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Plasma excitations in a single-walled carbon nanotube with an external transverse magnetic field

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Abstract. The effect of different uniform transverse external magnetic fields in plasma frequency when propagated parallel to the surface of the single-walled metallic carbon nanotubes is studied. The classical electrodynamics as well as Maxwell's equations are used in the calculations. Equations are developed for both short- and long-wavelength limits and the variations are studied graphically.

Keywords. Plasma frequency; plasma oscillations; transverse magnetic field; short wavelength; long wavelength; dispersion relation.

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1. Introduction

Plasmas resulting from the ionization of neutral gases generally contain equal numbers of positive and negative charge carriers. In this situation, the oppositely charged fluids are strongly coupled, and tend to electrically neutralize one another on macroscopic length scales. Such plasmas are termed quasineutral. Quasineutrality demands that $n_{\mathrm{i}} \approx n_{\mathrm{e}} = n$ where n is the number density (i.e., the number of particles per cubic metre) of the species. The plasma frequency, $\omega_{\mathrm{p}}^2 = n_{\mathrm{e}}^2/\varepsilon_0 m$ is the most fundamental time-scale in plasma physics. Clearly, there is a different plasma frequency for each species. However, relatively fast electron frequency means the electron plasma frequency. It is easily seen that ω_{p} corresponds to the typical electrostatic oscillation frequency of a given species in response to a small charge separation. However, the cold plasma model goes beyond the single-particle description, because it determines the electromagnetic fields self-consistently in terms of the charge and current densities generated by the motions of the constituent particles of the plasma.

Carbon nanotubes (CNTs) were first synthesized in 1991 by Iijima as graphitic carbon needles, ranging from 4 to 30 nm in diameter and up to 1 m in length. CNTs have remarkable electrical and mechanical properties. Collective electron excitations in carbon nanotubes, the so-called plasmon modes, can provide important information about their structural and electronic properties. Using electron-energy-loss spectroscopy, Pichler *et al* [1] experimentally studied the electron excitations in single-walled carbon nanotubes and measured the plasmon energies.

The study of CNTs is now an active area of research, which can lead to the development of advanced technology devices. One of the most fascinating aspects about CNTs is their surface mode excitations. In recent years, many experimental and theoretical works have been done to study the high-frequency excitations (electron oscillations) in these systems. Also, it is well-known that in such systems both the positive ions and the electrons oscillate under low-frequency disturbance.

Fetter [2] used a simple hydrodynamic model to study the electrodynamics of the electron–ion plasma in a periodic array and obtained an acoustic branch in addition to the optical branch. Wei and Wang [3] studied the dispersion relation of quantum ion acoustic wave (QIAW) oscillations in single-walled carbon nanotubes (SWCNTs) with the quantum hydrodynamic (QHD) model which was developed by Haas *et al* [4,5]. In the presence of a static magnetic field, we may expect a new excitation in CNTs, i.e., quantum magnetosonic wave (QMSW) oscillations. Let us note that the effects of a static magnetic field on the plasmon oscillations of an electron gas in CNTs have been investigated by several authors using various methods. Shyu *et al* [6] studied the magnetoplasmon of SWCNTs within the tight-binding model. The low-frequency single-particle and collective excitations of SWCNTs were studied in the presence of a magnetic field by Chiu *et al* [7,8]. Vedernikov *et al* [9] studied the collective oscillations of two-dimensional electrons in nanotubes in the presence of a magnetic field parallel to the tube axis. The energies of neutral and charged excitations and plasmon frequencies of nanotubes as functions of the magnetic field were analysed by Chaplik [10] and Gumbs [11], who calculated the dispersion relation of the collective magnetoplasmon excitations for an electron gas confined to the surface of a nanotube when a magnetic field is perpendicular to its axis. In particular, by using the hydrodynamic model and Maxwell's equations, Kobayashi [12] studied the magnetostatic plasma wave oscillations of a SWCNT in the Voigt configuration. Moradi and Khosrav [13,14] studied the dispersion relations of plasma waves which propagate parallel to the surface of the SWMCNTs. The dispersion relation for the plasma excitation in a SWCNT by the application of external magnetic field for short wavelength limit was explained by Vijayalakshmi *et al* [15].

Here, we are interested in the application of a set of transverse magnetic fields which propagate parallel to the surface of a SWCNT in the cold plasma approximation in short and long wavelength limits and concentrate on the excitations of the electron–ion system as two fluids confined to its surface. A static magnetic field B_0 that is normal to the cylindrical surface is assumed (Voigt configuration).

2. Basic equations

Consider an infinitely long and infinitesimally thin SWCNT with radius a. Take the cylindrical polar coordinate $x = (r, \varphi, z)$ for an arbitrary point in space. Let us consider the

CNT to consist of electron and ion fluids superimposed at $r = a$ with charges e and Ze, respectively.

The equilibrium densities (per unit area) of electrons n_e^0 and of ions n_i^0 satisfy $n_\mathrm{i}^0 = n^0$ and $n_\mathrm{e}^0 = Zn_\mathrm{i}^0 = Zn^0$

$$\frac{\partial n_\mathrm{e}}{\partial t}(x, t) + Zn_0 \nabla_\parallel u_\mathrm{e}(x, t) = 0, \tag{1}$$

$$\beta(x, t)\, \nabla_\parallel u_\mathrm{i}(x, t) = 0, \tag{2}$$

and equation of linearized momentum

$$\frac{\partial u_\mathrm{e}}{\partial t}(x, t) = -e/m\left[E_\parallel(x, t) + u_\mathrm{e}(x, t) \times B_0\right] - \frac{\infty}{zn_0}\nabla_\parallel n_\mathrm{e}(x, t) + \frac{\beta}{Zn_0}\nabla_\parallel\left[\nabla_\parallel^2 n_\mathrm{e}(x, t)\right] \tag{3}$$

$$\frac{\partial u_\mathrm{i}}{\partial t}(x, t) = Zm_\mathrm{e}/m_\mathrm{i}\left[E_\parallel(x, t) + u_\mathrm{i}(x, t) \times B_0\right], \tag{4}$$

where n_e (n_i) is the velocity of electron (ion) and

$$\nabla_\parallel = e^z(\partial/\partial z) + a^{-1}e^\varphi(\partial/\partial\varphi). \tag{5}$$

$E_\parallel$ is the tangential component of the electromagnetic field.

The three terms on the right-hand side of eq. (3) are respectively force due to electric and magnetic fields, force due to internal interaction which can be considered as classical pressure of electron fluid and the third term comes from the quantum diffraction effect which can be considered as quantum pressure.

The electric current density flowing on the surface of the cylinder is given by

$$J_\mathrm{e}(x, t) = \sigma_\mathrm{e} E_\parallel(x, t) \tag{6}$$

$$J_\mathrm{i}(x, t) = \sigma_\mathrm{i} E_\parallel(x, t), \tag{7}$$

where σ_e (σ_i) is the conductivity tensor of the electron (ion).

Now, we can define Fourier–Bessel transform $Am(q)$ of an arbitrary function $A(\varphi, z, t)$ by

$$A(\varphi, z, t) = \sum_{m=-\infty}^{m=+\infty} \int_{-\alpha}^{+\alpha} \mathrm{d}Am(q) \exp\left[i(m\varphi + qz - \omega t)\right]. \tag{8}$$

Using eqs (1)–(7) by eliminating terms n_e and n_i we get equations for σ_e and σ_i in terms of cyclotron frequency ω, i.e., $\omega_{ce} = eB_0/m_\mathrm{e}$ $(\omega_{ci} = Z_\mathrm{e}B_0/m_\mathrm{i})$ of electron (ion).

In the space above and below the electron–ion cylinder, the transverse electric wave satisfies

$$\begin{cases} E_{zm}(r) = E_{0_z} I_m(\kappa a) K_m(\kappa r), & r > a, \\ E_{zm}(r) = E_{0_z} K_m(\kappa a) I_m(\kappa r), & r < a, \end{cases} \tag{9}$$

where $I_m(x)$ and $K_m(x)$ are the modified Bessel functions and $K^2 = q^2 - \omega^2/c^2$, where c is the speed of light. When the speed of light is taken to be infinitely large, we have

$$\begin{aligned}\omega^4 &- \omega^2\left[\left(\alpha+\beta q_m^2\right)q_m^2 + \left(1+\frac{z^2 m_\mathrm{e}^2}{m_\mathrm{i}^2}\right)\omega_{ce}^2\right. \\ &+ \left. e^2 Z n_0 a \frac{e^2 Z n_0 a}{\varepsilon_0 m_\mathrm{e}}\left(1+\frac{Z m_\mathrm{e}}{m_\mathrm{i}}\right) q^2_{ml_m(qa)} K_{m(qa)}\right] \\ &+ \frac{Z m_\mathrm{e}}{m_\mathrm{i}}\left[\left(\alpha+\beta q_m^2\right)q_m^2\left(1+\frac{Z m_\mathrm{e}}{m_\mathrm{i}}\right)\omega_{ce}^2\right] \\ &\times \frac{e^2 Z n_0 a}{\varepsilon_0 m_\mathrm{e}} q_m^2 I_{m(qa)} K_{m(qa)} \frac{Z^2 m_\mathrm{e}^2}{m_\mathrm{i}^2}\omega_{ce}^2\left[\omega_{ce}^2 + \left(q\alpha+\beta q_m^2\right)q_m^2\right] = 0, \end{aligned} \quad (10)$$

which determines the normal electrostatic modes.

The roots of the above equation give (in the limit $Zm_\mathrm{e}/m_\mathrm{i} \ll 1$) separate equations for ω_+ and ω_-, where ω_+ is for high-frequency dispersion and ω_- is for low-frequency (QMSW – quantum magnetosonic wave) dispersion. Here, we are considering low-frequency disturbances. Let us consider cold plasma approximation in the long-wavelength limit. Then, roots of eq. (10) become

$$\omega_+^2(m,q) \approx \omega_{ce}^2 + \frac{e^2 Z n_0 a}{\varepsilon_0 m_\mathrm{e}} q_m^2 I_m(qa) K(qa) \quad (11)$$

$$\omega_-^2(m,q) \approx \frac{Z m_\mathrm{e}}{m_\mathrm{i}}\omega_{ce}^2\left[1+\frac{\omega_{ce}^2}{(e^2 Z n_0 a/\varepsilon_0 m_\mathrm{e}) q_m^2 I_m(qa) K_m(qa)}\right]^{-1}. \quad (12)$$

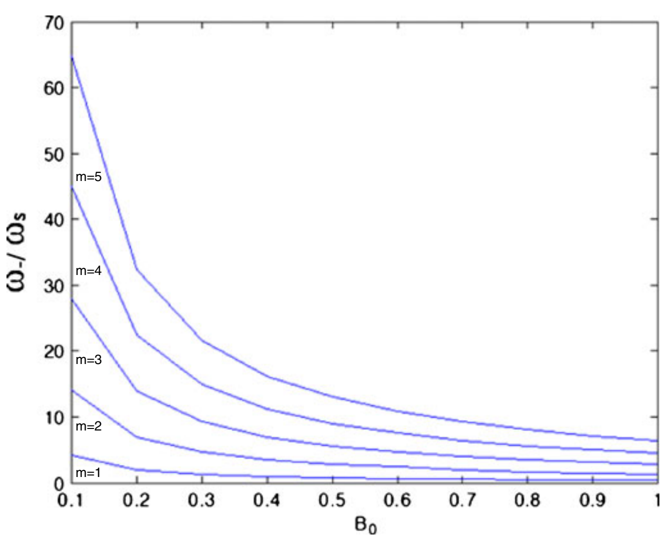

Figure 1. Dispersion relation ω_-/ω_s vs. applied magnetic field (long-wavelength limit).

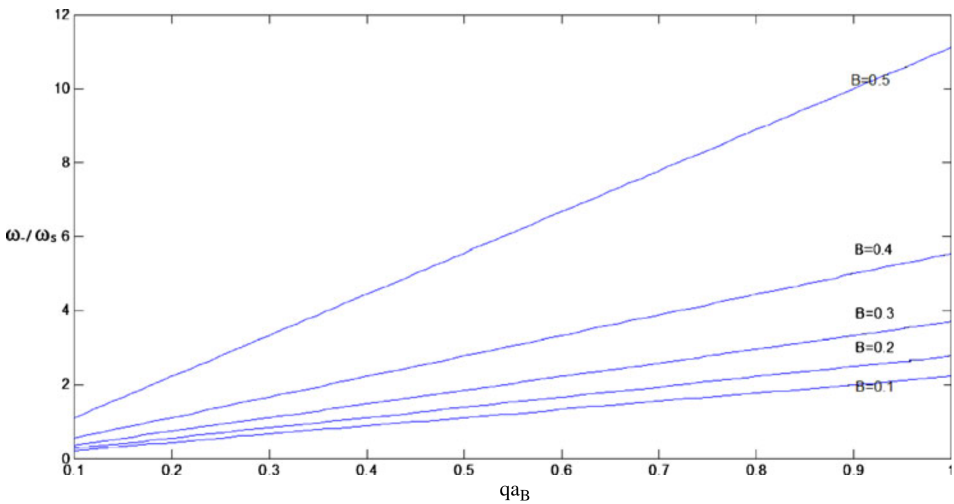

Figure 2. Dispersion relation ω_-/ω_s vs. the dimensionless variable qa_B (short-wavelength limit).

To study the variation of dimensionless frequency ω_-/ω_s with respect to the applied magnetic fields, we have taken $\omega_s = (Zm_e\omega_{ce}^2/m_i)^{1/2}$. In the long-wavelength limit, the effect of applied magnetic field is studied and plotted as shown in figure 1.

In short-wavelength limit, i.e., $qa \to \alpha$ we may use the asymptotic expression of the Bessel functions,

$$I_m(x) = e^x / (2\pi x)^{1/2}$$

and

$$K_m(x) = (\pi/2x)^{1/2}\, e^{-x} \tag{13}$$

so that the dispersion relation

$$\omega_-^2(q) \approx \frac{Zm_e}{m_i}\left(\alpha + \beta q_m^2\right) q_m^2 \left[1 + \frac{\alpha + \beta q_m^2}{\left(e^2 n_0/\varepsilon_0 m_e 2q\right)}\right]. \tag{14}$$

Using eq. (14), the variation of the dimensionless frequency ω_-/ω_s with respect to the variable qa_B for different values of magnetic fields applied for CNTs are studied. Here $\omega_s = (Zm_e\omega_{ce}^2/m_i)^{1/2}$. The dispersion curves obtained are as shown in figure 2.

3. Result

3.1 *In long-wavelength limit*

Using eq. (11), the variation of dimensionless plasma frequency with applied magnetic field for different m values of a CNT of radius $a = 5a_B$ is studied and plotted as shown in figure 1. It is seen that for the magnetosonic wave (MSW) oscillations, frequencies become almost similar and tend to be saturated for all m modes when the magnetic field increases. Only for lower magnetic fields, frequency dispersions are high for different m values.

3.2 *In short-wavelength limit*

In short-wavelength limit, it can be seen that the frequency is varied in accordance with the applied magnetic field. The dispersion relation for different values of B_0 as grouped in figure 2 was interpreted as, 'when the applied magnetic field is increased, the plasma dispersion exhibits stronger dispersions'.

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