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Planar electron beams in a wiggler magnet array

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Abstract. Transport of high current (∼kA range with particle energy ∼1 MeV) planar electron beams is a topic of increasing interest for applications in high-power (1–10 GW) and high-frequency (10–20 GHz) microwave devices such as backward wave oscillator (BWO), klystrons, gyro-BWOs, etc. In this paper, we give a simulated result for transport of electron beams with velocity $V_b = 5.23 \times 10^8$ cm s^{-1}, relativistic factor $\gamma = 1.16$, and beam voltage = ∼80 kV in notched wiggler magnet array. The calculation includes self-consistent effects of beam-generated fields. Our results show that the notched wiggler configuration with ∼6.97 kG magnetic field strength can provide vertical and horizontal confinements for a sheet electron beam with 0.3 cm thickness and 2 cm width. The feasibility calculation addresses to a system expected to drive for 13–20 GHz BWO with rippled waveguide parameters as width $w = 3.0$ cm, thickness $t = 1.0$ cm, corrugation depth $h = 0.225$ cm, and spatial periodicity $d = 1.67$ cm.

Keywords. Sheet beams; wiggler magnetic field; beam focussing.

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1. Introduction

Relativistic sheets or ribbon electron beams for microwaves have been of academic interest for several decades [1], but research was limited due to the difficulty associated with transportation of the beam and designing gun. Here, we are considering the issue of transportation of the beam. Previously [2], many experiments were conducted using uniform solenoidal focussing magnetic field. The most notable instability with solenoidal field is the shearing $E \times B$ drift that curls the edges of the beam. Due to shearing effects, a diocotron instability is formed in the beam. These problems can be avoided by employing very high magnetic field and short propagation length. Using an earlier 3D code, ARGUS [3], an 80 kV, 102 A sheet beam for a 120 GHz gyrotron oscillator was successfully designed by Read *et al*. Owing to the relatively low voltage, self-magnetic fields were

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not important, and curling of the beam was manageable because of the high (5 T) field that was inherent in the gyratron and because the beam shape was only required over a few centimetres. Hence, a solenoidal field with high strength is not appropriate because of economic reasons.

The goal of this paper is to report the technology developed for the transportation of beams in a wiggler magnet array. In §2, we provide the magnetostatic field pattern obtained from the notched wiggler magnetic array. Section 3 of the paper deals with the condition required for stable sheet beam transport; followed by calculation of peak magnetic field for 1 cm beam height using particle in cell (PIC) simulation code.

2. Magnetostatic field for wiggler magnet array

Magnetic field for a wiggler configuration is of the form $\vec{B} = -\nabla\chi_m$, where scalar potential can be defined as $\chi_m = (B_0/k_z)\cosh(k_x x)\cos(k_z z)\{a \sinh k_y y + b\cosh k_y y\}$,

$$k_x^2 + k_y^2 = k_z^2,$$

$$B_x = -\frac{k_x B_0}{k_z}\sinh(k_x x)\cos(k_z z) \times \{a\cosh(k_y y) + b\sinh(k_y y)\}, \tag{1}$$

$$B_y = -\frac{k_y B_0}{k_z}\cosh(k_x x)\cos(k_z z) \times \{a\cosh(k_y y) + b\sinh(k_y y)\}, \tag{2}$$

and

$$B_z = -B_0\cosh(k_x x)\sin(k_z z) \times \{a\cosh(k_y y) + b\sinh(k_y y)\}, \tag{3}$$

where a and b are the width and the length of the magnet in x and y directions, respectively. Wiggler field is visualized as a vertical field periodically varying with the beam slightly moving back and forth horizontally. If $a = 1$ and $b = 0$ in the above expression, the field is known as a wiggler field and if $a = 0$ and $b = 1$, then the field is known as periodic permanent magnet (PPM) field, which can be visualized as an axial field periodically varying with the beam slightly rocking back and forth between the vertical and the horizontal directions. Figure 1 shows the shape of the wiggler magnets used for horizontal focussing. The wiggler-magnet surfaces facing the beam have a notch giving a small step in the dipole field at the desired beam width [5]. Electrons that drift outward and cross the transition suffer an enhanced inward deflection, reversing the drift direction. This approach has the following advantages over other configurations:

1. All magnets are identical.
2. No extra parts or alignment is required beyond those of a uniform-field wiggler.
3. The effective force approximates a reflecting wall so that the confinement is not sensitive to the beam space-charge distribution or magnitude of the field transition.

We investigate the field due to wiggler magnet array that will provide wiggler motion to electrons of the sheet beam as follows. In principle, the fields for the above configuration

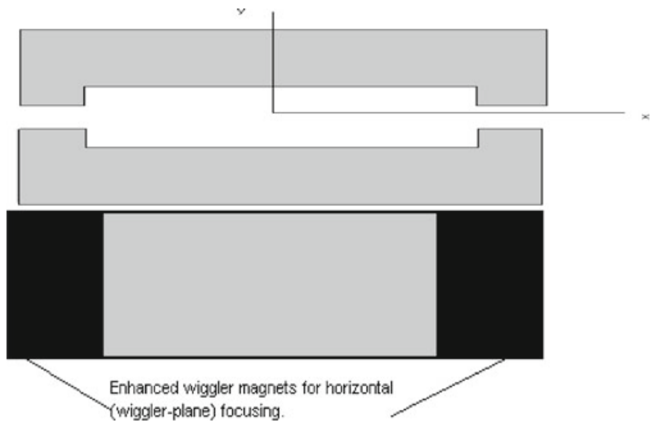

Figure 1. Shape of the wiggler magnets for horizontal (wiggler-plane) focussing.

can be obtained from the scalar magnetic field potential which satisfies Laplace's equation along with appropriate boundary conditions at the inside edges of the magnet,

$$\nabla^2 \phi_M(x, y, z) = 0, \quad |x| \leq \frac{a_\mathrm{m}}{2}, \quad |y| \leq \frac{b_\mathrm{m}}{2},$$

where a_m and b_m are the full width and full height of the magnet. We use the Biot savart formula in eqs (1)–(3) and calculate the fields numerically as shown in figure 2.

Field variation obtained in figure 2 is as expected through the wiggler field for seven full cells. Initial cell had an axial length of $L_\mathrm{c}/2 = 2.0$ mm. At locations away from the entrance, the peak magnetic field was 6.97 kG.

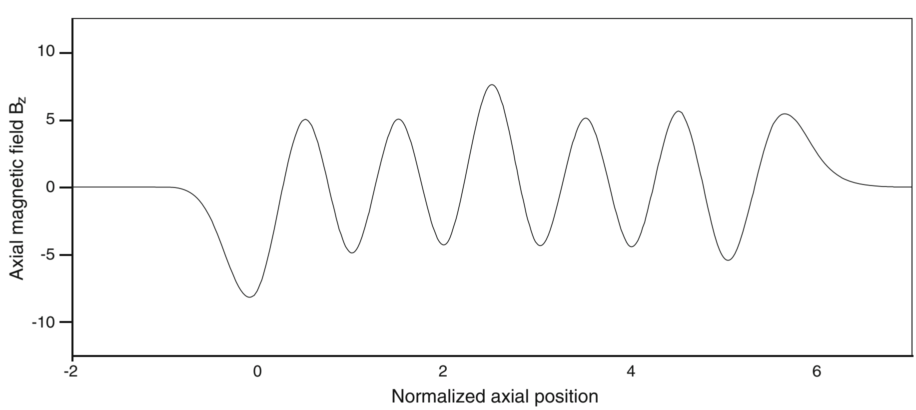

Figure 2. Numerically calculated axial magnetic field component B_z in kG of the notched wiggler magnet array as displayed in figure 1 for $x = 0$ and $y = 0$.

3. Stable sheet beam transport in the wiggler field

By following the development by Humphries *et al* [4], we limit our attention to vertical focussing and assume that the system has infinite length in x. The magnets are oriented in such a way that they can produce a dipole field B_y with alternating polarity along the axis. Let us consider that the dipole field variation is approximated as

$$B_y(x,z) = -\frac{k_y B_0}{k_z}\cosh(k_x x)\cos(k_z z) \times \{a\cosh(k_y y) + b\sinh(k_y y)\}. \tag{4}$$

The dipole fields (B_y) impart an oscillating velocity in the x direction. The resulting $v_x \times B_z$ force provides focussing along y. The B_z components have opposite polarity above and below the vertical axis. We can therefore, approximate the field component near $y = 0$ with the expression

$$B_z \cong y f(z)\,. \tag{5}$$

Since there is no current source, the condition $\nabla \times B = 0$ holds; or

$$\frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z}. \tag{6}$$

Substitution of eqs (4) and (5) in eq. (6) determines $f(z)$ and gives an expression for the axial component of the field

$$Bz(y,z) \cong -k_y B_0 \cosh(k_x x)\sin(k_z z) \times \{a\cosh(k_y y) + b\sinh(k_y y)\}y. \tag{7}$$

Canonical momentum is a conserved quantity along x direction.

$$P_x = \gamma m_e v_x - eA_x. \tag{8}$$

The condition $P_x = 0$ holds good if the electrons enter from a region magnetic flux. The vector potential is related to the dipole field by

$$B_y = \frac{\partial A_x}{\partial z}. \tag{9}$$

Integration of eq. (9) gives

$$A_x = \frac{B_0}{k_y}\cosh(k_x x)\sin(k_z z) \times \{a\cosh(k_y y) + b\sinh(k_y y)\}. \tag{10}$$

Combining eqs (8) and (10) gives an expression for the velocity along x.

$$v_x = \frac{eB_0}{\gamma m_e k_y}\cosh(k_x x)\sin(k_z z) \times \{a\cosh(k_y y) + b\sinh(k_y y)\}. \tag{11}$$

Taking the vertical magnetic force as $ev_x \times B_z$ and introducing beam field and emittance force [6], we find the following equation for the envelope half width Y:

$$\frac{\mathrm{d}^2 Y}{\mathrm{d}z^2} \cong -\left(\frac{eB_0\cosh(k_x x)\sin(k_z z) \times \{a\cosh(k_y y) + b\sinh(k_y y)\}^2}{\gamma m_e \beta c}\right)Y + K_y + \frac{\varepsilon_y^2}{Y^3}. \tag{12}$$

In eq. (12), K_y is the linear generalized perveance

$$K_y = \frac{eJ}{2\varepsilon_0 m_e (\gamma\beta c)^3}, \tag{13}$$

where ε_0 is the permittivity of the free space. The quantity J in (13) is the line current density of the sheet electron beam in A/m and the quantity ε_y is the vertical emittance.

$$\varepsilon_y \cong y\Delta\theta_y, \tag{14}$$

where $\Delta\theta_y$ is the amplitude of the vertical angular divergence. Averaging over z gives the envelope equation for a sheet beam in a 2D wiggler.

$$\frac{\mathrm{d}^2 Y}{\mathrm{d}z^2} \cong -\left(\frac{eB_0}{\gamma m_e \beta c}\right)^2 Y + K_y + \frac{\varepsilon_y^2}{Y^3}. \tag{15}$$

Setting $\left(\mathrm{d}^2 Y/\mathrm{d}z^2\right) = 0$ gives an approximate condition for a matched beam. For specified values of beam kinetic energy, linear current density and angular divergence, we can calculate the required value of B_0 for a desired beam height. Equation (15) is the envelope equation for planar beam focussing in a wiggler magnet array. The wiggler system has the following advantages:

(1) Vacuum fields are confined to the beam transport region.
(2) It is not necessary to place iron flux conductors between magnets.
(3) Field levels are relatively low in the flux conductors so that they are not saturated.

The implications are that higher values of B_0 can be obtained in a wiggler with the same volume of magnet material. Figure 3 shows a 1.5D simulation using the PIC code TUBE; it verifies the stable transport with the wiggler field of 6.97 kG, with no emittance growth. Here, the considered sheet beam has velocity $V_\mathrm{b} = 5.23 \times 10^8$ cm s^{-1} and the beam dimensions are: thickness = 0.3 cm and the width = 2 cm.

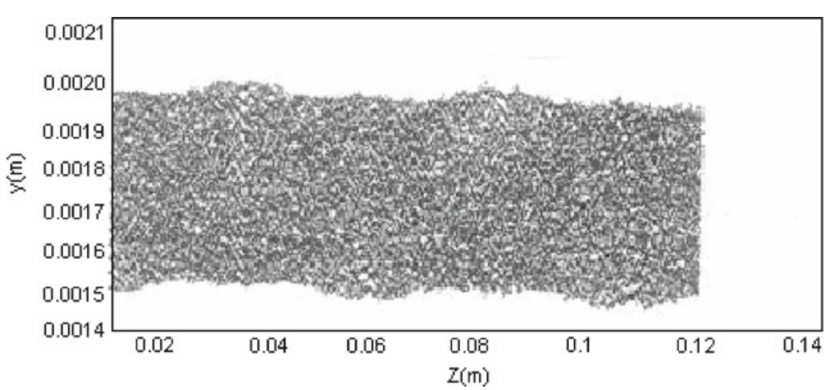

Figure 3. One and a half-dimensional PIC simulation of wiggler transport using TUBE.

4. Conclusion

In this paper, we have analysed the magnetic field pattern obtained from the notched wiggler magnet array system. Envelope equation for the sheet beam focussing in wiggler magnet array is developed, by which, the required value of the peak magnetic field can be calculated for focussing the beam with known kinetic energy, linear current and angular divergence for a desired beam height. Through simulation code we have shown that the magnetic field calculated by the envelope equation for a particular beam height is feasible to give stable transport to the electron beam for the system expected to drive 13–20 GHz BWO. Waveguide parameters used in the calculations are: width $b = 3$ cm, thickness $a = 1$ cm, corrugation depth $h = 0.225$ cm, and spatial periodicity $d = 1.67$ cm, sheet electron beam with thickness 0.3 cm, width 2.0 cm, velocity $V_\mathrm{b} = 5.23 \times 10^8$ cm s^{-1} and relativistic factor $\gamma = 1.16$, and beam voltage = ∼80 kV. During simulation, space-charge effects are ignored. Tailoring the edge profile to mitigate space-charge effects in a high current beam is in progress.

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