PRAMANA
— journal of
physics

© Indian Academy of Sciences

Vol. 80, No. 3
March 2013
pp. 429–437

Is quantum theory compatible with special relativity?

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MS received 19 July 2011; revised 26 September 2012; accepted 8 October 2012

Abstract. How a proposed quantum nonlocal phenomenon could be incompatible with the requirements of special relativity is studied. To show this, the least set of assumptions about the formalism and the interpretation of non-relativistic quantum theory is considered. Then, without any reference to the collapse assumption or any other stochastic processes, an experiment is proposed, involving two quantum systems, that interacted at an arbitrary time, with results which seem to be in conflict with requirements of special relativity.

Keywords. Quantum non-locality; no-signalling theorems.

PACS Nos 03.30.+p; 03.65.−w; 03.65.Ta

1. Introduction

Although the Schrödinger equation is characteristically non-relativistic, the non-relativistic quantum theory instrumentally complies with the requirement of special relativity (SR) in very peculiar ways. That is, those counterintuitive non-local phenomena predicted by non-relativistic quantum mechanics (e.g., collapse of wave function [1,2], non-local quantum correlations [3–15], the instantaneous spreading of a compact wave function [16], Aharonov–Bohm effect [17], and protective measurements [17]) do not imply the practical possibility of sending a true signal faster than the speed of light in vacuum. However, it is not trivial to show where those non-local proposals for the superluminal communication fail. In general, different arguments have been presented for each distinctive case. In the following, we review this issue in detail. Briefly, the superluminal communication schemes break down either due to the violation of rules of non-relativistic dynamics (in particular due to no-signalling theorems [3–15]), or because

DOI: 10.1007/s12043-012-0487-y; *e*Publication: 7 March 2013

they include arbitrary energies greater than the relativistic limit (e.g., see [17]), or because of the unavoidable stochastic noise processes [18].

Following the celebrated Bell's theorem [19], there is also a conceptually different approach to investigate the compatibility of a non-local theory (including quantum theory) with SR, which would not be pursued in this paper. In this approach, the aforementioned compatibility is discussed by analysing whether the locality is violated because of a violation of parameter independence or of outcome independence [20].

There are some arguments against superluminal signalling via quantum correlations which are usually referred to as quantum no-signalling theorems [3–13]. They argue that there are some implicit principles that constrain quantum non-locality in ways that finally prevent the superluminal communication schemes via quantum correlations. They assume that quantum non-locality can be demonstrated in a direct way, only if the local operations have observable effects (i.e., changing the probability distribution) in causally disjoint (i.e., space-like) regions. Hence, some of the no-signalling theorems show that for the schemes to communicate faster than light, they must involve the processes incompatible with the principles of quantum theory, e.g., the cloning of an unknown quantum state which is prevented by the linearity of quantum dynamics. Others demonstrate that the superluminal communication demands the statistical interdependence of spatially-separated subsystems of a composite system. These no-signalling theorems refute such statistical interdependency by considering the tensor-product formulation of composite systems and the unitary temporal evolutions. On this account, the outcomes of any measurement on one subsystem cannot be affected by operations performed on the other disjoint subsystem, and thus quantum correlations cannot make possible the transmission of signals of any kind (including the superluminal ones, too). However, there are objections claiming that the logical reasoning of these kinds of no-signalling theorems are 'question-begging' or even 'circular' [14,15].

Among other examples illustrating the non-local features of non-relativistic quantum theory, we could recall the collapse of wave function in the measurement process [20a] (e.g., see [2]). Although the mysterious reduction of wave function has sparkled off lots of heated discussions about the interpretation of quantum theory, it cannot be used for superluminal signalling due to the stochastic nature of the collapse. In general, it has been shown that proposed scenarios for the superluminal quantum communication involve stochastic processes that eventually cancel out the true sending of faster-than-light signal [18]. Nevertheless, the non-local features of the collapse can be particularly highlighted in the measurement of non-local variables [21]. However, by exploiting quantum entanglement, Aharonov and Albert introduced a framework to measure the non-local variables without contradicting the requirements of SR [22].

The free propagation of a compact wave function might also result in superluminal anomaly. If a wave function is non-zero only in a bounded region V_0 at the time $t = 0$, then by free evolution at all later times $t > 0$, its amplitude is non-zero even outside the light-cone of V_0. However, this superluminal propagation in non-relativistic quantum theory is usually considered as being 'of no great concern' [16]. To our best knowledge, the reason why this superluminal propagation in non-relativistic quantum theory has no importance is either due to the fact that the preparation of a strictly localized state requires the existence of infinite potentials, or because the amplitude to propagate outside the light-cone of V_0 is extremely small [22a]. However, one cannot argue that the non-relativistic nature of Schrödinger equation is at fault here, because only the few assumptions of "Hilbert space and positivity of the energy" are enough for obtaining the superluminal results [16]. A careful analysis by quantum field theory solves this problem, by introducing negative energy states and antimatters [22b, 23]. Hence, at least non-relativistic predictions (i.e., extremely small superluminal tails) closely approach implications of the relativistic quantum theory.

In non-relativistic quantum theory, we can fire a particle superluminally (e.g., there is no upper bound for the speed of a wavepacket's centroid), but obviously in this case we are far above the energy limit where the non-relativistic dynamics is expected to be valid. In addition, the superluminal signalling schemes can be made possible by protective measurement too [17], but they include arbitrary potentials greater than the rest mass of particles, which is clearly the relativistic limit where the non-relativistic dynamics is not expected to be valid.

Accordingly, in general, we may summarize the compatibility between the quantum non-local features with the requirements of SR as follows:

(1) The quantum non-local features are trivial because they include, e.g., relativistic energies.
(2) The superluminal proposals using quantum non-local features are refuted by laws of quantum physics, e.g., because they contradict the linearity of the dynamics.

Nevertheless, in this work we show that even in non-relativistic quantum regime, one can find situations in which the predictions of quantum theory go beyond the requirement of SR. We explicitly propose an experiment (without involving the stochastic processes such as collapse) in which an upper bound for all the involved energies and interactions is coherently imposed. It is straightforward to show that our experiment is far beyond the relativistic energy limit, and so the scope is deliberately limited to the non-relativistic energies. Finally, we argue that the results of this experiment seem to be in conflict with the requirement of SR. However, in contrast to the instantaneous spreading of a compact wave function (in which the non-relativistic results closely approach the results implied by SR), here the non-relativistic predictions are substantially different from the implications of SR.

The structure of this paper is as follows. In §2, the experiment used in this study is elaborated. In §3, some constraints into our experiment are introduced in order to keep all energies and interactions much less than the relativistic energy limit. Finally, in §4, the predicted results of our experiment have been compared in detail with the requirement of SR.

2. Description of the experiment

We have two one-dimensional harmonic oscillators of the same mass M whose frequencies of oscillations are ω_1 and ω_2, respectively. Their Hamiltonians are given as:

$$\hat{H}_1 = \hbar\omega_1(\hat{a}^\dagger\hat{a} + 1/2), \quad \hat{H}_2 = \hbar\omega_2(\hat{b}^\dagger\hat{b} + 1/2), \tag{1}$$

where $\hat{a}$ and $\hat{b}$ are annihilation operators of the first and second system, respectively. The observer can decide whether these systems remain interacting, or not interacting. The interaction between these two systems is assumed to be:

$$\hat{H}_{\rm int} = \hbar\chi(t)\,\hat{a}^\dagger\hat{a}({\rm e}^{-it\omega_2}\hat{b}^\dagger + {\rm e}^{it\omega_2}\hat{b}), \tag{2}$$

where $\chi(t)$ is the time-dependent coupling between two systems, which is only effective within the interval $0 \le t \le \tau$:

$$\int_{-\infty}^{+\infty}\chi(t){\rm d}t = \int_0^\tau \chi(t){\rm d}t = \chi_\tau. \tag{3}$$

The interaction in eq. (2) will couple the momentum of the second system with the energy of the first system. This interaction is also very appropriate for other purposes, which will be described in detail. Accordingly, the total Hamiltonian of the composite system is described by $\hat{H} = \hat{H}_1 + \hat{H}_1 + \hat{H}_{\rm int}$. Then, the interaction Hamiltonian in the interaction picture is given by

$$\hat{H}_{\rm I}(t) = \hbar\chi(t)\,\hat{a}^\dagger\hat{a}(\hat{b}^\dagger + \hat{b}). \tag{4}$$

On this account, the corresponding unitary time-evolution operator is

$$\hat{U}_{\rm I}(t) = \exp\left[-i\left(\int_0^t \chi(t'){\rm d}t'\right)\hat{a}^\dagger\hat{a}(\hat{b}^\dagger + \hat{b})\right]. \tag{5}$$

At the time $t = 0$, the two systems are prepared in coherent states as follows:

$$|\alpha\rangle_1 = \hat{D}_a(\alpha)|0\rangle = {\rm e}^{-\alpha^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle_1,$$
$$|\alpha\rangle_2 = \hat{D}_b(\alpha)|0\rangle = {\rm e}^{-\alpha^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle_2, \tag{6}$$

where $\hat{D}_a(\alpha) = \exp[\alpha\hat{a}^\dagger - \alpha^*\hat{a}]$ is the displacement operator, $|\alpha\rangle_1$ describes the state of the first system and $|\alpha\rangle_2$ the state of the second system. Here $\alpha > 0$ is assumed to be real. Now, considering eqs (5) and (6), the state of composite system in the interaction picture, at the time t, is described by

$$\begin{aligned}|\psi_{\rm I}(t)\rangle &= \hat{U}_{\rm I}(t)\,(|\alpha\rangle_1|\alpha\rangle_2)\\ &= {\rm e}^{-\alpha^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle_1\exp\big[(-in\chi_t)\hat{b}^\dagger - (-in\chi_t)^*\hat{b}\big]|\alpha\rangle_2\\ &= {\rm e}^{-\alpha^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle_1\hat{D}_b(-in\chi_t)\hat{D}_b(\alpha)|0\rangle_2\\ &= {\rm e}^{-\alpha^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n{\rm e}^{-in\alpha\chi_t}}{\sqrt{n!}}|n\rangle_1\hat{D}_b(\alpha - in\chi_t)|0\rangle_2\\ &= {\rm e}^{-\alpha^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n{\rm e}^{-in\alpha\chi_t}}{\sqrt{n!}}|n\rangle_1|\alpha - in\chi_t\rangle_2,\end{aligned} \tag{7}$$

where $\chi_t = \int_0^t \chi(t')\mathrm{d}t'$. For the inner product of the displaced vacuum states $|\alpha - in\chi_t\rangle$ and $|\alpha - im\chi_t\rangle$, we have [18]

$$\langle\alpha - im\chi_t|\alpha - in\chi_t\rangle = \exp\left[i(m-n)\chi_t - \chi_t^2(m-n)^2/2\right]. \tag{8}$$

Now suppose that at the time $t = \tau$ the following relation is satisfied:

$$\chi_\tau = \int_0^\tau \chi(t)\mathrm{d}t \gg 1 \tag{9}$$

which says that the initial momentum dispersion of the second system is much smaller than the least induced momentum due to the interaction. With this assumption in force, at the time $t = \tau$ the inner product (9) becomes

$$\langle\alpha - im\chi_t|\alpha - in\chi_t\rangle \simeq \delta_{m,n}. \tag{10}$$

3. Non-relativistic conditions

We require all of the involved energies (e.g., energy of the individual systems, interaction energy, potentials etc.) to be negligible in comparison with their rest mass [23a]. Otherwise, relativistic effects such as particle creation and annihilation become important. In other words, we want to avoid any trivial non-local effects, similar to the ones discussed in §1. Consequently, in order to work within the realm of the non-relativistic quantum theory, the following relation should also be fulfilled:

$$Mc \gg \sqrt{\langle p^2\rangle} \geq \Delta p \Rightarrow Mc \gg \Delta p, \tag{11}$$

where Δp is the dispersion in momentum of the system. Regarding the Heisenberg uncertainty relation, a minimum amount of dispersion in the position of the particle can be calculated as follows:

$$\Delta x \gg \frac{\hbar}{2Mc}, \tag{12}$$

where $\hbar/Mc$ is the Compton wavelength of the particle. In other words, since we are working in the domain of non-relativistic quantum theory, we cannot compress a wavepacket to an arbitrary small volume less than its Compton wavelength. With these assumptions in force, we have bounded the velocity of the wavepacket (i.e., velocity of the probability flux), and also the speed of its spreading, by the speed of light in vacuum. Now we introduce the aforementioned conditions into our systems.

We first assume the initial position dispersions of systems to be much smaller than the corresponding Compton wavelengths. The initial states are Gaussian wavepackets, centred around the position $x_i = \alpha\sqrt{2\hbar/M\omega_i}$, with the dispersion $\Delta x_i = \sqrt{\hbar/2M\omega_i}$ where $i = 1, 2$. Let us suppose $\alpha \gg 1$. Combining this with eq. (12), it yields

$$\alpha\sqrt{\frac{2\hbar}{M\omega_i}} \gg \Delta x_i = \sqrt{\frac{\hbar}{2M\omega_i}} \gg \frac{\hbar}{2Mc}. \tag{13}$$

We also suppose that the initial energies of these systems are much smaller than their rest energies

$$\frac{Mc^2}{\hbar\omega_1} \gg \sqrt{\langle(\hat{a}^\dagger\hat{a})^2\rangle} = \sqrt{\alpha^4+\alpha^2}; \quad \frac{Mc^2}{\hbar\omega_2} \gg \sqrt{\langle(\hat{b}^\dagger\hat{b})^2\rangle} = \sqrt{\alpha^4+\alpha^2}. \tag{14}$$

For the interaction energy, we require that the overall interaction energy should not exceed the relativistic limit, that is,

$$Mc^2 \gg \frac{1}{\tau}\int_0^\tau \mathrm{d}t\sqrt{\langle \hat{H}_I^2(t)\rangle} = \frac{\hbar\chi_\tau}{\tau}\sqrt{(1+4\alpha^4)(\alpha^4+\alpha^2)}. \tag{15}$$

We also assume that the induced momenta on the second system remain in the non-relativistic limit. So, regarding eqs (7), (8) and (10), we require the average of the final momentum and also its spreading to be much smaller than the relativistic limit:

$$\frac{Mc^2}{\hbar\omega_2} \gg \chi_\tau^2\alpha^4; \quad \frac{Mc^2}{\hbar\omega_1} \gg \chi_\tau^2\alpha^4. \tag{16}$$

4. Comparison with the requirements of SR

We have managed the setting of the above-mentioned experiment in ways that any non-local anomaly cannot be attributed to the trivial relativistic effects. Then, in this section we shall compare the predicted results with the requirements of SR. We should mention that we have taken the relativistic considerations into account concerning the scale of the distances between the disjoint regions of space.

With regard to the first system, we consider the situation as depicted in figure 1, in which V is the region between $\alpha\sqrt{(2\hbar/M\omega_1)} - 20\Delta x_1$ and $\alpha\sqrt{(2\hbar/M\omega_1)} + 20\Delta x_1$ and V' is the space interval on the left side of the origin

$$V: 2(\alpha+10)\Delta x_1 \geq x_1 \geq 2(\alpha-10)\Delta x_1, \tag{17}$$

$$V': 0 \geq x_1. \tag{18}$$

We assume $\alpha \gg 10$. Thus, the two regions have no overlap. Since the initial state of the first system (eq. (6)) is a normal Gaussian distribution, at the time $t = 0$ the probability of finding the first system out of V is given by $1-\mathrm{erf}(20/\sqrt{2})$, where 'erf' is the error

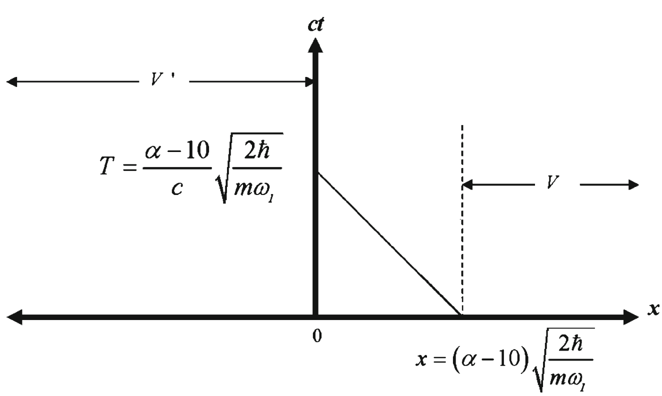

Figure 1. Space-time description of regions V and V'.

function. Hence, this quantity is less than 10^{-87}. Likewise, the probability of finding the first system in V' at time $t = 0$ is

$$P(V', t = 0) = \int_{-\infty}^{0} |\langle x_1|\alpha\rangle|^2 \mathrm{d}x_1 = \frac{1}{\sqrt{2\pi}(\Delta x_1)} \int_{-\infty}^{0} \mathrm{e}^{-(1/2\Delta x_1)(x_1 - 2\alpha\Delta x_1)^2} \mathrm{d}x_1 \tag{19}$$

which is rigorously close to zero because $\alpha \gg 10$. For example, in the case $\alpha = (\hbar c/e^2) \sim 137$, the order of magnitude of $P(V', t = 0)$ is less than 10^{-10^4}.

Let us suppose that at the time $t = \tau$, the region V' is outside the space-time domain of the region V, that is,

$$2(\alpha - 10)\Delta x_1 \geq c\,\tau. \tag{20}$$

Our analysis in the previous section shows that there are no quantum mechanical limitations on τ other than the one described in eq. (9). Substituting eq. (20) into eq. (15) yields

$$\frac{Mc^2}{\hbar\omega_1} \gg \frac{2\chi_\tau^2(1 + 4\alpha^2)(\alpha^4 + \alpha^2)}{(\alpha - 10)^2}. \tag{21}$$

Now, we can consistently introduce conditions (20), (21) into constraints (9), (13)–(16), and show that all of them are in agreement for many sets of parameters in the regimes in which the vibrational amplitude of the second system is much smaller than the wavelength of the first system, e.g.

$$\frac{Mc^2}{\hbar\omega_1} = 10^{18}; \quad \frac{Mc^2}{\hbar\omega_2} = 10^{10}; \quad \alpha = \chi_\tau = \frac{\hbar c}{e^2} \simeq 137. \tag{22}$$

SR imposes that the probability flux should have a finite propagation speed, bounded by the speed of light. Thus, the probability of finding a system in one space-time region cannot raise unless the probability flux travelling or spreading at the speed of light (or less) can reach this region from other regions. Since at the time τ the region V' is outside the forward light-cone of V (refer to figure 1), the probability of finding the first system at the time τ in V' should be less than or equal to that of finding it outside V at the time $t = 0$. In other words, we expect

$$1 - P(V, t = 0) \geq P(V', t = \tau). \tag{23}$$

Therefore, the probability of finding the first system in V' at the time τ should not exceed 10^{-87}. This is the value that we expect if the prediction of non-relativistic quantum theory is compatible with the requirement of SR. However, combining the orthogonality condition in eq. (10) with eq. (7), the probability of finding the first system in V' at the time τ is given by

$$P(V', \tau) = \mathrm{e}^{-\alpha^2} \sum_{n=0}^{\infty} \frac{\alpha^{2n}}{n!} \int_{-\infty}^{0} |\langle x_1|n\rangle|^2 \mathrm{d}x_1. \tag{24}$$

The functions $\langle x_1|n\rangle$ are the normalized eigenfunctions of harmonic oscillator. Taking into the account the parity properties of these normalized eigenfunctions, we have

$$\int_{-\infty}^{0} |\langle x_1|n\rangle|^2 \mathrm{d}x_1 = \int_{0}^{\infty} |\langle x_1|n\rangle|^2 \mathrm{d}x_1 = \frac{1}{2}. \tag{25}$$

Introducing eq. (25) into eq. (24) yields

$$P(V', \tau) = \frac{1}{2}\mathrm{e}^{-\alpha^2}\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{n!} = \frac{1}{2}. \tag{26}$$

5. Discussion and concluding remarks

There are clear arguments as to why the quantum non-locality in the collapse of wave function or in the quantum correlations cannot lead to any contradiction with the requirement of SR [2–15,17]. However, in our experiment, we basically manipulate neither the quantum correlation nor any other stochastic quantum processes such as collapse process. Our analysis has been done with a minimal set of assumptions about the formalism and the interpretation of quantum theory, without any reference to the collapse assumption. First, a quantum mechanical pure state of the two systems was described by a unit vector $|\alpha\rangle_1|\alpha\rangle_2$ in a Hilbert space $\mathcal{H}$, where $\mathcal{H}$ is the tensor product of individual Hilbert spaces $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. Second, the time evolution of the state vector is given by the Schrödinger equation. Third, Born's probabilistic rule is assumed. With these assumptions in force, we have shown that the results of measurement in one region of space (V) can be changed dramatically due to some sort of operations in a causally disjoint region (V'). In other words, the results of our proposed experiment (which are consistently taken from non-relativistic quantum theory) are in conflict with the requirement of SR.

Because our results are contracting the requirements of SR, one might argue that we are in a relativistic regime and thus we cannot use the Schrödinger equation. Two important issues within this line of reasoning are in order. First, in this reasoning our results are refuted not because of any conflict with predictions of quantum theory (in contrast with other proposals discussed in §1), but because they are in conflict with the requirements of SR. In other words, the implicit premise is that the special relativity is the guiding principle of nature and any other physical theory should be consistent with it. Second, such reasoning simply claims that the empirically verifiable predictions of quantum theory are wrong. We should also remember that the non-local character of our experiment does not derive from the involvement of supeluminal energies. In other word, we conclude that this new non-local feature is not trivial like some other non-local features discussed in §1. We may conclude that our experiment shows that there is a new kind of non-local feature in non-relativistic quantum theory. To the best of our knowledge, no fundamental features of quantum theory restrict our experiment in a way to reconcile its compatibility with SR. However, some unknown deeper principles underling quantum theory may do so. Accordingly, the connection between the predictions of quantum theory and requirements of SR are far from a common ground and much more theoretical and experimental work is needed to clarify this issue. Our proposed experiment can open a new perspective in this regard.

Acknowledgement

M Bahrami wants to thank Prof. G C Ghirardi of Trieste University, Italy for his valuable comments on this paper.

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