

Mass shift of σ -meson in nuclear matter

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Abstract. The propagation of σ -meson in nuclear matter is studied in the Walecka model, by assuming that the sigma couples to a pair of nucleon–antinucleon states and to particle–hole states. The in-medium effect of σ – ω mixing is also studied. For completeness, the coupling of sigma to two virtual pions was also considered. It is found that the σ -meson mass decreases with respect to its value in vacuum and that the contribution of the σ – ω mixing effect on the mass shift is relatively small.

Keywords. Scalar mesons; hadrons in dense matter; spectral function; dense nuclear matter.

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1. Introduction

The study of matter under extreme conditions of density and temperature has become very important because it prepares one to understand the physics for some interesting topics, such as the conditions in the early Universe, the physics of processes in stellar evolution and in heavy-ion collision. Particularly, the study of properties of mesons in hot and dense matter is significant for comprehending the possible signature for detecting the quark-gluon plasma (QGP) state in heavy-ion collision, as well as to obtain information about the signal of the presence of QGP and also to know which symmetries are restored [1].

The study of meson propagation in a medium is essential to interpret the properties of nuclear matter [2]. In this work, we use the Walecka model to investigate the σ -meson propagation in nuclear matter. The Walecka model is a renormalizable relativistic quantum field theory for nuclear matter where the degrees of freedom are nucleons

that interact through the exchange of scalar and vector mesons: the sigma and the omega, respectively [3,4]. This model has proved to be very successful in explaining several processes in nuclear theory [5].

Much studies were done on hadrons in dense nuclear matter in the last few years. One of the interesting phenomena of nuclear physics that has been recently explained, fitting the data by the shift-down hypothesis of the meson masses in nuclear matter, is the anomalous long lifetime of the C-14 nucleus [6].

One of the phenomenological reasons to include the σ -meson as a $\bar{q}q$ degree of freedom is the fact that the linear sigma model has been very successful in reproducing the data for many processes in πN and $\pi\pi$ interactions. On the other hand, theoretical predictions and models which have been successful require the existence of the σ -meson [7]. It plays a fundamental role in understanding the symmetry-breaking mechanism in QCD and also in the confinement problem [8].

The existence of σ -meson with a mass of about 500 MeV has been the subject of a long-standing controversy. However, recently it was announced that the σ -meson has been detected in $\pi\pi \rightarrow \pi\pi$ processes and also in the decay products of heavy quark systems [9]. Thus, after the experimental announcement of the existence of σ -meson, the controversy is not about its existence but about its structure [9]. The composition of σ -meson as a $\bar{q}q$ -meson or as a $\pi\pi$ resonant particle is still a subject of debate [10–13]. We assume in this paper, that the σ -meson is a two-pion resonance, according to the Particle Data Group Collaboration [14].

It is still an open question whether σ is a fundamental particle, or a dynamically generated particle. Today, the discussion about σ has moved from the speculation about its existence, to its structure or dynamics [7].

The relevance of σ -meson in hadron and nuclear physics can be seen from the following phenomena [15]: (a) The σ -meson is responsible for the attractive part, in the intermediate range, of the nuclear force [4,16]. Without this force, we cannot even explain the existence of nuclei, the bound state of nucleons. (b) The σ -meson participates as an intermediate particle in several nuclear processes and in pion–nucleon scattering [16,17]. (c) In meson–meson interaction, the σ -meson occurs as a resonance, showing up as a pole in the T -matrix [16,18,19]. In the sigma model, developed by Gell-Mann and Levy [16,20], the σ -meson plays a role similar to the Higgs particle in the standard electroweak theory [16,20,21], giving mass to the nucleon when the $SU(2)_L \otimes SU(2)_R$ symmetry is spontaneously broken. Additionally it can be proved, from the quark structure, that the σ -meson is the chiral partner of the pion [22].

According to ref. [9], the identification of σ -meson was realized in the experiment $D^+ \rightarrow \pi^- \pi^+ \pi^+$. The experimental results of this three-body decay of charm meson can be fitted by assuming an intermediate scalar–isoscalar state which can be identified with the σ -meson.

In this work we study the scalar σ -meson in nuclear medium when the meson couples to two nucleons through a Yukawa-type coupling. The interaction of the meson with nucleons includes interactions with the Fermi sea producing nucleon–hole states and interactions with the Dirac sea producing nucleon–antinucleon states [3,4]. The σ – ω mixing is also considered in the calculations.

We have associated the mass of sigma to the peak position of its spectral function. The σ -meson self-energy is calculated in the one-loop level and the propagator is computed by

summing over ring diagrams, in the so-called random phase approximation (RPA) [23]. To carry out the summation, we use the Dyson's equation.

2. Formalism

Starting from the Walecka model, we study the possible modifications of the σ -meson mass in the nuclear matter. For this purpose, interactions of σ -meson with particle-hole states, nucleon-antinucleon excitations in the Dirac sea, and the σ - ω mixing effect [3-5] were considered. The σ - ω mixing is a purely density effect which occurs when a particle-hole pair is excited in the medium, and, as presented in figure 1, the scalar meson decays into a longitudinal ω -meson. After this, the inverse process send it back to the original σ -meson. In ref. [24], the authors studied the mixing effect in dilepton production in relativistic heavy-ion experiments.

The study of meson propagation in nuclear medium requires the inclusion of the σ - ω mixing effect. In order to evaluate the mass of σ -meson, we need, firstly, to calculate its dressed propagator. The mixing effect results in the coupling between the Dyson's equation for the sigma and omega propagators. The expression for the full σ - ω propagator can be written as

$$i \Delta(k) = i \Delta_0(k) + i \Delta_0(k) [-i \Sigma(k)] i \Delta(k), \quad (1)$$

where $i \Delta_0(k)$ is the non-interacting σ -meson propagator and is written as [3,5]

$$i \Delta_0(k) = \frac{i}{k^2 - (m_\sigma^0)^2 + i\epsilon} \quad (2)$$

and $\Sigma(k)$ is the σ -self-energy. The self-energy $\Sigma(k)$ contains all the information about the interaction between the σ -meson with nucleons and with ω -mesons in nuclear matter. In order to determine the self-energy $\Sigma(k)$, we must specify the dynamical content of our model.

We start with the Lagrangian density for the Walecka model, which is given as [4]

$$L = \bar{\Psi}[\gamma_\mu(i\partial^\mu - g_v V^\mu) - (M - g_s \Phi)]\Psi + \frac{1}{2}(\partial_\mu \Phi \partial^\mu \Phi - (m_\sigma^0)^2)\Phi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu + \delta L, \quad (3)$$

where Ψ is the nucleon field with mass M , V^μ is the neutral vector meson field (ω) with mass m_v , the tensor field of the vector meson is $F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$, Φ is the scalar (σ) meson field with the bare mass m_σ^0 ; g_s and g_v are the coupling constants, and finally δL contains renormalization counter terms.

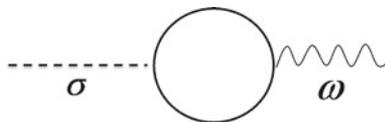


Figure 1. Sigma-omega mixing diagram.

The interaction Lagrangian density which describes the σ - π dynamics [25], is given by

$$L_{\sigma\pi\pi}^{\text{eff}} = \frac{1}{2} g_{\sigma\pi\pi} m_\pi \vec{\pi} \cdot \vec{\pi} \Phi. \tag{4}$$

The influence of the interaction of σ -mesons with nucleons and ω -mesons in nuclear matter is introduced through the modification of the free propagator in the one-loop approximation. We shall calculate the full propagator in the chain approximation, which consists of an infinite summation of the one-loop self-energy diagrams [5]. The diagrammatical representation of the modified propagator is shown in figure 2, and the analytical expression is given by $i\Delta(k)$.

The solution for $\Delta(k)$ in the Dyson's eq. (1) is given by

$$\Delta(k) = \frac{1}{\Delta_0^{-1}(k) - \Sigma(k)} = \frac{1}{k^2 - (m_\sigma^0)^2 - \Sigma(k)}. \tag{5}$$

The analytical expression for the self-energy $\Sigma(k)$, is given as [25,26]:

$$\Sigma(k) = \Sigma_{\sigma\pi}(k) + \Sigma_{\sigma N}(k) + \Sigma_\mu^m(k), \tag{6}$$

where the calculation of $\Sigma_{\sigma\pi}(k)$ and $\Sigma_{\sigma N}(k)$ are given in ref. [16] and the self-energy of the σ - ω mixing is given by

$$i\Sigma_\mu^m(k) = i g_s g_v \int \frac{d^4q}{(2\pi)^4} \text{Tr}[G(q)\gamma_\mu G(q+k)]. \tag{7}$$

The scalar-vector mixing effect between σ and ω is a purely medium effect, occurring when a longitudinal ω -meson produces a particle-hole state which decays into a σ , or vice versa. $G(q) = G_F(q) + G_D(q)$ is the full nucleon propagator [4]:

$$G(q) = (\gamma_\mu q^{*\mu} + M^*) \left[\frac{1}{q^{*2} - M^{*2} + i\varepsilon} + \frac{i\pi}{E^*(q)} \delta(q_0 - E^*(q)) \Theta(k_F - |\vec{q}|) \right], \tag{8}$$

where $q^{*\mu} \equiv (q^0 - g_v V^0, \vec{q})$, $E^*(q) \equiv \sqrt{\vec{q}^2 + M^{*2}}$, k_F is the Fermi momentum and M^* is the effective mass of the nucleon.

We take as the definition of the mass of a particle, the magnitude of the four-momentum K for which the spectral function gets its maximum, a definition well-established and used

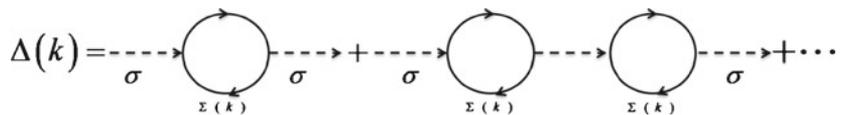


Figure 2. Sigma-meson propagator.

extensively in literature [27–29]. For this reason, this definition is utilized in this work for calculating the meson mass in nuclear matter.

3. Calculation of the σ - ω mixing term

Integrating eq. (8) with respect to q^0 we obtain

$$\Sigma_\mu^m(k) = 8\pi g_s g_v \int \frac{d^3q}{(2\pi)^4} \frac{[4k^\mu q^2 + 4k \cdot q q^\nu] \Theta(k_F - |\vec{q}|)}{[(k^2)^2 - 4(q \cdot k)^2] E^*(q)}. \quad (9)$$

On the other hand, as the baryonic current is conserved, it follows that

$$k^\mu \Sigma_\mu^m(k) = 0. \quad (10)$$

Taking the condition as for low energies [3] and taking \vec{k} in the positive direction of the z -axis, we find

$$\Sigma_\mu^{Rm}(k) = \frac{g_s g_v M^*}{\pi^2} \left[2k_F - C_0 \varepsilon_F \ln \left| \frac{C_0 \varepsilon_F + k_F}{C_0 \varepsilon_F - k_F} \right| \right], \quad (11)$$

where

$$C_0 = \frac{k_0}{|\vec{k}|} \quad \text{and} \quad \varepsilon_F \equiv \sqrt{k_F^2 + M^{*2}}.$$

The propagator takes the form

$$\Delta(k^2) = \frac{1}{k^2 - m_\sigma^2 - \text{Re}\Sigma^R(k^2) - i\text{Im}\Sigma^R(k^2)} \quad (12)$$

with

$$\text{Re}\Sigma^R(k) = \text{Re}[\Sigma_{\sigma\pi}^R(k) + \Sigma_{\sigma N}^R(k) + \Sigma_\mu^{Rm}(k)] \quad (13)$$

$$\text{Im}\Sigma(k) = \text{Im}\Sigma_{\sigma\pi}(k). \quad (14)$$

The parameters in eq. (12) are the reported m_σ value for the σ -meson mass, which we take in the range $450 \text{ MeV} < m_\sigma < 550 \text{ MeV}$, the bare $\sigma\pi\pi$ coupling constant $g_{\sigma\pi\pi} = 12.8$, the bare σN coupling constant $g_s^2 = 54.289$ and the bare ωN coupling constant $g_v^2 = 102.770$ [5].

The spectral function of the σ -meson at twice the normal nuclear matter density is plotted in figure 3. The nucleon mass was fixed at the physical value $M = 939 \text{ MeV}$. The effective nucleon mass M^* is the appropriate value at nuclear-matter saturation density. This relation was taken as $M^*/M = 0.730$, and the bare σN coupling constant $g_s^2 = 54.289$ [5].

As shown in figure 3, the mass of σ -meson in a medium suffers a down-shift with respect to its mass in vacuum. It is worth saying that there is no unique result concerning the shift mass of mesons in medium. In ref. [2] they obtained an increase in the effective mass of the σ -meson. On the other hand, authors who have used different models such

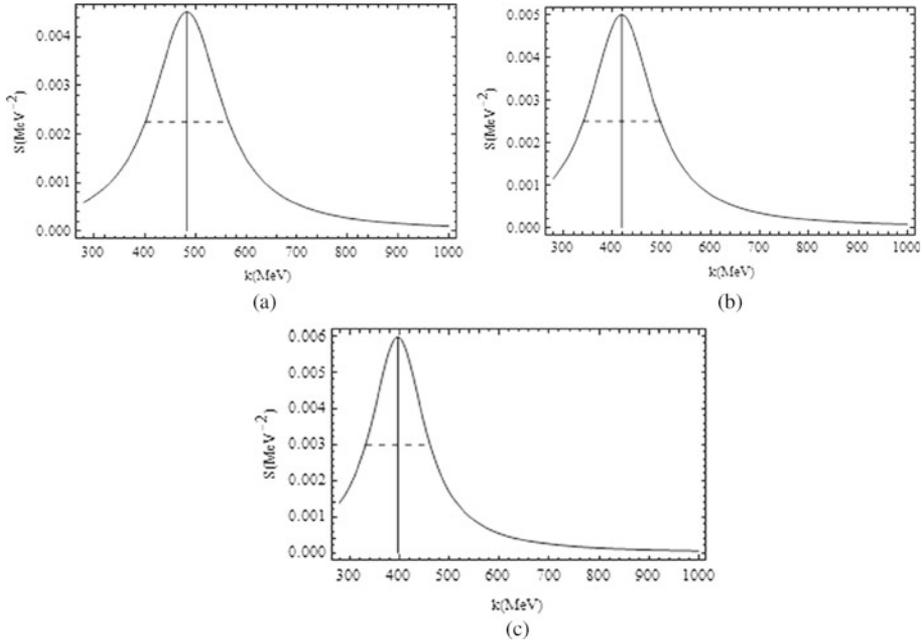


Figure 3. Spectral function of the σ -meson at twice the normal nuclear matter: (a) in vacuum, (b) without mixing effect and (c) with mixing effect.

as Nambu–Jona–Lasinio or QCD sum rules, have reported a reduction of the σ -meson effective mass in medium.

4. Conclusion

In the study of the propagation of scalar σ -meson in nuclear matter, we have found that the effective mass of the meson decreases with respect to its value in vacuum. Assuming that the σ -meson couples to a particle–hole pair in nuclear matter, that it interacts with nucleon–antinucleon states in the Dirac sea, and also by including, for completeness, the coupling of sigma to two pions, we obtained 13% reduction in the value of the σ -meson mass without considering the scalar-vector mixing and 18% reduction by considering the mixing effect. The mixing effect between σ -mesons and ω -mesons occurs only in a medium, when a longitudinal ω -meson produces a particle–hole state which decays into a σ -meson, or vice versa. When this σ – ω mixing is included, we found that the effect is small on the effective mass of the σ -meson. The effective mass of σ -meson when the σ – ω mixing effect is included, decreases in a small amount, a difference of 5% at twice the normal nuclear matter density. These results are in agreement with those obtained by using other models, such as the Nambu–Jona–Lasinio [30,31] and the chiral perturbation theory [18,19], which predict a decrease in the mass of the σ -meson in a medium. This result can be interpreted as a partial restoration of chiral symmetry [32,33]. On the other

hand, by taking the width at one half of the maximum value of the spectral function, a reduction of 23% with respect to the width in the vacuum is obtained.

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