem and P K Shukla, *Phys. Lett.* **A374**, 290 (2009)
[39] P Chatterjee, U N Ghosh, K Roy, S V Muniandy, C S Wong and B Sahu, *Phys. Plasmas* **17**, 122314 (2010)
[40] E F El-Shamy, *Phys. Plasmas* **16**, 113704 (2009)
[41] E F El-Shamy, R Sabry, W M Moslem and P K Shukla, *Phys. Plasmas* **17**, 082311 (2010)
[42] M Akbari-Moghanjoughi, *Phys. Plasmas* **17**, 072101 (2010)
[43] S K El-Labany, E F EL-Shamy, W F El-Taibany and P K Shukla, *Phys. Lett.* **A374**, 960 (2010)
[44] Yan-Xia Xu, Zing-ming Liu, Mai-Mai Lin, Yu-Ren Shi, Jian- Min Chen and Wen-Shan Duan, *Phys. Plasmas* **18**, 052301 (2011)