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Quadrupole moments of low-lying baryons with spin-$\frac{1}{2}^+$, spin-$\frac{3}{2}^+$, and spin-$\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ transitions

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Abstract. The chiral constituent quark model (χCQM) with general parametrization (GP) method has been formulated to calculate the quadrupole moments of the spin-$\frac{3}{2}^+$ decuplet baryons and spin-$\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ transitions. The implications of such a model have been investigated in detail for the effects of symmetry breaking and GP parameters pertaining to the two- and three-quark contributions. It is found that the χCQM is successful in giving a quantitative and qualitative description of the quadrupole moments.

Keywords. Electromagnetic moments; phenomenological quark models.

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1. Introduction

One of the main challenges in the theoretical and experimental hadronic physics is to understand the structure of hadrons within the quantum chromodynamics (QCD) [1], which can be obtained from the measurements of electromagnetic form factors. Following the discoveries that quarks and antiquarks carry only 30% of the total proton spin [2], the orbital angular momentum of quarks and gluons is expected to make significant contribution. In addition to this, there is a significant contribution coming from the strange quarks in the nucleon which are otherwise not present in the valence structure. It therefore becomes interesting to discuss the interplay between the spin of non-valence quark and the orbital angular momentum in understanding the spin structure of baryons. Further, the experimental developments [3,4], providing information on the radial variation of the charge and magnetization densities of the proton, give evidence for the deviation of charge distribution from spherical symmetry. On the other hand, it is well-known that the quadrupole moment of the nucleon should vanish on account of its spin-$1/2$ nature. This observation has naturally turned to be the subject of intense theoretical and experimental activity.

The $\Delta(1232)$ resonance is the lowest-lying excited state of the nucleon in which the search for quadrupole strength has been carried out [5,6]. The spin and parity selection rules in the $\gamma + p \to \Delta^+$ transition allow three contributing photon absorption amplitudes, the magnetic dipole $G_{\rm M1}$, the electric quadrupole moment $G_{\rm E2}$, and the charge quadrupole moment $G_{\rm C2}$. The $G_{\rm M1}$ amplitude gives us information on magnetic moment, whereas information on the intrinsic quadrupole moment can be obtained from measurements of $G_{\rm E2}$ and $G_{\rm C2}$ amplitudes [7]. If the charge distribution of the initial and final three-quark states were spherically symmetric, the $G_{\rm E2}$ and $G_{\rm C2}$ amplitudes of the multipole expansion would be zero [8]. However, the recent experiments at Mainz, Bates, Bonn, and JLab Collaborations [9,10] reveal that these quadrupole amplitudes are clearly non-zero [7]. The ratio of the electric quadrupole amplitude to the magnetic dipole amplitude is at least $(E2/M1) \equiv -0.025 \pm 0.005$ and a comparable value of the same sign and magnitude has been measured for the $C2/M1$ ratio [7]. Further, the quadrupole transition moment ($Q^{\Delta^+ N}$) measured by LEGS [5] and Mainz [6] Collaborations ($-0.108 \pm 0.009 \pm 0.034$ fm^2 and -0.0846 ± 0.0033 fm^2, respectively) also leads to the conclusion that the nucleon and the Δ^+ are intrinsically deformed.

There have been many theoretical investigations on understanding the implications of the $C2/M1$ and $E2/M1$ ratios in finding out the exact sign of deformation in the spin-$\frac{1}{2}^+$ octet baryons. However, there is little consensus between the results even with respect to the sign of the nucleon deformation. Some of the models predict the deformation in nucleon as oblate [11], some predict nucleon deformation as prolate [12,13] whereas others speak about 'deformation' without specifying the sign. It is important to mention here that the deformation of the octet baryons is an intrinsic phenomenon and has to be coupled with the orbital angular momentum to compare with the spectroscopic value. The electromagnetic structures of the $\frac{3}{2}^+$ decuplet baryons have also been studied using the variants of the constituent quark model (CQM) [14,15], chiral quark soliton model (χQSM) [16], spectator quark model [17], slow rotator approach (SRA) [18], skyrme model [19], general parametrization method [20], light cone QCD sum rules (QCDSR) [21], large $N_{\rm c}$ [22], chiral perturbation theory (χPT) [23], lattice QCD (LQCD) [24], etc. In this case also, the results for different theoretical models are not consistent in terms of sign and magnitude with each other.

One of the important non-perturbative approaches in the low-energy regime of QCD is the chiral constituent quark model (χCQM) [25]. The χCQM coupled with the 'quark sea' generation through the chiral fluctuation of a constituent quark into a Goldstone boson (GB), successfully explains the 'proton spin crisis' [26], hyperon β decay parameters [27], strangeness content in the nucleon [28], and in the N and Δ magnetic moments [29]. Further, the χCQM coupled with the 'quark sea' polarization and orbit angular momentum of the quark sea (referred to as the Cheng–Li mechanism) is able to give an excellent fit to the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet baryon magnetic moments and a perfect fit to the violation of Coleman–Glashow sum rule [30]. The model has been successfully extended to explain the magnetic moment of charmed baryons with spin-$\frac{1}{2}^+$, spin-$\frac{3}{2}^+$, spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ and spin-$\frac{1}{2}^+ \to \frac{1}{2}^+$ radiative decays [31]. In view of the above developments, it becomes desirable to calculate the quadrupole moment of the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet baryons within the framework of χCQM.

The purpose of the present communication is to present a unified approach to calculate the quadrupole moment of the spin-$\frac{3}{2}^+$ decuplet baryons including the spin-$\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ transitions within the framework of χCQM using a general parametrization (GP) method [32]. In particular, we aim to predict the sign of deformation in the baryons as prolate or oblate.

2. Quadrupole moment

The charge radii (r_B^2) and quadrupole moments (Q^B) are the lowest order moments of the charge density ρ in a low-momentum expansion. The charge radii contain fundamental information about the possible 'size' of the baryons whereas the 'shape' of a spatially extended particle is determined by its quadrupole moment [33]. With respect to the body frame of axis, the intrinsic quadrupole moment is defined as

$$Q_0 = \int \mathrm{d}^3\mathbf{r}\rho(\mathbf{r})(3z^2 - r^2)\,. \tag{1}$$

For the charge density concentrated along the z-direction, the term proportional to $3z^2$ dominates, Q_0 is positive, and the particle is prolate-shaped. If the charge density is concentrated in the equatorial plane perpendicular to z-axis, the term proportional to r^2 dominates, Q_0 is negative and the particle is oblate-shaped.

The intrinsic quadrupole moment Q_0 must be distinguished from the spectroscopic quadrupole moment Q measured in the laboratory frame. A simple example will illustrate this point. Suppose, one has determined the quadrupole moment Q_0 of a classical charge distribution $\rho(\mathbf{r})$ with symmetry axis z and angular momentum J in the body-fixed frame according to eq. (1). The quadrupole moment of the same charge distribution with respect to the laboratory frame is then given by

$$Q = P_2(\cos\theta)Q_0 = \frac{1}{2}(3\,\cos^2(\theta) - 1)Q_0 = \left(\frac{3J_{z'}^2 - J(J+1)}{2J(J+1)}\right)Q_0, \tag{2}$$

where θ is the angle between the body-fixed z- and the laboratory frame z'-axes, and P_2 is the second Legendré polynomial. The latter arises when transforming the spherical harmonic of rank 2 in Q_0 from body-fixed to laboratory coordinates. The third equality in eq. (2) is obtained when $\cos(\theta)$ is expressed in terms of the spin projection $J_{z'}$ on the laboratory frame z'-axis and the total spin J of the system as $\cos(\theta) = J_{z'}/\sqrt{J^2}$. Thus, in the laboratory, one does not measure the intrinsic quadrupole moment directly but only its projection onto the z'-axis. In the quantum mechanical analogue of eq. (2), the denominator of the projection factor is changed into $(2J+3)(J+1)$. The projection factor shows that $J = 0$ and $J = 1/2$ systems have vanishing spectroscopic quadrupole moments even though they may be deformed and their intrinsic quadrupole moments are non-zero. Some information on the shape of nucleon or any other member of the octet baryon can be obtained by electromagnetically exciting baryon to the spin $J = 3/2$ or higher spin state.

The most general form of the quadrupole operator composed of a two- and three-quark term in spin-flavour space can be expressed as [32]

$$\widehat{Q} = B' \sum_{i \neq j}^{3} e_i (3\sigma_{i\,z}\sigma_{j\,z} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) + C' \sum_{i \neq j \neq k}^{3} e_i (3\sigma_{j\,z}\sigma_{k\,z} - \boldsymbol{\sigma}_j \cdot \boldsymbol{\sigma}_k), \quad (3)$$

where $\sigma_{i\,z}$ is the z component of the spin operator of the quark i and e_i is the quark charge. The GP method constants B' and C' parametrize the orbital and colour matrix elements. The order of GP parameters B, and C, corresponding to the two- and three-quark terms, decreases with the increasing complexity of terms and obey the hierarchy $|B| > |C|$. These are fitted by using the available experimental values for the charge radii and quadrupole moment of nucleon as input.

The quadrupole moment operators for the spin-$\frac{1}{2}^+$, spin-$\frac{3}{2}^+$ baryons, and spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions can be calculated from the operator in eq. (3) and are expressed as

$$\begin{aligned} \widehat{Q}^B &= B' \left(3 \sum_{i \neq j} e_i \boldsymbol{\sigma}_{iz} \boldsymbol{\sigma}_{jz} - 3 \sum_i e_i \boldsymbol{\sigma}_{iz} + 3 \sum_i e_i \right) \\ &\quad + C' \left(3 \sum_{i \neq j \neq k} e_i \boldsymbol{\sigma}_{jz} \boldsymbol{\sigma}_{kz} + 3 \sum_i e_i \boldsymbol{\sigma}_{iz} \right), \quad (4) \\ \widehat{Q}^{B^*} &= B' \left(3 \sum_{i \neq j} e_i \boldsymbol{\sigma}_{iz} \boldsymbol{\sigma}_{jz} - 5 \sum_i e_i \boldsymbol{\sigma}_{iz} + 3 \sum_i e_i \right) \\ &\quad + C' \left(3 \sum_{i \neq j \neq k} e_i \boldsymbol{\sigma}_{jz} \boldsymbol{\sigma}_{kz} + 5 \sum_i e_i \boldsymbol{\sigma}_{iz} - 6 \sum_i e_i \right), \quad (5) \\ \widehat{Q}^{B^* B} &= 3B' \sum_{i \neq j} e_i \boldsymbol{\sigma}_{iz} \boldsymbol{\sigma}_{jz} + 3C' \sum_{i \neq j \neq k} e_i \boldsymbol{\sigma}_{jz} \boldsymbol{\sigma}_{kz}. \quad (6) \end{aligned}$$

It is clear from the above equations that the determination of quadrupole moment basically reduces to the evaluation of the flavour ($\sum_i e_i$), spin ($\sum_i e_i \boldsymbol{\sigma}_{iz}$), and tensor terms ($\sum_i e_i \boldsymbol{\sigma}_{iz} \boldsymbol{\sigma}_{jz}$) and ($\sum_i e_i \boldsymbol{\sigma}_{jz} \boldsymbol{\sigma}_{kz}$) for a given baryon.

Using the three-quark spin-flavour wave functions for the spin-$\frac{1}{2}^+$ octet and spin-$\frac{3}{2}^+$ decuplet baryons, the quadrupole moment can now be calculated by evaluating the matrix elements of operators in eqs (4), (5) and (6). We now have

$$\begin{aligned} Q^B &= \langle B | \widehat{Q}_B | B \rangle, \\ Q^{B^*} &= \langle B^* | \widehat{Q}_{B^*} | B^* \rangle, \\ Q^{B \to B^*} &= \langle B^* | \widehat{Q}_{B^* B} | B \rangle, \quad (7) \end{aligned}$$

where $|B\rangle$ and $|B^*\rangle$ respectively, denote a spin-flavour baryon wave function for the spin-$\frac{1}{2}^+$ octet and spin-$\frac{3}{2}^+$ decuplet baryons [34].

3. Naive quark model

The appropriate operators for the spin and flavour structure of baryons in the naive quark model (NQM) [35] are defined as

$$\sum_i e_i = \sum_{q=u,d,s} n_q^B q + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} n_{\bar{q}}^B \bar{q} = n_u^B u + n_d^B d + n_s^B s + n_{\bar{u}}^B \bar{u} + n_{\bar{d}}^B \bar{d} + n_{\bar{s}}^B \bar{s}\,, \quad (8)$$

and

$$\sum_i e_i \sigma_{iz} = \sum_{q=u,d,s} (n_{q_+}^B q_+ + n_{q_-}^B q_-) = n_{u_+}^B u_+ + n_{u_-}^B u_- + n_{d_+}^B d_+ + n_{d_-}^B d_- + n_{s_+}^B s_+ + n_{s_-}^B s_-\,, \quad (9)$$

where n_q^B ($n_{\bar{q}}^B$) is the number of quarks with charge q ($\bar{q}$) and $n_{q_+}^B$ ($n_{q_-}^B$) is the number of polarized quarks q_+ (q_-). For a given baryon $u = -\bar{u}$ and $u_+ = -u_-$, with similar relations for the d and s quarks.

For the spin-$\frac{1}{2}^+$ octet baryons, the quadrupole moment of p and Σ^+ in NQM can be expressed as

$$Q^p = 3B'(2u + d - 2u_+ - d_+) + C'(-4u + d + 4u_+ - d_+), \quad (10)$$

$$Q^{\Sigma^+} = 3B'(2u + s - 2u_+ - s_+) + C'(-4u + s + 4u_+ - s_+). \quad (11)$$

For the spin-$\frac{3}{2}^+$ decuplet baryons, the quadrupole moment of Δ^+ and Ξ^{*-} can be expressed as

$$Q^{\Delta^+} = B'(6u + 3d + 2u_+ + d_+) + C'(-6u - 3d + 10u_+ + 5d_+), \quad (12)$$

$$Q^{\Xi^{*-}} = B'(3d + 6s + d_+ + 2s_+) + C'(-3d - 6s + 5d_+ + 10s_+). \quad (13)$$

Table 1. Quadrupole moments of the octet baryons in NQM using GP method.

Baryons	NQM
p	$3B'\,(2u + d - 2u_+ - d_+) + C'\,(-4u + d + 4u_+ - d_+)$
n	$3B'\,(u + 2d - u_+ - 2d_+) + C'\,(u - 4d - u_+ + 4d_+)$
Σ^+	$3B'\,(2u + s - 2u_+ - s_+) + C'\,(-4u + s + 4u_+ - s_+)$
Σ^-	$3B'\,(2d + s - 2d_+ - s_+) + C'\,(-4d + s + 4d_+ - s_+)$
Σ^0	$3B'\,(u + d + s - u_+ - d_+ - s_+)$ $+ C'\,(-2u - 2d + s + 2u_+ + 2d_+ - s_+)$
Ξ^0	$3B'\,(u + 2s - u_+ - 2s_+) + C'\,(u - 4s - u_+ + 4s_+)$
Ξ^-	$3B'\,(d + 2s - d_+ - 2s_+) + C'\,(d - 4s - d_+ + 4s_+)$
Λ^0	$3B'\,(u + d + s - u_+ - d_+ - s_+) + 3C'\,(-s + s_+)$

Similarly, for the spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions, the quadrupole moment of the $\Delta^+ p$ and $\Sigma^{*-}\Sigma^-$ transitions can be expressed as

$$Q^{\Delta^+ p} = 2\sqrt{2}B'(u_+ - d_+) + 2\sqrt{2}C'(-u + d), \tag{14}$$

$$Q^{\Sigma^{*-}\Sigma^-} = 2\sqrt{2}B'(d_+ - s_+) + 2\sqrt{2}C'(-d + s). \tag{15}$$

Table 2. Quadrupole moments of the decuplet baryons in NQM and χCQM using GP method.

Baryon	NQM	χCQM
Δ^{++}	$B'(9u + 3u_+) + C'(-9u + 15u_+)$	$8B' + 4C' - (B' + 5C')\frac{a}{3}(9 + 3\alpha^2 + 2\beta^2 + 4\zeta^2)$
Δ^+	$B'(3(2u + d) + 2u_+ + d_+)$ $+ C'(-3(2u + d) + 5(2u_+ + d_+))$	$4B' + 2C' - (B' + 5C')\frac{a}{3}(6 + \beta^2 + 2\zeta^2)$
Δ^0	$B'(3(u + 2d) + u_+ + 2d_+)$ $+ C'(-3(u + 2d) + 5(u_+ + 2d_+))$	$(B' + 5C')a(-1 + \alpha^2)$
Δ^-	$B'(9d + 3d_+) + C'(-9d + 15d_+)$	$-4B' - 2C' + (\mathrm{B}' + 5B')\frac{a}{3}(6\alpha^2 + \beta^2 + 2\zeta^2)$
Σ^{*+}	$B'(3(2u + s) + 2u_+ + s_+)$ $+ C'(-3(2u + s) + 5(2u_+ + s_+))$	$4B' + 2C' - (B' + 5C')\frac{a}{3}(6 + \alpha^2 + 2\zeta^2)$
Σ^{*-}	$B'(3(2d + s) + 2d_+ + s_+)$ $+ C'(-3(2d + s) + 5(2d_+ + s_+))$	$-4B' - 2C' + (B' + 5C')\frac{a}{3}(5\alpha^2 + 2\beta^2 + 2\zeta^2)$
Σ^{*0}	$B'(3(u + d + s) + u_+ + d_+ + s_+)$ $+ C(-3(u + d + s) + 5(u_+ + d_+ + s_+))$	$(B' + 5C')\frac{a}{3}(-3 + 2\alpha^2 + \beta^2)$
Ξ^{*0}	$B'(3(u + 2s) + u_+ + 2s_+)$ $+ C'(-3(u + 2s) + 5(u_+ + 2s_+))$	$(B' + 5C')\frac{a}{3}(-3 + \alpha^2 + 2\beta^2)$
Ξ^{*-}	$B'(3(d + 2s) + d_+ + 2s_+)$ $+ C'(-3(d + 2s) + 5(d_+ + 2s_+))$	$-4B' - 2C' + (B' + 5B')\frac{a}{3}(4\alpha^2 + 3\beta^2 + 2\zeta^2)$
Ω^-	$B'(9s + 3s_+) + C'(-9s + 15s_+)$	$-4B' - 2C' + (B' + 5C')\frac{a}{3}(3\alpha^2 + 4\beta^2 + 2\zeta^2)$

Table 3. Quadrupole moments of the spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions in NQM and χCQM using GP method.

Baryon	NQM	χCQM
$\Delta^+ p$	$2\sqrt{2}B'(u_+ - d_+) + 2\sqrt{2}C'(-u + d)$	$2\sqrt{2}B'\left(1 - \frac{a}{3}(3 + 3\alpha^2 + \beta^2 + 2\zeta^2)\right) - 2\sqrt{2}C'$
$\Sigma^{*+}\Sigma^+$	$2\sqrt{2}B'(u_+ - s_+) + 2\sqrt{2}C'(-u + s)$	$2\sqrt{2}B'\left(1 - \frac{a}{3}(3 + 2\alpha^2 + 2\beta^2 + 2\zeta^2)\right) - 2\sqrt{2}C'$
$\Sigma^{*-}\Sigma^-$	$2\sqrt{2}B'(d_+ - s_+) + 2\sqrt{2}C'(-d + s)$	$\frac{2\sqrt{2}}{3}B'a(\alpha^2 - \beta^2)$
$\Sigma^{*0}\Sigma^0$	$\sqrt{2}B'(u_+ + d_+ - 2s_+)$ $+ \sqrt{2}C'(-u - d + 2s)$	$\sqrt{2}B'\left(1 - a(1 + \frac{\alpha^2}{3} + \beta^2 + \frac{2}{3}\zeta^2)\right) - \sqrt{2}C'$
$\Xi^{*0}\Xi^0$	$2\sqrt{2}B'(u_+ - s_+) + 2\sqrt{2}C'(-u + s)$	$2\sqrt{2}B'\left(1 - \frac{a}{3}(3 + 2\alpha^2 + 2\beta^2 + 2\zeta^2)\right) - 2\sqrt{2}C'$
$\Xi^{*-}\Xi^-$	$2\sqrt{2}B'(d_+ - s_+) + 2\sqrt{2}C'(-d + s)$	$\frac{2\sqrt{2}}{3}B'a(\alpha^2 - \beta^2)$
$\Sigma^{*0}\Lambda$	$\sqrt{6}B'(u_+ - d_+) + \sqrt{6}C'(-u + d)$	$\sqrt{6}B'\left(1 - \frac{a}{3}(3 + 3\alpha^2 + \beta^2 + 2\zeta^2)\right) - \sqrt{6}C'$

The expressions for quadrupole moment of the other $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet baryons, and for spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions in NQM can similarly be calculated. The results are presented in tables 1, 2, and 3.

4. Chiral constituent quark model

In light of the recent developments and successes of the χCQM in explaining the low-energy phenomenology [26–30], we formulate the quadrupole moments for the $\frac{3}{2}^+$ decuplet baryons and spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions. The basic process in the χCQM [25] is the GB emission by a constituent quark which further splits into a $q\bar{q}$ pair as

$$q_\pm \to \mathrm{GB}^0 + q'_\mp \to (q\bar{q}') + q'_\mp\,, \tag{16}$$

where $q\bar{q}' + q'$ constitute the 'quark sea' [36].

The effective Lagrangian describing the interaction between quarks and a nonet of GBs is

$$\mathcal{L} = g_8 \bar{q}\Phi q, \tag{17}$$

with

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} & \pi^+ & \alpha K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \beta\frac{\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} & \alpha K^0 \\ \alpha K^- & \alpha\bar{K}^0 & -\beta\frac{2\eta}{\sqrt{6}} + \zeta\frac{\eta'}{\sqrt{3}} \end{pmatrix}$$

and

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \tag{18}$$

where $\zeta = g_1/g_8$, g_1 and g_8 are the coupling constants for the singlet and octet GBs, respectively. If the parameter $a(=|g_8|^2)$ denotes the transition probability of chiral fluctuation of the splitting $u(d) \to d(u) + \pi^{+(-)}$, then $\alpha^2 a$, $\beta^2 a$ and $\zeta^2 a$ respectively, denote the probabilities of transitions of $u(d) \to s + K^{-(o)}$, $u(d,s) \to u(d,s) + \eta$, and $u(d,s) \to u(d,s) + \eta'$. $SU(3)$ symmetry breaking is introduced by considering $M_s > M_{u,d}$ as well as by considering the masses of GBs to be nondegenerate ($M_{K,\eta} > M_\pi$ and $M_{\eta'} > M_{K,\eta}$) [26,36,37].

A redistribution of the flavour and spin structure takes place in the interior of baryons due to the chiral symmetry breaking and the modified flavour and spin structure can be calculated by substituting for every constituent quark

$$q \to P_q q + |\psi(q)|^2\,, \tag{19}$$

$$q_\pm \to P_q q_\pm + |\psi(q_\pm)|^2\,, \tag{20}$$

where P_q is the transition probability of no emission of GB from any of the q quarks and $|\psi(q)|^2$ ($|\psi(q_\pm)|^2$) is the transition probability of the q ($q_\pm$) quark, which have been detailed in ref. [26].

After the inclusion of 'quark sea', the quadrupole moment for the spin-$\frac{1}{2}^+$ octet baryons vanishes on account of the effective cancellation of contribution coming from the 'quark sea' and the orbital angular momentum as observed spectroscopically. For the spin-$\frac{3}{2}^+$ decuplet baryons, the quadrupole moment in χCQM can be obtained by substituting eqs (19) and (20) for each quark in eqs (10) and (11). The quadrupole moment of Δ^+ and Ξ^{*-} in χCQM can be expressed as

$$Q^{\Delta^+} = 4B' + 2C' - (B' + 5C')a\left(2 + \frac{\beta^2}{3} + \frac{2\zeta^2}{3}\right), \tag{21}$$

$$Q^{\Xi^{*-}} = -4B' - 2C' + (B' + 5C')a\left(\frac{4\alpha^2}{3} + \beta^2 + \frac{2\zeta^2}{3}\right). \tag{22}$$

Similarly, the quadrupole moment of $\Delta^+ p$ and $\Sigma^{*-}\Sigma^-$ transitions in χCQM can be expressed as

$$Q^{\Delta^+ p} = 2\sqrt{2}\left[B'\left(1 - a\left(1 + \alpha^2 + \frac{\beta^2}{3} + \frac{2\zeta^2}{3}\right)\right) - C'\right], \tag{23}$$

$$Q^{\Sigma^{*-}\Sigma^-} = 2\sqrt{2}B'a\left(\frac{\alpha^2}{3} - \frac{\beta^2}{3}\right). \tag{24}$$

The expressions for the quadrupole moment of other $\frac{3}{2}^+$ decuplet baryons and spin-$\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ transitions in χCQM can be calculated similarly. The results are presented in tables 2 and 3.

5. Results and discussion

The quadrupole moment calculations involve two sets of parameters, the GP method parameters (B' and C') and the $SU(3)$ symmetry-breaking parameters of χCQM (a, $a\alpha^2$, $a\beta^2$, and $a\zeta^2$). In order to obtain a simultaneous fit to charge radii and quadrupole moments, the GP parameters were fitted to the presently available data for the charge radii of nucleon and quadrupole moment of $\Delta^+ N$ transition as input. The set of parameters obtained after χ^2 minimization are as follows:

$$B' = -0.047, \quad C' = -0.008, \tag{25}$$

obeying the hierarchy $|B'| > |C'|$ [38] corresponding to the two- and three-quark contribution. For the χCQM parameters, a best fit is obtained by carrying out a fine-grained analysis of the spin and flavour distribution functions of proton [26–30] leading to

$$a = 0.12, \quad \alpha = 0.7, \quad \beta = 0.4, \quad \zeta = -0.15. \tag{26}$$

It would be interesting to mention here that in the present work, we have used the same set of χCQM parameters which were used to calculate the magnetic moment of baryons [26,30].

Using the set of parameters discussed above, we have calculated the numerical values for the quadrupole moment for the $\frac{3}{2}^+$ decuplet baryons in χCQM and presented the results in table 4. The results of the spin-$\frac{3}{2}^+ \rightarrow \frac{1}{2}^+$ transitions are presented in table 5.

Table 4. Quadrupole moments of the spin-$\frac{3}{2}^+$ decuplet baryons in χCQM using GP method and $SU(3)$ symmetry breaking.

Baryon	NQM (fm^2)	CQM [15] 10^{-2} fm^2	χPT [23] 10^{-1} fm^2	SRA [18] 10^{-1} fm^2	Skyrme [19] 10^{-2} fm^2	GPM [20] (fm^2)	χCQM with $SU(3)$ Symmetry (fm^2)	χCQM with $SU(3)$ Symmetry-breaking (fm^2)
Δ^{++}	−0.409	−9.3	-0.8 ± 0.5	−0.87	−8.8	−0.12	−0.3437	−0.3695
Δ^{+}	−0.204	−4.6	-0.3 ± 0.2	−0.31	−2.9	−0.06	−0.1719	−0.1820
Δ^{0}	0.0	0.0	0.12 ± 0.05	0.24	2.9	0.0	0.0	0.0055
Δ^{-}	0.204	4.6	0.6 ± 0.3	0.80	8.8	0.06	0.1719	0.1930
Σ^{*+}	−0.204	−5.4	-0.7 ± 0.3	−0.42	−7.1	−0.069	−0.1719	−0.1808
Σ^{*-}	0.204	4.0	4.0 ± 0.2	0.52	7.1	0.039	0.1719	0.1942
Σ^{*0}	0.0	−0.7	-0.13 ± 0.07	0.05	0.0	0.014	0.0	0.0067
Ξ^{*0}	0.0	−1.3	-0.35 ± 0.2	−0.07	−4.6	−0.1719	0.0	0.0079
Ξ^{*-}	0.204	3.4	0.2 ± 0.1	0.35	4.6	0.024	0.1719	0.1954
Ω^{-}	0.204	2.8	0.09 ± 0.05	0.24	0.0	0.014	0.1719	0.1966

Table 5. Quadrupole moments of the spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ decuplet to octet transitions in χCQM using GP method and $SU(3)$ symmetry-breaking.

Baryon	NQM (fm^2)	Skyrme [19] 10^{-2} fm^2	GPM [20] (fm^2)	χCQM with $SU(3)$ Symmetry (fm^2)	χCQM with $SU(3)$ Symmetry-breaking (fm^2)
$\Delta^{+}p$	−0.110	−5.2	−0.082	−0.0608	−0.0846
-0.0846 ± 0.0033					
$\Sigma^{*+}\Sigma^{+}$	−0.110	−0.93	−0.076	−0.0608	−0.0864
$\Sigma^{*-}\Sigma^{-}$	0.0	0.93	0.014	−0.0608	−0.0018
$\Sigma^{*0}\Sigma^{0}$	−0.055	0.0	−0.031	0.0	−0.0441
$\Xi^{*0}\Xi^{0}$	−0.110	2.91	−0.031	−0.0608	−0.0864
$\Xi^{*-}\Xi^{-}$	0.0	−2.91	0.007	0.0	−0.0018
$\Sigma^{*0}\Lambda$	−0.096	−4.83	−0.041	−0.0526	−0.0733

To understand the implications of chiral symmetry breaking and 'quark sea', we have also presented the results of NQM. Since the calculations in χCQM have been carried out using the GP method, the NQM results have also been presented by including the two- and three-quark term contributions of the GP parameters so that the contribution of the 'quark sea' effects can be calculated explicitly. For spin-$\frac{1}{2}^+$ octet baryons, we find that the quadrupole moments are zero for all the cases in NQM. Even if we consider the contribution coming from two-quark terms with the inclusion of 'quark sea', and $SU(3)$ symmetry breaking effects, the quadrupole moments still remain zero. Because of the angular momentum conservation, the quadrupole moments of particles with total angular momentum $1/2$ are zero and remain zero, no matter how complicated are the operator or wave functions. However, the non-zero value can be obtained for the 'intrinsic' quadrupole moments. Further, the inclusion of orbital angular momentum contribution would lead to a zero spectroscopic quadrupole moment because the 'quark sea' spin and orbital angular momentum contribute with the opposite sign. This also agrees with the experimental observations. In the case of $\frac{3}{2}^+$ decuplet baryons, quadrupole moments of the charged baryons are equal whereas all neutral baryons have zero quadrupole moment.

For the $\frac{3}{2}^+$ decuplet, and radiative decays of baryons, it can be easily shown that in the $SU(3)$ symmetric limit, the magnitude of quadrupole moments can be expressed by the following relations:

$$\frac{Q^{\Delta^{++}}}{2} = Q^{\Delta^+} = Q^{\Delta^-} = Q^{\Sigma^{*+}} = Q^{\Sigma^{*-}} = Q^{\Xi^{*-}} = Q^{\Omega^-}, \tag{27}$$

$$Q^{\Delta^+ p} = Q^{\Sigma^{*+}\Sigma^+} = 2Q^{\Sigma^{*0}\Sigma^0} = Q^{\Xi^{*0}\Xi^0} = \frac{2}{\sqrt{3}} Q^{\Sigma^{*0}\Lambda}. \tag{28}$$

The inclusion of $SU(3)$ symmetry breaking changes this pattern considerably and we obtain

$$Q^{\Omega^-} > Q^{\Xi^{*-}} > Q^{\Sigma^{*-}} > Q^{\Delta^-} > \frac{Q^{\Delta^{++}}}{2} > Q^{\Delta^+} > Q^{\Sigma^{*+}},$$

$$2Q^{\Sigma^{*0}\Sigma^0} > Q^{\Xi^{*0}\Xi^0} = Q^{\Sigma^{*+}\Sigma^+} > Q^{\Delta^+ p} = \frac{2}{\sqrt{3}} Q^{\Sigma^{*0}\Lambda}. \tag{29}$$

Also, we have

$$Q^{\Xi^{*0}} = Q^{\Sigma^{*0}} = Q^{\Delta^0}, \tag{30}$$

which has its importance in the isospin limit where the three-quark core in neutral baryons does not contribute to the quadrupole moment. In the limit of $SU(3)$ symmetry breaking, a non-vanishing value for the neutral baryons quadrupole moment is generated by the 'quark sea' through the chiral fluctuations of constituent quarks leading to

$$Q^{\Xi^{*0}} > Q^{\Sigma^{*0}} > Q^{\Delta^0}. \tag{31}$$

In the $SU(3)$ limit, the transition moments involving the negatively charged baryons are zero, that is

$$Q^{\Xi^{*-}\Xi^-} = Q^{\Sigma^{*-}\Sigma^-} = 0. \tag{32}$$

This is because, if flavour symmetry is exact, U-spin conservation forbids such transitions. The exact order of $SU(3)$ symmetry breaking effects can be easily found from tables 4 and 5. Since there is no experimental or phenomenological information available for any of these quadrupole moments, the accuracy of these relations can be tested in future experiments.

For the spin-$\frac{3}{2}^+$ decuplet baryons presented in table 4, quadrupole moments result in NQM using the GP method to predict an oblate shape for all positively charged baryons (Δ^{++}, Δ^+, and Σ^{*+}) and prolate shape for negatively charged baryons (Δ^-, Σ^{*-}, Ξ^{*-}, and Ω^-). It is important to mention here that the NQM is unable to explain the deformation in neutral baryons (Δ^0, Σ^{*0}, and Ξ^{*0}). On incorporating the effects of chiral symmetry breaking and 'quark sea' in the χCQM, a small amount of prolate deformation in neutral baryons (Δ^0, Σ^{*0}, and Ξ^{*0}) is observed. The trend of deformations is however the same for the positively and negatively charged baryons in χCQM and NQM. The other phenomenological models also observe a similar trend, for example, light cone QCD sum rules [21], spectator quark model [17], lattice QCD [24], χPT [23], chiral quark soliton model (χQSM) [16], etc.

For the case of spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions in table 5, it is observed that quadrupole moments of all the transitions are oblate in shape. This result is further endorsed by the predictions of Skyrme model [19]. The effects of chiral symmetry breaking can further be substantiated by a measurement of the other transition quadrupole moments.

6. Summary and conclusion

To summarize, χCQM is able to provide a fairly good description of the quadrupole moments of spin-$\frac{3}{2}^+$ decuplet baryons and spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions using the GP method. For the spin-$\frac{3}{2}^+$ decuplet baryons, prolate shape is observed for the neutral decuplet baryons (Δ^0, Σ^{*0}, and Ξ^{*0}). This trend of deformation is the same for positively and negatively charged baryons in χCQM as well as other phenomenological models, for example, light cone QCD sum rules [21], spectator quark model [17], lattice QCD [24], χPT [23], chiral quark soliton model (χQSM) [16], etc. In the case of spin-$\frac{3}{2}^+ \to \frac{1}{2}^+$ transitions, the oblate shape of the quadrupole moments of all the transitions are endorsed by the predictions of Skyrme model [19]. New experiments aimed at measuring the quadrupole moments of other baryons as well as transitions would provide us with valuable information on the structure of the baryons which is needed for a profound understanding of the nonperturbative regime of QCD.

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