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Heat transfer in MHD flow of dusty viscoelastic (Walters' liquid model-B) stratified fluid in porous medium under variable viscosity

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Abstract. The paper investigates the effects of heat transfer in MHD flow of viscoelastic stratified fluid in porous medium on a parallel plate channel inclined at an angle θ. A laminar convection flow for incompressible conducting fluid is considered. It is assumed that the plates are kept at different temperatures which decay with time. The partial differential equations governing the flow are solved by perturbation technique. Expressions for the velocity of fluid and particle phases, temperature field, Nusselt number, skin friction and flow flux are obtained within the channel. The effects of various parameters like stratification factor, magnetic field parameter, Prandtl number on temperature field, heat transfer, skin friction, flow flux, velocity for both the fluid and particle phases are displayed through graphs and discussed numerically.

Keywords. Walter's liquid model-B; stratified fluid; porous medium; variable viscosity.

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1. Introduction

There are many viscoelastic fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. One such fluid is Walters' (model B) viscoelastic fluid which is used in chemical technology and industry. Walters [1] reported that the mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5 g of polymer per litre with density 0.98 g per litre behaves very similar to the Walters' (model B) viscoelastic fluid. This class of fluids is used in the manufacture of spacecrafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastic engineering equipments, contact lens etc. Walters' (model B) viscoelastic fluid forms the basis for the manufacture of many such important and useful products.

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The problems of the fluid mechanics involving dust, gas particles mixture arise in many processes of practical importance. The flow of a binary mixture of viscous fluid and dust particles is a subject of interest for engineers and scientists because such flow occurs in powder technology, transport of liquid slurries in chemical processing, nuclear processing and in different geophysical situations. The non-Newtonian (Walters' liquid model-B) fluid embedded with symmetrically distributed uniform, non-conducting, solid spherical dust particles is said to be a dusty fluid. The influence of dust particles on the viscoelastic fluid flow has many applications such as in the production of plastic products like rayon and nylon, in the purification of crude oil, in the pulp and paper industry, in the textile industry, in treating environment pollution, in the petroleum industry, in the purification of rain water etc.

Several authors have carried out the study of dusty viscoelastic fluids under different physical conditions. Walters [2] proposed a theoretical model for elastoviscous fluids. Renowned mathematician Saffman [3] has proposed equations of motion for binary mixture of fluid and dust particles. Lal and Johri [4] and Purkait [5] studied MHD transient flow of second-order Rilvin–Erickesen fluid down an inclined channel. MHD flow of unsteady viscoelastic (Walters' liquid model-B) conducting fluid between two porous concentric circular cylinders was discussed by Chakraborty and Sengupta [6]. Sharma and Kumar [7] worked on the stability of two superposed Walters'-B viscoelastic liquids. Combined effects of heat and mass transfer by natural convection in a saturated thermally stratified porous medium were analysed by Angirasa *et al* [8]. Sharma and Rana [9] discussed the thermosolutal instability of Walters' (model B) viscoelastic rotating fluid permeated with suspended particles and variable gravity field in porous medium. Rayleigh–Taylor instability of Walter B elastoviscous fluid through porous medium was discussed by Sharma *et al* [10]. Shapiro and Fedorovich [11] have explained unsteady convectively-driven flow along a vertical plate immersed in a stably stratified fluid. The effects of chemical reaction on MHD flow of dusty viscoelastic (Walters' liquid model-B) liquid with heat source/sink were discussed by Kumar and Srivastava [12]. Magyari *et al* [13] worked on unsteady free convection along an infinite vertical flat plate embedded in a stably stratified fluid-saturated porous medium. Heat transfer in a Walters' liquid model-B fluid over an impermeable stretching sheet with non-uniform heat source/sink and elastic deformation was discussed by Nandeppanavar *et al* [14]. Prakash *et al* [15] investigated Kumar and Srivastava [12] with thermal diffusion. Kumar and Singh [16] worked on the stability of two straitified Walters' B viscoelastic superposed fluids. Stability of stratified elastoviscous Walters' model B fluid in the presence of horizontal magnetic field and rotation was explained by Sharma and Gupta [17]. Chang *et al* [18] carried out the numerical results for transient-free convective mass transfer in a Walters'-B viscoelastic flow with wall suction.

In the present paper we are introducing Walters' liquid model-B instead of Rivlin–Ericksen fluid under the same conditions and assumptions taken by Chakarvarty [19]. In the present note, the problem is formulated, solved and pertinent results for velocity, skin friction, flow flux and Nusselt number are discussed in detail using graphs.

2. Formulation of the problem

We consider fully developed flow of an incompressible, dusty (Walters' liquid model-B) fluid of electrically conducting material through a parallel plate channel. The dusty fluid is assumed to be flowing between two infinitely long plates separated by $2h$, inclined horizontally by an angle θ. The centre line of the channel coincides with x-axis while y-axis is perpendicular to it. A uniform magnetic field B_0 is applied normal to the plates. The inertial force experienced by the dust particles is equal and opposite to that due to fluid motion. Since the plates are infinitely long, the velocity of fluid and particles are functions of y and t. The dust particles are assumed to be solid, electrically non-conducting, spherical, uniformly distributed throughout the fluid, identical and symmetrical in size. The number density of the dust particles is constant and its value is small throughout the motion. There is no chemical reaction, mass transfer and heat radiation among the dust particles. Hall effect, polarization effect and the effect due to buoyancy are negligible. The value of magnetic Reynolds number (Rm) is small enough so that the induced field is negligible. The fluid is conducting so that the Joule effect due to the presence of external magnetic field is negligible. Initially (at time $t = 0$) there is no flow and plates are at two different temperatures, i.e. T_0 for lower plate and T for upper plate, $T > T_0$. The fluid density and viscosity under consideration are varying along y-axis throughout the channel and are given as

$$\rho = \rho_0 \mathrm{e}^{-n(y/h+1)}, \tag{1}$$

$$\mu = \mu_0 \mathrm{e}^{-n(y/h+1)}, \tag{2}$$

where n, ρ_0 and μ_0 are stratification factor, the coefficient of density and the viscosity of fluid respectively on the line of channel at $y = -h$. A schematic diagram of the problem is shown in figure 1.

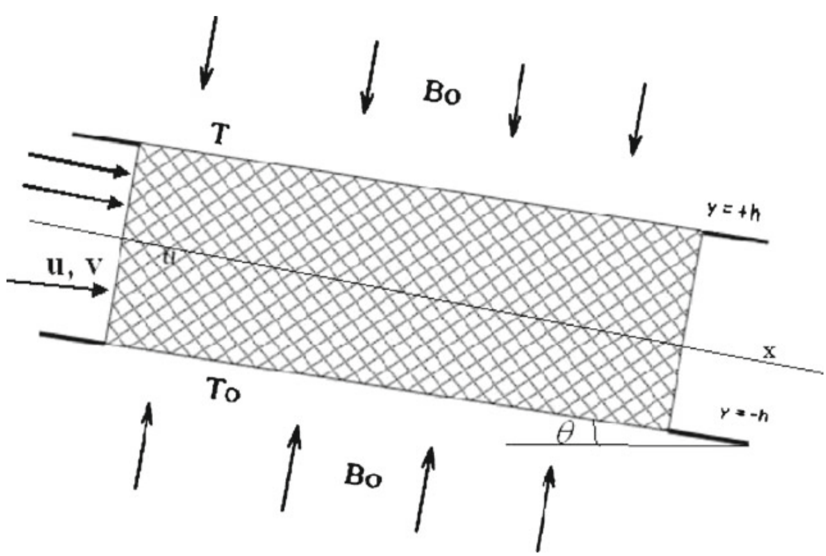

Figure 1. Schematic diagram of the problem.

3. Governing equations

Under the assumptions described already, the equations governing the flow are:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \left(n\frac{\upsilon_1}{h}\right)\frac{\partial u}{\partial y} + \upsilon_1\frac{\partial^2 u}{\partial y^2} - \upsilon_2\left\{\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right)\right\}$$
$$-\frac{\upsilon_1}{k_1}u + g\sin\theta + K\frac{N}{\rho}(v-u) + \frac{\sigma}{\rho}\left(B_0^2 u\right) \quad (3)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} + g\cos\theta = 0, \quad (4)$$

$$m\frac{\partial v}{\partial t} - K(u-v) = 0, \quad (5)$$

$$C_{\mathrm{p}}\frac{\partial T}{\partial t} = \alpha C_{\mathrm{p}}\frac{\partial^2 T}{\partial y^2} + \upsilon_1\left(\frac{\partial u}{\partial y}\right)^2 + \upsilon_2\left\{\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2\right\}, \quad (6)$$

where m is the mass of the dust particle, C_{p} is the specific heat under constant pressure, p is the fluid pressure, K is the proportionality constant, T is the fluid temperature, υ_1 and υ_2 are respectively the kinematic coefficient of fluid viscosity and viscoelasticity and α is the thermal conductivity.

The fluid pressure is

$$p = \rho g\{x\sin\theta - y\cos\theta\} + \rho x a(t) + A, \quad (7)$$

where A is a constant.

Equation (3) with (7) is

$$\frac{\partial u}{\partial t} = -a(t) - n\frac{\upsilon_1}{h}\frac{\partial u}{\partial y} + \upsilon_1\frac{\partial^2 u}{\partial y^2} - \upsilon_2\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{\upsilon_1}{k_1}u$$
$$+ K\frac{N}{\rho}(v-u) + \frac{\sigma}{\rho}(B_0^2 u), \quad (8)$$

where N is the number density of dust particles.

The boundary conditions are

$$u = 0, \quad v = 0, \quad T = T_0 \mathrm{e}^{-2nt}, \quad \text{at } y = -h,$$
$$u = u_0 \mathrm{e}^{-nt}, \quad v = v_0 \mathrm{e}^{-nt}, \quad T = T_1 \mathrm{e}^{-2nt}, \quad \text{at } y = +h, \quad (9)$$

where T_0 and T_1 are the temperatures at the plates $y = -h$ and $y = +h$ respectively.

The following are non-dimensional parameters

$$u^* = \frac{u}{u_0}, \quad v^* = \frac{v}{v_0}, \quad y^* = \frac{y}{h}, \quad t^* = \frac{t u_0}{h}$$
$$a^* = \frac{ah}{u_0^2}, \quad T^* = \frac{T}{T_0}, \quad k^* = \frac{h}{\sqrt{k_1}}, \quad \lambda = \frac{u_0}{v_0}. \quad (10)$$

Here k_1 is the porosity of the medium. Introducing these non-dimensional quantities in eqs (5), (6) and (8) and omitting stars, we get

$$\frac{\partial u}{\partial t} = -a(t) - \frac{n}{R}\frac{\partial u}{\partial y} + \frac{1}{R}\frac{\partial^2 u}{\partial y^2} + \eta\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) - \frac{K^2}{R}u + \frac{C}{R_t}\left(\frac{v}{\lambda} - u\right) + \frac{M^2}{R}u, \quad (11)$$

$$R_t \frac{\partial v}{\partial t} - (u\lambda - v) = 0, \tag{12}$$

$$\frac{\partial^2 T}{\partial y^2} = R\,\mathrm{Pr}\,\frac{\partial T}{\partial t} - \left\{E\,\mathrm{Pr}\left(\frac{\partial u}{\partial y}\right)^2\right\} + \eta E R\,\mathrm{Pr}\left\{\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial y}\right)^2\right\}, \tag{13}$$

where $R = (u_0 h/\upsilon_1)$ – Reynolds number, $\eta = \left(-\upsilon_2 h/h^2\right)$ – viscoelastic parameter, $C = (mN/\rho)$ – dust particles concentration, $R_t = \{mu_0/Kh\}$ – relaxation time parameter for dust particles, $M = \sqrt{\left\{B_0^2 h^2 \sigma/\rho \upsilon_1\right\}}$ – Hartmann number, $E = \left\{u_0^2/C_\mathrm{p} T_0\right\}$ – Eckert number and $\mathrm{Pr} = \upsilon_1/\alpha$ – Prandtl number.

The boundary conditions reduce to:

$$\begin{aligned} &u = v = 0, \quad T = \mathrm{e}^{-2nt}, \quad \text{at } y = -1, \\ &u = v = \mathrm{e}^{-nt}, \quad T = \chi \mathrm{e}^{-2nt}, \quad \text{at } y = +1, \end{aligned} \tag{14}$$

where $\chi = (T_1/T_0)$ is a constant temperature.

4. Solution of the problem

For the solution of eqs (11)–(13) under the boundary conditions (14), we assume

$$u = f(y)\mathrm{e}^{-nt}, \quad v = g(y)\mathrm{e}^{-nt}, \quad T = F(y)\mathrm{e}^{-2nt}, \quad a = a_0 \mathrm{e}^{-nt}. \tag{15}$$

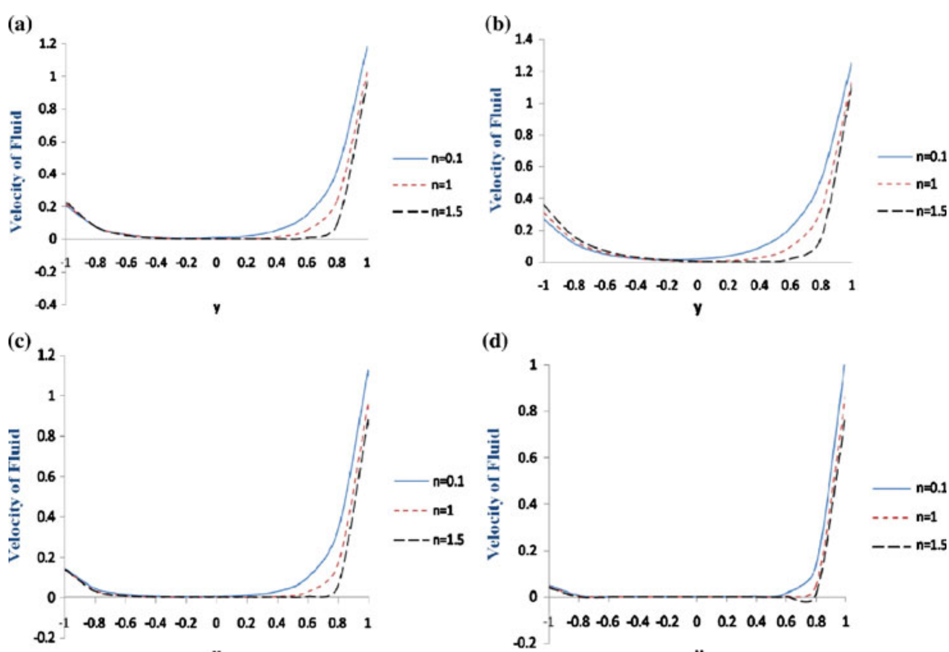

Figure 2. Dimensionless velocity profile of fluid for different values of η, M, K ($t = 1$, $n = 0.5$, $R = 5$, $a_0 = 1$, $C = 0.1$, $R_t = 0.1$, $\lambda = 1.5$). (**a**) $K = 5$, $M = 0.5$, (**b**) $K = 5$, $M = 2.5$, (**c**) $M = 1$, $K = 6$, (**d**) ($M = 1$, $K = 10$).

On solving eqs (11)–(13) with (15), we get

$$\frac{\partial^2 f(y)}{\partial y^2} - A_1 \frac{\partial f(y)}{\partial y} + A_2 f(y) - A_3 = 0 \tag{16}$$

$$g(y) - \lambda \frac{f(y)}{(1 - nR_t)} = 0 \tag{17}$$

$$\frac{\partial F(y)}{\partial y^2} + A_4 F(y) + A_5 \left(\frac{\partial f(y)}{\partial y} \right)^2 = 0. \tag{18}$$

The corresponding boundary conditions change to:

$$\begin{aligned} &f(-1) = g(-1) = 0, \quad F(-1) = 1, \\ &f(1) = g(1) = 1, \quad F(1) = \chi. \end{aligned} \tag{19}$$

On solving eqs (16)–(18) under the boundary conditions (19), we get

$$f(y) = C_1 e^{m_1 y} + C_2 e^{m_2 y} + \frac{A_3}{A_2} \tag{20}$$

$$g(y) = \frac{1}{A_6} \left[C_1 e^{m_1 y} + C_2 e^{m_2 y} + \frac{A_3}{A_2} \right] \tag{21}$$

$$F(y) = C_3 \cos \sqrt{A_4} y + C_4 \sin \sqrt{A_4} y - \left[S_1 e^{2m_1 y} + S_2 e^{2m_2 y} + S_3 e^{(m_1+m_2)y} \right]. \tag{22}$$

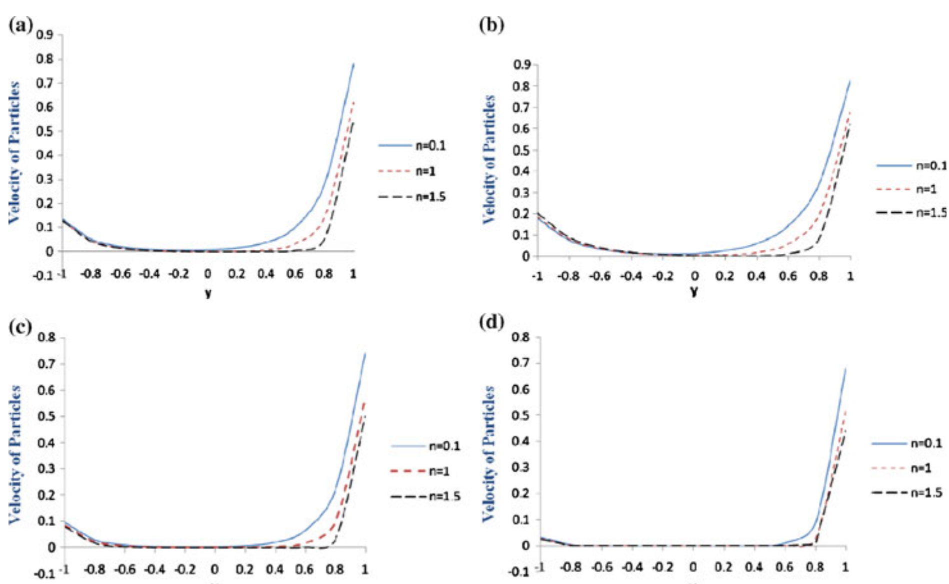

Figure 3. Dimensionless velocity profiles of dust particles for different values of η, M, K. (**a**) $K = 5$, $M = 0.5$, (**b**) $K = 5$, $M = 2.5$, (**c**) $M = 1$, $K = 6$ and (**d**) $M = 1$, $K = 10$.

From eqs (15) with the help of eq. (20), velocity of the fluid is given by

$$u(y,t) = \left[C_1 e^{m_1 y} + C_2 e^{m_2 y} + \frac{A_3}{A_2} \right] e^{-nt}. \tag{23}$$

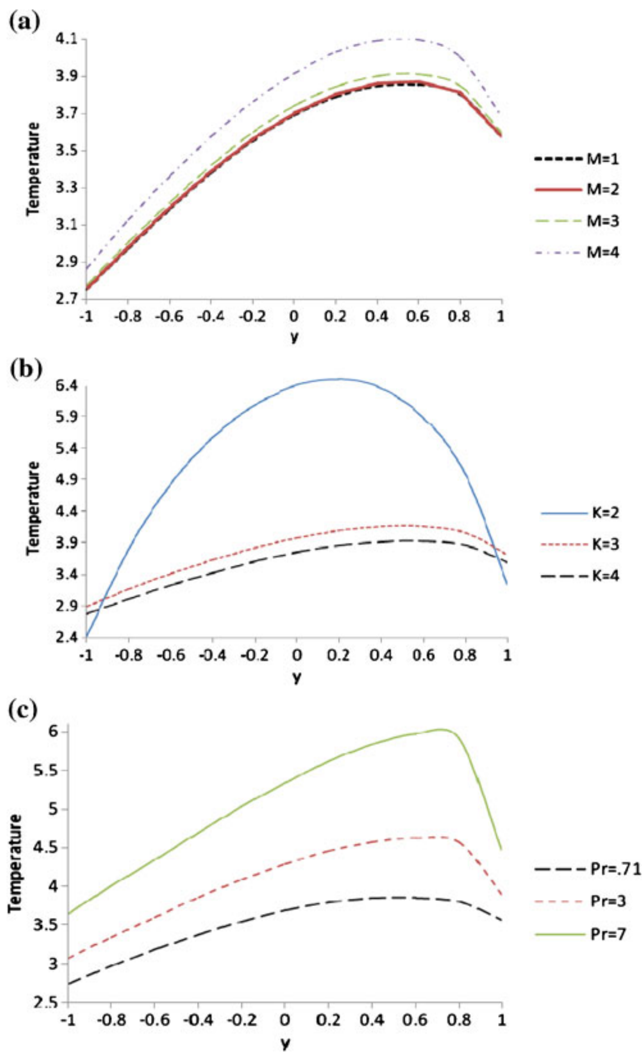

Figure 4. Dimensionless temperature field for different values of M, K, Pr ($t = 1$, $n = 0.5$, $R = 5$, $a_0 = 1$, $\eta = 0.1$, $C = 0.1$, $R_t = 0.1$, $E = 0.5$, $\lambda = 1.5$). (**a**) Temperature field for different values of M, (**b**) temperature field for different values of K and (**c**) temperature field for different values of Pr.

Similarly, from eqs (15) with the help of eq. (21), velocity of the dust particles is given by

$$v\,(y,t) = \frac{1}{A_6}\left[C_1 \mathrm{e}^{m_1 y} + C_2 \mathrm{e}^{m_2 y} + \frac{A_3}{A_2}\right]\mathrm{e}^{-nt}. \tag{24}$$

Temperature distribution within the channel is given by

$$\begin{aligned} T\,(y,t) = &\Big\{C_3 \cos\sqrt{A_4}y + C_4 \sin\sqrt{A_4}y \\ &-\left[S_1 \mathrm{e}^{2m_1 y} + S_2 \mathrm{e}^{2m_2 y} + S_3 \mathrm{e}^{(m_1+m_2)y}\right]\Big\}\,\mathrm{e}^{-2nt}. \end{aligned} \tag{25}$$

5. Skin friction

The skin friction (viscous drag) at the plates for fluid (τ_{f}) and for dust particles (τ_{p}) are given by

$$\begin{aligned} \tau_{\mathrm{f}} &= \left[\left(\frac{1}{R} - \eta\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial y}\right)\right]_{y=\pm 1} \\ &= \left(\frac{1}{R} + n\eta\right)\left[C_1 m_1 \mathrm{e}^{\pm m_1} + C_2 m_2 \mathrm{e}^{\pm m_2}\right]\mathrm{e}^{-nt} \end{aligned} \tag{26}$$

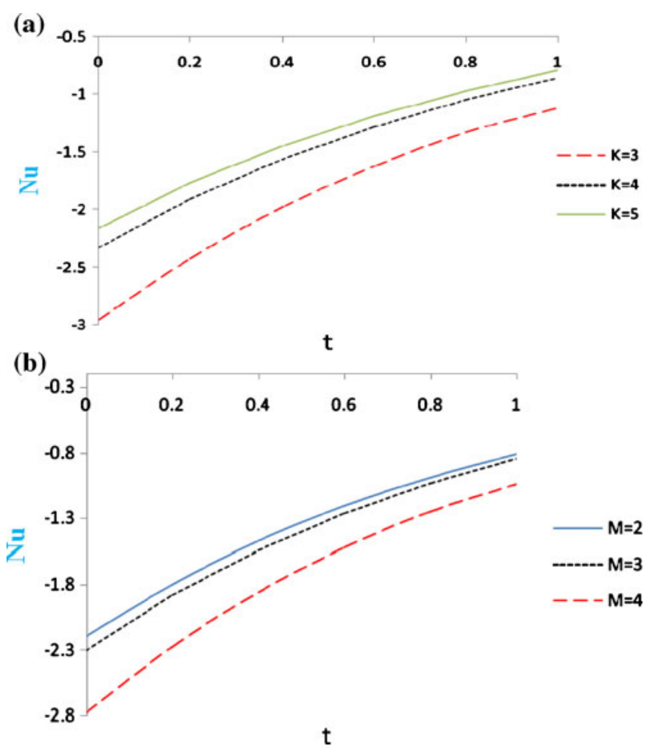

Figure 5. Dimensionless rate of heat transfer for different values of M, K ($t = 1$, $n = 0.5$, $R = 5$, $a_0 = 1$, $\eta = 0.1$, $C = 0.1$, $R_t = 0.1$, $E = 0.5$, $\lambda = 1.5$, $\mathrm{Pr} = 0.71$). (**a**) Rate of heat transfer for different values of K and (**b**) rate of heat transfer for different values of M.

$$\tau_{\mathrm{p}} = \left[\left(\frac{1}{R} - \eta\frac{\partial}{\partial t}\right)\left(\frac{\partial v}{\partial y}\right)\right]_{y=\pm 1}$$
$$= \frac{1}{A_6}\left(\frac{1}{R} + n\eta\right)\left[C_1 m_1 \mathrm{e}^{\pm m_1} + C_2 m_2 \mathrm{e}^{\pm m_2}\right]\mathrm{e}^{-nt}. \quad (27)$$

6. Flow flux for fluid and particles

The flow flux for fluid (ϕ_{f}) and particles (ϕ_{p}) within the channel are given as

$$\phi_{\mathrm{f}} = \int_{-1}^{1} u\,\mathrm{d}y = \mathrm{e}^{-nt}\int_{-1}^{1} f(y)\mathrm{d}y$$
$$= 2\left[\frac{C_1}{m_1}\sinh m_1 + \frac{C_2}{m_2}\sinh m_2 + \frac{A_3}{A_2}\right]\mathrm{e}^{-nt} \quad (28)$$

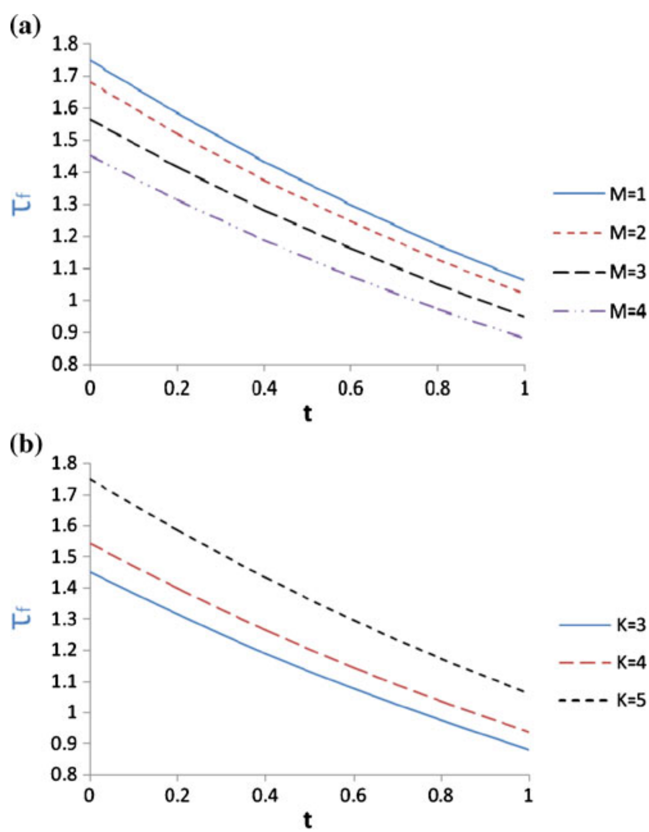

Figure 6. Dimensionless skin friction of fluid for different values of M, K ($n = 0.5$, $R = 5$, $a_0 = 1$, $\eta = 0.1$, $C = 0.1$, $R_t = 0.1$, $\lambda = 1.5$). (**a**) Skin friction of fluid for different values of M and (**b**) skin friction of fluid for different values of K.

$$\phi_{\mathrm{p}} = \int_{-1}^{1} v\,\mathrm{d}y = \mathrm{e}^{-nt}\int_{-1}^{1} g(y)\,\mathrm{d}y = \frac{2}{A_6}\left[\frac{C_1}{m_1}\sinh m_1 + \frac{C_2}{m_2}\sinh m_2 + \frac{A_3}{A_2}\right]\mathrm{e}^{-nt}. \quad (29)$$

7. Heat transfer

It may be necessary to evaluate the rate of heat transfer. The heat transfer coefficient in terms of Nusselt number (N_u) at the wall surfaces is given as

$$N_u = \left[\frac{\partial T}{\partial y}\right]_{y=\pm 1} = \mathrm{e}^{-2nt}\left[\frac{\partial F}{\partial y}\right]_{y=\pm 1}$$

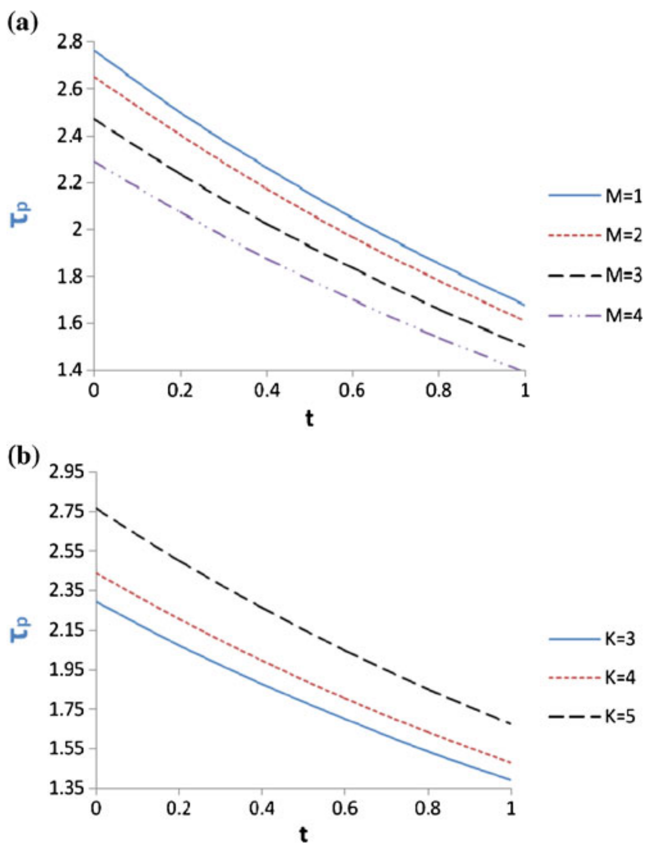

Figure 7. Dimensionless skin friction of dust particles for different values of M, K ($n = 0.5$, $R = 5$, $a_0 = 1$, $\eta = 0.1$, $C = 0.1$, $R_t = 0.1$, $\lambda = 1.5$). (**a**) Skin friction of dust particles for different values of M and (**b**) skin friction of dust particles for different values of K.

$$= \Big\{-C_3\sqrt{A_4}\sin\sqrt{A_4} + C_4\sqrt{A_4}\cos\sqrt{A_4} - \big[2S_1 m_1 e^{\pm 2m_1} + 2S_2 m_2 e^{\pm 2m_2} + S_3(m_1+m_2)e^{\pm(m_1+m_2)}\big]\Big\} e^{-2nt}. \quad (30)$$

8. Results and discussions

The present investigation highlights the behaviour of dusty viscoelastic (Walters' liquid model-B) stratified fluid subject to a variable viscosity. The system of momentum and heat transfer equation are solved analytically and analysed using the stratification factor (η), magnetic field parameter (M), porous parameter (K) and Prandtl number (Pr). The lower plate of the channel is kept at lower temperature T_0 and upper plate is kept at high temperature T to see the effect of temperature on the velocity of the fluid near the plate. The numerical calculation for skin friction and Nusselt number have been carried out at the upper wall of the channel. The velocity, flow flux and skin friction for both the fluid and dust particles as well as the rate of heat transfer for constant values of $t = 1$, $n = 0.5$, $R = 5$, $a_0 = 1$, $C = 0.1$, $R_t = 0.1$, $E = 0.5$, $\chi = 1.5$, $\lambda = 1.5$ are presented in various graphs.

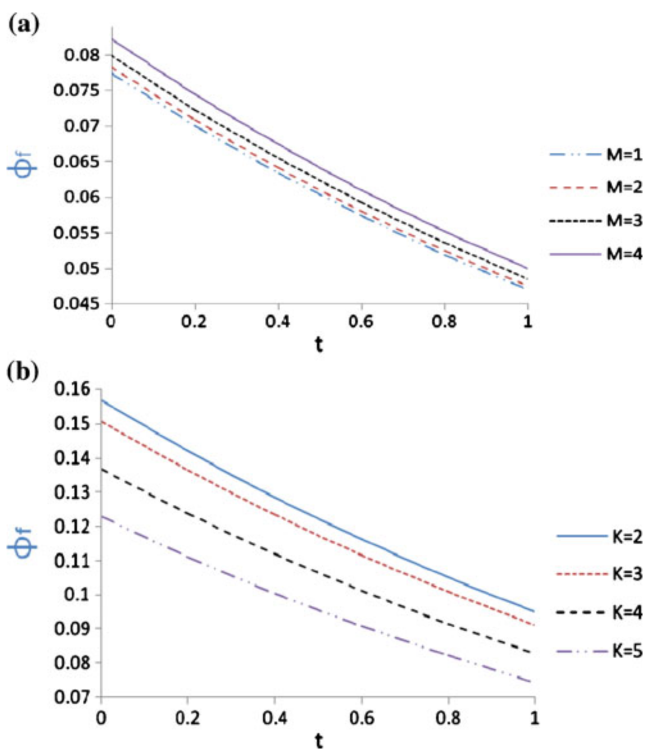

Figure 8. Dimensionless flow flux of fluid for different values of M, K ($n = 0.5$, $R = 5$, $a_0 = 1$, $C = 0.1$, $R_t = 0.1$, $\lambda = 1.5$). (**a**) Flow flux of fluid for different values of M and (**b**) flow flux of fluid for different values of K.

Figures 2 and 3 elucidate the velocity of the fluid and dust particles for different values of stratification factor n with increasing magnetic field parameter M and porous parameter K. It is observed that the velocity of the fluid and dust particles increases for increasing values of magnetic field parameter M (0.5 to 2.5) while increasing values of porous parameter (K = 6 to 10) for all values of n between n = 0.1 and n = 1.5 decreases the same. Velocity of the fluid and that of the particles is decreasing for increasing values of stratification factor n. At the lower plate, the velocity of the fluid and particles is low while it is high at the upper plate because the lower plate is kept at lower temperature and upper plate at high temperature. As the temperature increases, the velocity of the fluid as well as the fluid particles increases. Increasing the value of temperature reduces the viscosity of fluid resulting in low skin friction. Due to low skin friction, the velocity of the fluid increases. Figures 2c, 3c are plotted for velocity of the fluid and fluid particles for porous parameter (K = 6) while figures 2d, 3d are plotted for porous parameter, K = 10. Increasing values of porous parameter decreases the velocity of the fluid. Therefore, figures 2d, 3d show low velocity in comparison to figures 2c, 3c when $y \sim 0.8$.

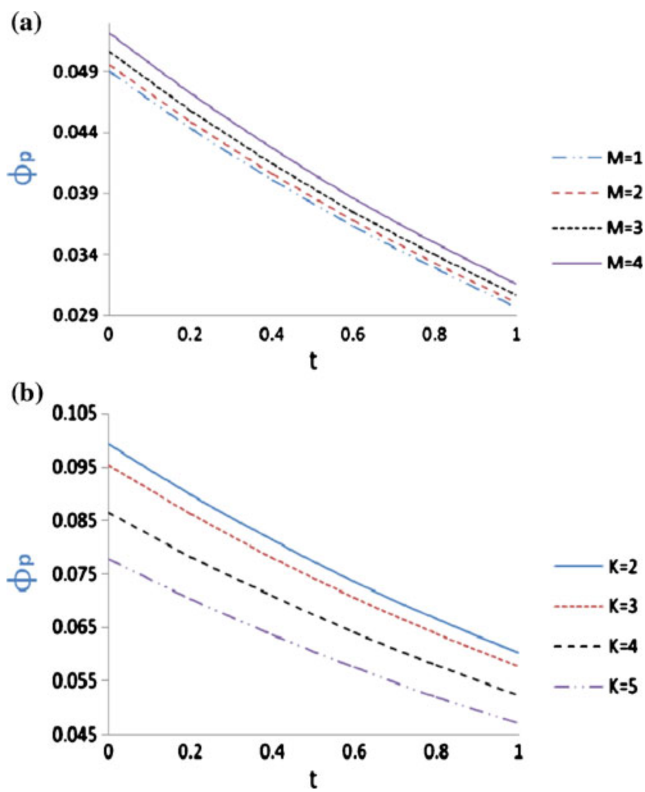

Figure 9. Dimensionless flow flux of dust particles for different values of M, K (n = 0.5, $R = 5$, $a_0 = 1$, $C = 0.1$, $R_t = 0.1$, $\lambda = 1.5$). (**a**) Flow flux of dust particles for different values of M and (**b**) flow flux of dust particles for different values of K.

Figure 4 illustrates the temperature field for different values of magnetic field parameter M, porous parameter K and Prandtl number Pr. The temperature increases for increasing values of magnetic field parameter and Prandtl number while it decreases for increasing values of porous parameter. Temperature increases linearly from the lower plate towards the upper plate, but near the upper plate it decreases rapidly. Temperature is maximum between the centre of the channel and upper plate.

Effects of porous parameter and magnetic field parameter on rate of heat transfer, i.e. Nusselt number (N_u) are shown in figures 5a and 5b. Increasing values of porous parameter and magnetic field parameter increase and decrease the rate of heat transfer respectively. The presence of porous parameter and magnetic field parameter increases the rate of heat transfer against time t.

Figures 6 and 7 depict the skin friction for fluid and particle phase respectively. It is observed that both increasing magnectic field parameter and decreasing porous parameter decrease the skin friction for both dusty fluid and dust particles. From figures 6 and 7 it is also observed that the skin friction is low at upper plate because the upper plate has high temperature in comparison to lower plate. Increasing values of temperature reduce the viscosity of fluid leading to low skin friction at the upper plate.

Figures 8 and 9 illustrate the effects of magnetic field and porous parameter on flow flux. Figures show that the flow flux increases for increasing values of magnetic field parameter while decreases for increasing values of porous parameter.

9. Conclusions

On the basis of the obtained results the following are the observations:

(1) Velocity profile for both the dusty fluid and dust particles is minimum at the centre of the channel and maximum at the upper plate for all values of sratification factor, magnetic filed parameter and porous parameter. It is due to the high temperature at the upper plate which reduces the viscosity of fluid resulting in high velocity.
(2) Velocity profiles for dusty fluid and particle phase increase rapidly near the upper plate for all values of stratification factor, magnetic field parameter and porous parameter.
(3) Velocity profile for the dust particles behaves the same as the dusty fluid which is the source velocity for dust particles.
(4) Skin friction for the dusty fluid is less than the dust particles.
(5) Flow flux of the dusty fluid is greater than the particle phase.
(6) Temperature at the centre of the channel increases very fast for $K = 2$ in comparison to $K = 3, 4$.
(7) Effect on temperature of magnetic field parameter ($M = 4$) is very high in comparison to $M = 1, 2, 3$.

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Appendix

$$A_1 = \frac{n}{1-n\eta R}, \quad A_3 = \frac{a_0}{1-n\eta R}, \quad A_4 = (2nR\,\mathrm{Pr}), \quad A_6 = \frac{\lambda}{(1-nR_t)}$$

$$A_2 = \frac{R}{1-n\eta R}\left[n - \frac{\left(K^2-M^2\right)}{R} + \frac{C\,n}{1-nR_t}\right], \quad A_5 = E\,\mathrm{Pr}\,(1+2nR\eta),$$

$$m_1 = \frac{A_1+\sqrt{A_1^2-4A_2}}{2}, \quad m_2 = \frac{A_1-\sqrt{A_1^2-4A_2}}{2},$$

$$A_7 = \frac{\sinh m_1}{\mathrm{e}^{-m_1}\sinh m_2 - \mathrm{e}^{-m_2}\sinh m_1}, \quad C_2 = A_7\left\{\frac{A_3}{A_2} + \mathrm{e}^{-m_1}\sinh m_2\right\},$$

$$C_1 = \frac{1}{\sinh m_1}\left\{\frac{1}{2} - C_2\sinh m_2\right\}, \quad S_1 = \frac{A_5C_1^2m_1^2}{4m_1^2+A_4}, \quad S_2 = \frac{A_5C_2^2m_2^2}{4m_2^2+A_4},$$

$$S_3 = \frac{A_5 2C_1C_2m_1m_2}{(m_1+m_2)^2+A_4},$$

$$C_4 = \frac{1}{\sin\sqrt{A_4}}\left[\frac{\chi-1}{2} + S_1\sinh 2m_1 + S_2\sinh 2m_2 + S_3\sinh(m_1+m_2)\right].$$

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