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Role of energy cost in the yield of cold ternary fission of ^{252}Cf

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Abstract. The energy costs in the cold ternary fission of ^{252}Cf for various light charged particle emission are calculated by including Wong's correction for Coulomb potential. Energy cost is found to be higher in cold fission than in normal fission. It is found that energy cost always increases with decrease in experimental yield in all the light charged particle emissions. The higher ground state deformation of the fragments, the odd–even effect and the enhanced yield in the octupole region observed in cold fission are found to be consistent with the concept of energy cost.

Keywords. General properties of fission; spontaneous fission.

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1. Introduction

The cold ternary fission was discovered with the developments of modern experimental techniques such as Gammasphere using triple coincidence method which allowed the fine resolution of the mass, charge and angular momentum content of the fragments. The fact that the light charged particles (LCP) emitted in ternary fission including the cold splitting are focussed mainly onto the equatorial plane perpendicular to the fission axis seems to indicate that most of the ternary clusters originate from the neck region between the two heavier fragments [1]. Since the energy and angular distribution show that they come between the regions of the heavy fragments at a time close to scission, the third particle can be used as a probe of the scission process and these rare processes give new insights into the fission process and its relationship with the theory of multifragmentation of nuclei. Although large numbers of particles are emitted as third particles, α-particles are the most prominent. Recently, direct experimental evidence was presented for the cold neutronless ternary spontaneous fission of ^{252}Cf in which the third particle was an α-particle, ^{10}Be or ^{14}C [2]. The yields of the light charged particles are different for different splittings and ^{10}Be yields in the cold ternary fission of ^{252}Cf are expected to be much smaller than α-particle yield [3].

No theories are available which could reproduce the individual yields and energy distributions of ternary fission fragments from first principles [4]. The yield is found to depend on several parameters, such as the charge and mass number of the fissioning species, its excitation energy, and angular momentum [5]. In the study of the ternary α emission probabilities for five spontaneously fissioning plutonium isotopes, Serot and Wagemans [6] found that the α-emission probability depends on the fissility parameter and the cluster preformation probability factor. The α-emission probability increases with increasing fissility parameter which is proportional to Z^2/A in terms of liquid drop model [7]. In the present study, we are going to calculate the energy cost using the method proposed by Halpern [8] in the various light charged particle emission from the cold ternary splitting of ^{252}Cf and to see what extent this quantity is consistent with the experimentally measured yield in different contexts.

The formation and emission of light charged particle in ternary fission is closely connected with the dynamics from saddle to scission. For the mechanism of the formation and escape of light charged particle during scission, theories have been proposed by Halpern [8] and Carjan [9]. According to Halpern, a sudden collapse of newly distorted fragments to more spherical shapes might enable individual particle to unbound by acquiring sufficient energy from the rapidly changing nuclear potentials in the neck region. The model developed by Carjan describes another mechanism for the emission of light charged particle. According to him, a preformed cluster cloud, through the one-body dissipation mechanism, acquire sufficient energy to be emitted during the descent of the fissioning nucleus towards the scission point. These theories imply that the ternary fission proceeds at the expense of deformation energy. Halpern [8], in his model, tried to give an explanation for the yield for various light charged particles in terms of the quantity energy cost E_c. The idea of Halpern is simple since there are several parameters associated with other theories. Accordingly, E_c is the estimated energy cost for the emission of the designated third particle which is the energy needed for the production and release of the ternary particle. The larger the amount of energy that must be transferred in this way from the fragments to the third particle, the smaller the expected likelihood for the transfer process. That is, the yields of various third particles are expected to be a decreasing function of E_c. Therefore, fission yield can be estimated by calculating the mean extra energy cost (E_c) required for emitting a ternary particle.

2. Energy cost

Formally, energy cost E_c is the amount of energy one would have to supply to remove some third particle from one of the fragments, which are freshly divided, and place it midway between the two fragments and it is computed as the sum of B, ΔV and K. Here B are the binding energies to both the light and heavy fragment which are computed from mass tables and then averaged for the mass divisions. ΔV is the measure of average difference in Coulomb potential energy between the corresponding binary and ternary configurations and K is the average kinetic energy with which the third particle is born. ΔV is calculated as the sum of the Coulomb potential energy of the third particle in the field of its mother fragment plus one fourth its potential energy in the field of the other fragment given by

$$\Delta V = \left(\frac{Z_2 Z_3}{d} + \frac{Z_1 Z_3}{4d}\right) e^2, \tag{1}$$

where $2d$ is the distance between centres of the heavy fragments (in the binary configuration) having atomic numbers Z_1 and Z_2; and Z_3 is the atomic number of the light charged particle.

The cold ternary fission events are characterized by ground state energy or very low total excitation energy (TXE) of the final fragments and high total kinetic energy (TKE) tending to the Q value. In these splittings, the fragments will have compact shapes and ground state deformation at the scission configuration in order to achieve high TKE. Hence the initial scission configurations are better known for cold ternary fission than for normal fission. When we move to cold fission scenario from normal fission, the following changes happen. For the computation of initial kinetic energy in normal fission, there is no consensus among various theories since different theories such as statistical theory and dynamical theories provide different values. The statistical theory predicts an initial kinetic energy of the order of 0.5 MeV and the nondissipative dynamical theory predicts an initial energy of around 20 MeV [10]. In cold fission, Misicu *et al* [11] approximated the potential of light charged particle, V_{LCP}, with a harmonic potential in the y-direction, centred at the saddle point,

$$V_{\mathrm{LCP}} \approx V_{\mathrm{saddle}} + \frac{1}{2} C y^2, \tag{2}$$

where $V_{\mathrm{saddle}} = V_{\mathrm{LCP}}\,(z = z_{\mathrm{saddle}},\, y = 0)$ and $C = (\partial^2 V_{\mathrm{LCP}}/\partial y^2)|_{y=0}$ is the stiffness constant. The elastic stiffness constant can be computed by the expression

$$C = \sum_{i=\mathrm{L,H}} \frac{1}{R^0_{\alpha i}} \sum_{\lambda \geq 0} \left(\left.\frac{\partial V_{\lambda 0 \lambda}(R_{\alpha i})}{\partial R_{\alpha i}}\right|_0 - \frac{\lambda\,(\lambda+1)}{2} \frac{V_{\lambda 0 \lambda}(R_{\alpha i})}{R_{\alpha i}} \right), \tag{3}$$

where λ represents the multipolarity. The distance of light and heavy fragment $R^0_{\alpha\mathrm{L}}$ and $R^0_{\alpha\mathrm{H}}$ respectively to the LCP is given by

$$R^0_{\alpha\mathrm{L}} = \frac{D}{1 + \sqrt{(Z_\mathrm{L}/Z_\mathrm{H})}}, \tag{4}$$

$$R^0_{\alpha\mathrm{H}} = \frac{D}{1 + \sqrt{(Z_\mathrm{H}/Z_\mathrm{L})}}. \tag{5}$$

Here Z_L and Z_H are the atomic numbers of the light and heavy fragments respectively and D is the interfragment distance. The initial kinetic energy of the light charged particle, K, is identified as zero-point energy, E_{LCP}, in the harmonic oscillator potential well given as

$$E_{\mathrm{LCP}} = \frac{1}{2}\hbar \sqrt{\frac{C}{m_{\mathrm{LCP}}}}. \tag{6}$$

Here m_{LCP} is the mass of the light charged particle. Another modification that we have to make in the context of cold fission is in the computation of ΔV. In normal fission, the fragments are highly deformed [12] and hence a deformation correction has to be applied for the Coulomb potential and in the present case we have used the Wong's correction [13] given as

$$V(R,\theta) = \frac{Z_1 Z_2 e^2}{R} + \left[\frac{9}{20\pi}\right]^{1/2} \left[\frac{Z_1 Z_2 e^2}{R^3}\right] \sum_{i=1}^{2} R_{0i}^2 \beta_{2i} P_2(\cos\alpha_i) + \left[\frac{3}{7\pi}\right] \left[\frac{Z_1 Z_2 e^2}{R^3}\right] \sum_{i=1}^{2} R_{0i}^2 \left[\beta_{2i} P_2(\cos\alpha_i)\right]^2, \quad (7)$$

where $R_{0i} = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3}$ and R is the distance between the fragment centres. Here $P_2(\cos\alpha_i)$ is a Legendré polynomial and α_i is the angle between the symmetry axis and the axis connecting the centres of the fragments. The tip distance at scission in normal fission is large (∼15–20 fm) which is much greater than in cold fission [14].

3. Results and discussion

The approximate tip distance at scission in cold fission can be computed using the method suggested by Misicu *et al* [11]. It corresponds to the distance at which the light charged particle (LCP) pocket disappears. This distance for α particle emission is approximately 7 fm, 6 fm for ^{10}Be and 5 fm for ^{14}C. The energy cost in various LCP emissions in the present case of cold neutronless splitting are calculated using the formalism described above by taking the finite size and deformation of the fragments. The deformation parameters and the masses for calculating binding energies are taken from the mass table [15]. The typical values of energy cost in the normal ternary fission are nearly 21 MeV for α-particle and 43 MeV for ^{10}Be [8]. It is found that the energy cost computed in the present case are higher than in normal ternary fission in all light charged particle emissions. In the case of α emission its average value is 26.50 MeV, for ^{10}Be emission it is 48.71 MeV and for ^{14}C emission it is 66.23 MeV. The calculated energy cost for ^{4}He, ^{10}Be and ^{14}C for the various cold ternary fission fragmentations are tabulated in table 1 and is found to be nearly the same for a given light charged particle and increases with the mass number of the light charged particle. In normal ternary fission, even though the estimation of average binding energy B and ΔV are simple, estimation of average initial kinetic energy K is considered as a difficult step. Halpern made its estimation from the duration of the surface collapse time τ. By assuming this time τ corresponds to one quarter of a vibrational period of the fragment, its value has been found to be 2 MeV. The estimated value of $B + \Delta V$ was 19 MeV, giving a total of 21 MeV for the energy cost of α emission. On the other hand, for the corresponding cold α emission the average value of $B + \Delta V$ is 26.45 MeV and that of the initial kinetic energy is 1.70 MeV. It is to be noted that much of the contribution to E_{c} comes through ΔV. On comparison of energy costs with the experimentally measured yield per 100 fission event it is found that for ^{4}He, ^{10}Be and ^{14}C emissions the yield decreases with increase of energy cost. It is also found that in the ^{10}Be and ^{14}C emissions from the splitting of ^{252}Cf, the energy cost for emitting third particle is very high compared to α emission which implies that yield is much lower than α

Table 1. Energy cost and yield in the cold ternary fragmentation of ^{252}Cf isotope.

Splitting	B (MeV)	ΔV (MeV)	K (MeV)	E_c (MeV)	$Y_{expt.}$	Y_{Theory}
^{92}Kr + ^{156}Nd + ^{4}He	8.85	18.26	1.72	28.83	0.002	0.022
^{98}Sr + ^{150}Ce + ^{4}He	9.29	18.86	1.70	29.76	0.014	0.018
^{100}Sr + ^{148}Ce + ^{4}He	9.23	18.06	1.70	28.99	0.021	0.021
^{100}Zr + ^{148}Ba + ^{4}He	8.48	17.68	1.69	27.85	0.038	0.027
^{101}Zr + ^{147}Ba + ^{4}He	3.57	17.70	1.70	22.97	0.082	0.067
^{103}Zr + ^{145}Ba + ^{4}He	1.19	17.70	1.71	20.60	0.084	0.104
^{142}Ba + ^{100}Sr + ^{10}Be	11.31	33.88	2.27	47.46	0.0023	0.0064
^{142}Xe + ^{100}Zr + ^{10}Be	8.44	33.06	2.24	43.47	0.011	0.011
^{140}Xe + ^{102}Zr + ^{10}Be	11.86	33.29	2.21	47.36	0.007	0.0065
^{138}Xe + ^{104}Zr + ^{10}Be	13.72	33.45	2.20	49.37	0.0046	0.0049
^{136}Te + ^{106}Mo + ^{10}Be	15.37	32.74	2.15	50.26	0.0045	0.0044
^{100}Sr + ^{138}Xe + ^{14}C	11.31	50.84	2.79	64.94	0.011	0.011
^{102}Zr + ^{136}Te + ^{14}C	12.75	49.78	2.71	65.24	–	0.010
^{104}Zr + ^{134}Te + ^{14}C	15.75	50.05	2.70	68.50	0.0032	0.0032
^{106}Mo + ^{132}Sn + ^{14}C	17.00	48.95	2.65	68.60	0.0028	0.0031
^{108}Mo + ^{130}Sn + ^{14}C	16.47	49.01	2.64	68.12	0.0045	0.0038
^{98}Sr + ^{140}Xe + ^{14}C	8.65	50.52	2.80	61.97	0.0018	0.031

emission as expected. The experimental value of the yield of α-particle is taken from Ramayya *et al* [16] and that of ^{10}Be and ^{14}C from Hamilton *et al* [2]. Figure 1 represents the graph plotted between natural logarithm of the experimental yield and negative energy

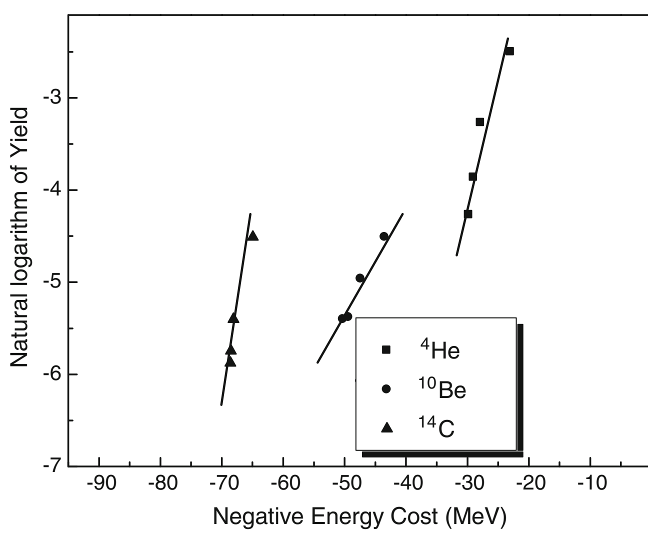

Figure 1. Plot for logarithm of yield with negative energy cost for the emissions of ^{4}He, ^{10}Be and ^{14}C from ^{252}Cf isotope.

Table 2. Constants A and B for yield–energy cost relation.

LCP	A	B
^{4}He	4.949	0.187
^{10}Be	4.580	0.138
^{14}C	6.077×10^{7}	0.345

cost which shows a linear relation in all the three emissions. From the observed variation of yield with energy cost we arrived at an equation for yield which is given as

$$Y = A\mathrm{e}^{-BE_C}. \tag{8}$$

The constants A and B are given in table 2. Using this relation we have computed the yield for various ternary fragmentations which are given in column 6 of table 1. In plotting the graph connecting energy cost vs. yield we have used linear regression and omitted some data points in figure 1 which are not in the straight line, but the straight lines represent the general trend which are evident from table 1. The values of constants A and B show a drastic change in the case of ^{14}C compared to ^{4}He and ^{10}Be particle emissions. This drastic change in the case of ^{14}C implies a possible change in the mechanism of ternary fission as the size of the third particle increases. In other words, when the third fragments become heavier, emission by three fission fragment formation at scission is suggested instead of emission from neck snapping.

Whether the ternary fission process is dynamical or statistical was controversial for a long period. According to the statistical model of ternary fission, developed by Fong [17,18], the fissioning system is assumed to remain in statistical equilibrium and the initial conditions for ternary fission events are governed by phase space arguments. All the distributions can be determined by counting the available states and the relative chance for emission probability can be determined. Due to the high energy cost (nearly 20 MeV) for the emission of the ternary α particle, statistical theory has been ruled out in normal ternary fission [19] since it led to emission probabilities that are orders of magnitude lower than the experimentally observed value for α particle. The high energy cost obtained in the present calculation also points against statistical process and supports the fact that the energy required to produce ternary fission comes from a dynamical process.

In the $^{144-147}$Ba mass region there are evidences for stable octupole deformation [20]. Due to the octupole deformation, penetrability also increases implying higher yield. We have also observed a low value of the calculated energy cost when the heavier partners are Ba isotopes (table 1). Therefore, the low energy cost implying a higher yield according to calculation is consistent with experimental fact.

The enhanced yield of even–even fragment over the odd–odd fragmentation is called odd–even effect. One feature of the cold fission yields is the fact that many odd–odd splitting have values larger than or equal to the neighbouring even–even fragmentations [21]. The increased yields in these cases may be attributed to the differences in level densities near the ground states in odd–odd and the even–even nuclei. The calculated energy cost data show that in the case of odd fragmentation it is lower than that of even fragmentation in few of the cold α-ternary fragmentations presented in table 1 indicating higher yield for odd–odd fragmentation. It is to be noted that this dependence on the odd–even effect is due to the presence of the binding energy term in the expression of energy cost.

4. Conclusions

The energy cost is computed in the cold ternary emission of ^{4}He, ^{10}Be and ^{14}C and compared with the experimental yield. There exists an inverse relationship with the experimental yield. Thus energy cost can be used to have an estimate of the yield of the ternary particle. An overview of the data shows that when the change in energy cost is large, the corresponding change in yield is very small. This sensitivity points to the extent to which this concept is reliable. It is found that nuclear deformations, especially the quadrupole deformations, have provided significant contribution to energy cost. The contribution to energy cost due to deformation is nearly 1% of the total value. Obviously, yield is affected by deformation. Many features of cold fission yield in the context of cold ternary fission are found to be reflected in the value of the energy cost also. The fact that energy cost is an increasing function of the size of mass of the light charged particle and hence lower yield for higher masses points to the limitation in their detection of LCP with higher mass. Since there are too many unknown parameters associated with the initial scission configurations in normal fission compared to cold fission, the uncertainty in the computation of energy cost is less in cold fission than in normal fission.

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