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Quantitative assessment of target dependence of pion fluctuation in hadronic interactions – estimation through erraticity

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Abstract. Event-to-event fluctuation pattern of pions produced by proton and pion beams is studied in terms of the newly defined erraticity measures $\chi(p, q)$, χ_q' and μ_q' proposed by Cao and Hwa. The analysis reveals the erratic behaviour of the produced pions signifying the chaotic multiparticle production in high-energy hadron–nucleus interactions (π^-–AgBr interactions at 350 GeV/c and p–AgBr interactions at 400 GeV/c). However, the chaoticity does not depend on whether the projectile is proton or pion. The results are compared with the results of the VENUS-generated data for the above interactions which suggests that VENUS event generator is unable to reproduce the event-to-event fluctuations of spatial patterns of final states. A comparative study of p–AgBr interactions and p–p collisions at 400 GeV/c from NA27, with the help of a quantitative parameter for the assessment of pion fluctuation, indicates conclusively that particle production process is more chaotic for hadron–nucleus interactions than for hadron–hadron interactions.

Keywords. Hadron–nucleus interaction; erraticity; entropy index.

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1. Introduction

A large collection of experimental and theoretical data exist on density fluctuation in multihadron production processes in high-energy physics ranging from hadron–hadron collisions to nucleus–nucleus collisions [1–9]. The investigation of multiplicity fluctuation in particle production has recently entered into a new phase. In multipion production process, the density of the emitted pion spectra fluctuates from phase-space bin-to-bin for each interaction (namely, spatial fluctuation) and these fluctuations also fluctuate from one

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event to the another (namely, event space fluctuation). Up to now, little attention has been paid to capture both the fluctuations simultaneously for the better understanding of the production dynamics. The method of factorial moment F_q does not completely take care of both the fluctuations at a time. In the case of vertically averaged horizontal moments, only the spatial fluctuation is taken into account neglecting the event space fluctuation. On the other hand, in the case of horizontally averaged vertical moments, the spatial fluctuation is neglected and only the fluctuations from event-to-event is taken into account. Thus, the common methods lead to the loss of information on the complete chaotic nature of the multiparticle production processes.

New moments $C_{p,q}$ have been introduced by Hwa and Cao [10–12] which are the moments of factorial moment distributions and take into account the spatial fluctuation as well as the event space fluctuation and thus are able to probe the production dynamics more deeply than factorial moments. Consequently, $C_{p,q}$ moments are sensitive to the fluctuations of factorial moments from event to event. Those fluctuations depend on the bin size because factorial moment F_q itself is a description of spatial pattern that varies according to the resolution. Thus, if those fluctuations scale with bin size, then the fluctuation is erratic in nature and the measure of these erratic fluctuations, 'the entropy index', can be extracted.

Many phenomena in nature ranging from phase transition in condensed matter to galactic clustering in astrophysics involve spatial patterns. Therefore, erraticity analysis has a wide range of applicability. It is a general measure of event-to-event fluctuation.

The study on erratic fluctuation originated in an attempt to understand possible chaotic behaviours in quark and gluon jets [10]. Further, the erraticity analysis was applied to classical chaos [13], multiparticle production in simulated data of hadronic collisions whose parameters are tuned to that of NA22 data [14], in 400 GeV/c p–p collision data from NA27 [15,16], in π^+–p and K^+–p collisions at 250 GeV/c [17], in ^{32}S–AgBr interactions data at 200 A GeV [18], in phase transition [19] and in heartbeat irregularity [20].

The moments $C_{p,q}$, however, do not show simple power-law dependences on the bin size when $p \geq 1.5$ [21]. New measures of erraticity are proposed by Cao and Hwa that generalize the bin size dependence [22].

This paper reports a study on event-to-event fluctuation pattern of pion production in π^-–AgBr interactions at 350 GeV/c and p–AgBr interactions at 400 GeV/c using the new generalized methodology proposed by Cao and Hwa [22]. The results of the experimental data have been compared with that of the VENUS-generated data for both the cases. To study target dependence of chaos in particle production process, we have performed a comparative analysis of hadron–nucleus interactions data (p–AgBr) and hadron–hadron interactions data (p–p, NA27 [23]) at 400 GeV/c.

2. Experimental details

Stacks of G5 nuclear emulsion plates were horizontally exposed to a π^- beam of 350 GeV incident energy from CERN SPS at CERN and to a proton beam of 400 GeV incident energy at Fermilab. Each plate was scanned by two independent observers to increase the scanning efficiency.

The events were chosen according to the following criteria:

- (i) The incident beam track should not exceed 3° from the main beam direction in the pellicle. This is done to ensure that we have taken the real projectile beam.
- (ii) Events showing interactions within 20 μm from the top and bottom surface of the pellicle were rejected. This is done to reduce the loss of tracks as well as to reduce the error in angle measurement.
- (iii) The incident particle tracks, which induced interactions, were followed in the backward direction to ensure that they indeed were projectile beams starting from the beginning of the pellicle.

According to the emulsion terminology, the particles emitted from interactions are classified as:

1. *Black particles*: They are target fragments with ionization greater or equal to $10I_0$, I_0 being the minimum ionization of a singly charged particle. Their range is less than 3 mm, the velocity less than $0.3c$ and the energy less than 30 MeV, where c is the velocity of light in vacuum.
2. *Grey particles*: They are mainly fast target recoil protons with energy up to 400 MeV. They have ionization $1.4I_0 \leq I < 10I_0$. Their ranges are greater than 3 mm and their velocities (v) are $0.7c \geq v \geq 0.3c$.
3. *Shower particles*: The relativistic shower tracks with ionization I less than or equal to $1.4I_0$ are mainly produced by pions and are not generally confined within the emulsion pellicle.
4. *The projectile fragments*: They are a different class of tracks with constant ionization, long range and small emission angle.

To ensure that the targets in the emulsion are silver or bromine nuclei, we have chosen only the events with at least eight heavily ionizing tracks (black+grey). Details about scanning and measurement of the datasets may be found in refs [24–28].

The emission angle (θ) was measured for each track by taking the coordinates (X_0, Y_0, Z_0) of the interaction point, coordinates (X_1, Y_1, Z_1) at the end of the linear portion of each secondary track and coordinate (X_i, Y_i, Z_i) of a point on the incident beam. The accuracy in pseudorapidity is of the order of 0.1 pseudorapidity units.

It is worthwhile to mention that the emulsion technique possesses very high spatial resolution, which makes it a very effective detector for studying the erratic behaviour of the produced pions in high-energy hadron–nucleus interaction.

3. Method of analysis

Cao and Hwa [10] defined event factorial moments for spatial patterns of a multiparticle system as

$$F_q^{(e)}(M) = \left(\frac{1}{M}\sum_{i=1}^{M} n_i\,(n_i-1)\cdots(n_i-q+1)\right) \times \left(\frac{1}{M}\sum_{i=1}^{M} n_i\right)^{-q},$$

where M is the partition number in phase space, n_i is the number of particles in the ith bin for eth event and $q = 2, 3, 4, \ldots$ is the order of the moment. Since $F_q^e(M)$ fluctuates from event to event, to quantify the degree of that fluctuation, a new normalized moment is defined as [10]

$$C_{p,q}(M) = \langle \Phi_q^p(M) \rangle, \tag{1}$$

where $\Phi_q(M) = F_q^e(M)/\langle F_q^e(M) \rangle$ and p is a positive real number. If $C_{p,q}(M)$ has a power-law behaviour as the division number M goes to infinity

$$C_{p,q}(M) \propto M^{\Psi_q(p)}, \quad M \to \infty, \tag{2}$$

then the behaviour is referred to as erraticity and $\Psi_q(p)$ as erraticity exponent.

If the power-law behaviour of eq. (2) is not strictly satisfied by the experimental data, since the general behaviours of $C_{p,q}(M)$ are rather similar in shape, one can regard $C_{2,2}(M)$ as the reference that carries the typical dependence on M and examine $C_{p,q}(M)$ as a function of $C_{2,2}(M)$ when M is varied as an implicit variable.

If $C_{p,q}(M)$ follows the scaling behaviour with $C_{2,2}(M)$ as [22]

$$C_{p,q}(M) \propto C_{2,2}^{\chi(p,q)}(M) \tag{3}$$

then the exponent $\chi(p, q)$ will be the measure of erraticity property.

Since $\chi(p, q)$ is an increasing function of p with increasing slope, an efficient way to characterize erraticity with one number is simply to use the slope of $\chi(p, q)$ at $p = 1$, i.e.,

$$\chi_q' = \frac{\mathrm{d}}{\mathrm{d}p} \chi(p, q)\big|_{p=1}. \tag{4}$$

Again, if the moments $C_{p,q}(M)$ do not show scaling behaviour in M but have similar nonlinear dependences on M, one can consider a generalized form of scaling

$$C_{p,q}(M) \propto g(M)^{\Psi_q'(p)}, \tag{5}$$

where $\ln g(M) = (\ln M)^a$ and $\Psi_q'(p)$ is the newly defined erraticity exponent. If eq. (5) is approximately valid for a common $g(M)$ for all p and q, it then follows from eq. (3) that

$$\chi(p, q) = \frac{\Psi_q'(p)}{\Psi_2'(2)} \tag{6}$$

and the newly defined entropy index,

$$\mu_q' = \frac{\mathrm{d}}{\mathrm{d}p} \Psi_q'(p)\big|_{p=1}. \tag{7}$$

Using eqs (4) and (6), one can have

$$\mu_q' = \Psi_2'(2) \chi_q'. \tag{8}$$

4. Results and discussion

The method of erraticity analysis described in the previous section is applied to π^-–AgBr interactions data at 350 GeV/c and p–AgBr interactions data at 400 GeV/c. We have performed this study in pseudorapidity space. To eliminate the effect of non-flat average distribution, the pseudorapidity phase-space variable η is transformed into the corresponding cumulant form [29] $X(\eta)$ as usual. After the transformation, the phase-space region $X(\eta)$ becomes [0,1].

To find whether the results from our experimental data could be reproduced by the standard generators of particle production in heavy-ion collisions, we simulated 2000 π^-–AgBr interactions at 350 GeV/c and 10000 p–AgBr interaction at 400 GeV/c using the VENUS generator.

The factorial moment $F_2^e(M)$ describes the pattern of the distribution of the produced pions of the eth event. As the pattern changes from event to event, $F_2^e(M)$ also changes, resulting in a distribution $P(F_2)$ as shown in figures 1a and 1b respectively for the experimental data of π^-–AgBr interactions at 350 GeV/c and p–AgBr interactions at 400 GeV/c as well as for the corresponding VENUS datasets. It is evident from figure 1 that for experimental data as well as for VENUS-generated data, $F_2^e(M)$ fluctuates greatly from event to event and its distribution is wide, which gives the hint that the pattern can change erratically. To quantify the event-to-event fluctuations

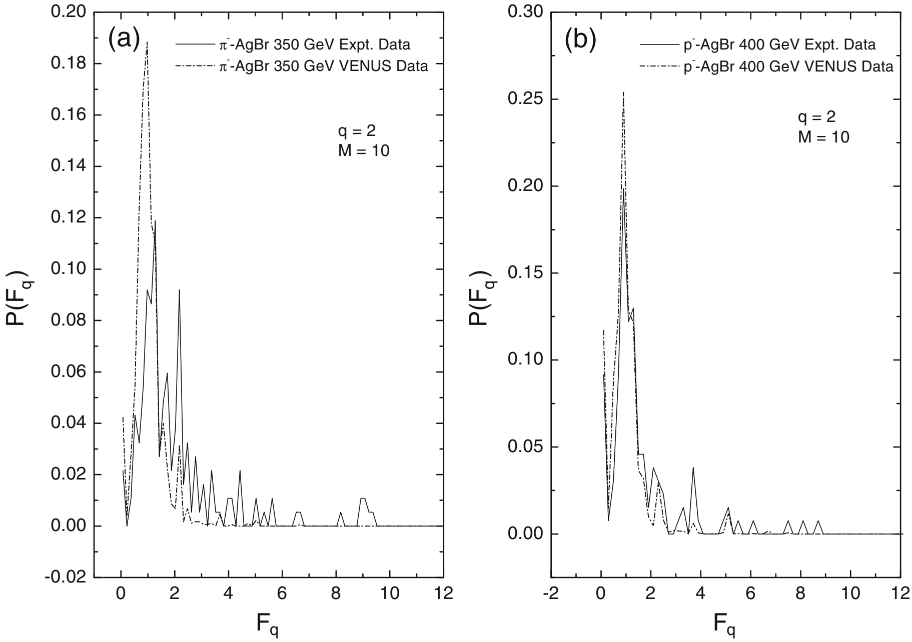

Figure 1. Probability distribution of event factorial moment F_q for both experimental data and VENUS datasets for (**a**) π^-–AgBr interactions at 350 GeV/c and (**b**) p–AgBr interactions at 400 GeV/c.

of factorial moments, we calculate the moment of the factorial moments $C_{p,2}(M)$ using eq. (1), where p is the order for event-to-event fluctuation. Figures 2a and 2b represent the ln $C_{p,2}(M)$ vs. ln M plots for $p = 2, 3, 4$ for experimental data of π^-–AgBr interactions at 350 GeV/c and p–AgBr interactions at 400 GeV/c respectively. The same for VENUS

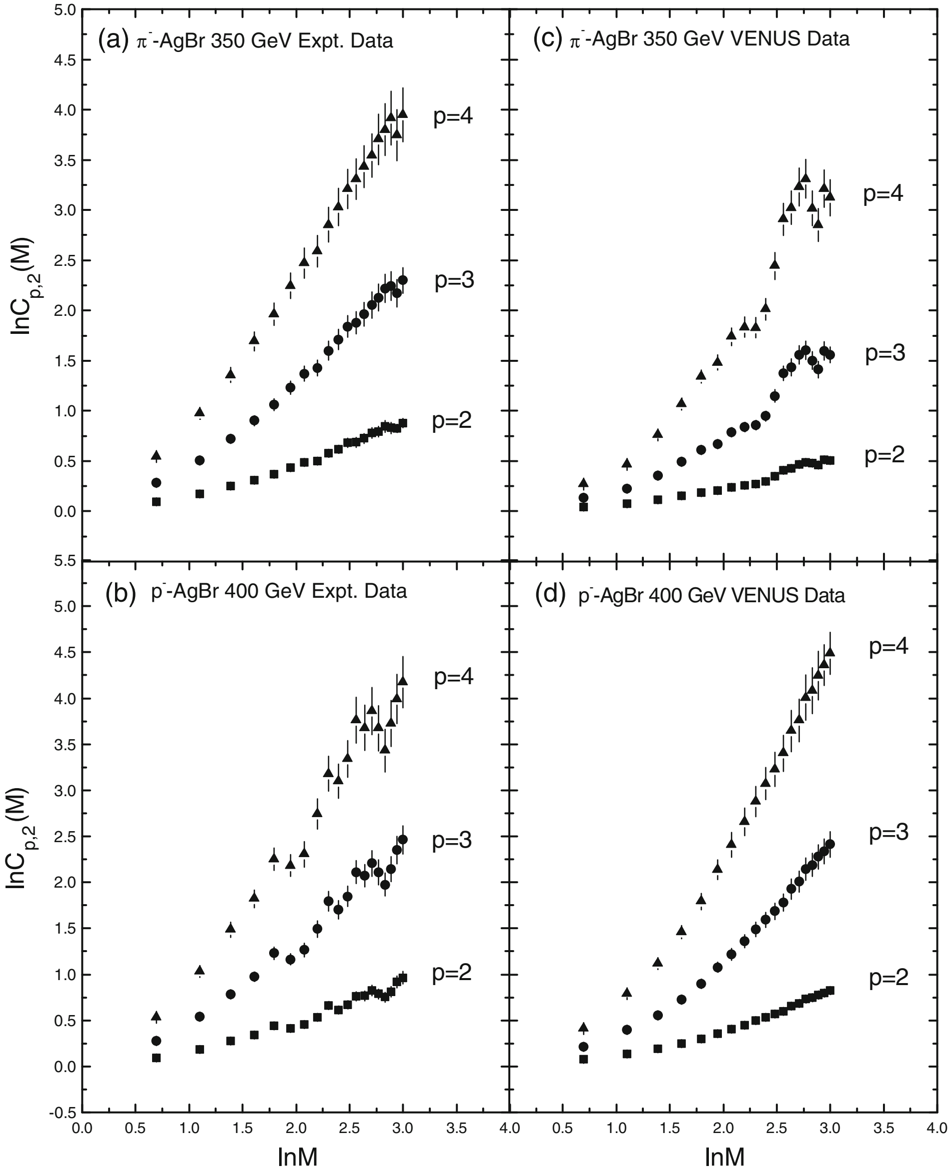

Figure 2. The dependence of $\ln C_{p,2}(M)$ on $\ln M$ for $p = 2$, 3 and 4 for both experimental and VENUS datasets.

data have also been calculated and plotted in figures 2c and 2d for comparison. These plots are not strictly linear. It may be said that the power-law behaviour of eq. (2) is not well satisfied. As all $C_{p,2}(M)$ follow the same trend, we have considered $C_{2,2}(M)$ as the reference and have plotted ln $C_{p,2}(M)$ vs. ln $C_{2,2}(M)$ for $p = 2, 3, 4$ in figures 3a and 3b

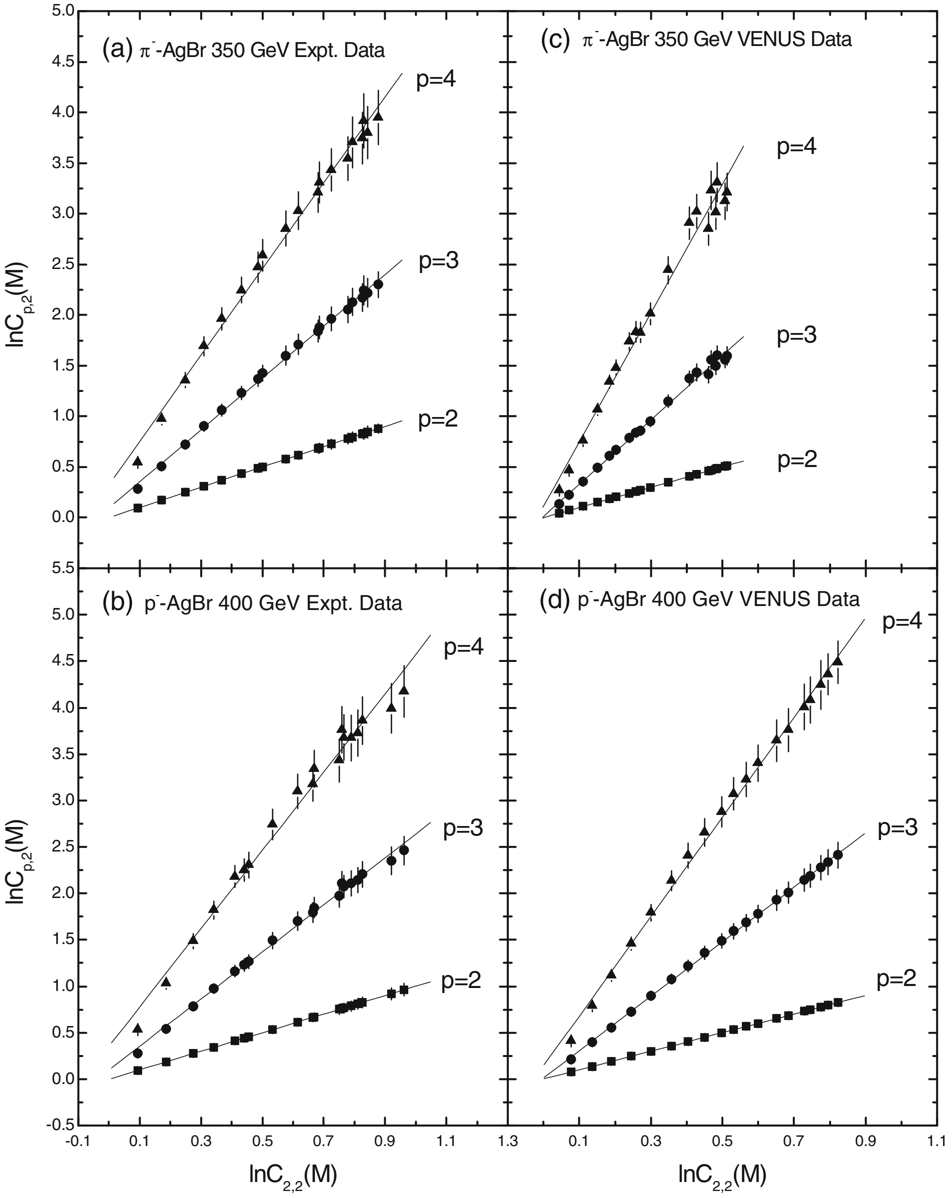

Figure 3. The plots of ln $C_{p,2}(M)$ vs. ln $C_{2,2}(M)$ for both experimental and VENUS datasets.

respectively for interactions initiated by pions and protons. The same for VENUS data have also been plotted in figures 3c and 3d. Now it is evident that power-law behaviour of eq. (3) is well satisfied. The linear best fits to $\ln C_{p,2}(M)$ vs. $\ln C_{2,2}(M)$ are performed. The confidence level of the best fits never falls below 90%. The slopes $\chi(p, 2)$ of the linear fits are shown in figure 4 against p. It can be seen from figure 4 that $\chi(p, 2)$ is an increasing function of p with varying slope. To get the slopes of these plots at $p = 1$, we calculated $\ln C_{p,2}(M)$ for $p = 0.9$ and 1.1 and plotted in figures 5a and 5b respectively for experimental data of π^-–AgBr and p–AgBr interactions and in figure 5c and 5d for the corresponding VENUS data. The erraticity measure, χ_2', is calculated using the slopes of these plots at $p = 1$. The values of χ_2' are given in table 1.

Since $C_{p,2}(M)$ does not show proper power-law behaviour with M, we shall search for a more general form as eq. (5). To obtain a good linear behaviour of $\ln C_{2,2}(M)$ vs. $\ln g(M)$ plot, we vary a and the best-fitted plot of $\ln C_{2,2}(M)$ vs. $\ln g(M)$ is shown in figure 6 for all the datasets. The slopes of the plots give $\Psi_2'(2)$ values, which are also included in table 1. The confidence level of the best fits never fall below 95%. Using eqs (6)–(8), we estimate the event-to-event fluctuation of factorial moments by calculating the newly defined entropy index, μ_2', and tabulate in table 1. The positive values of μ_2' signifies that the fluctuation of spatial patterns for hadron–nucleus interactions is definitely erratic suggesting chaotic multiparticle production in π^-–AgBr interactions at 350 GeV/c and p–AgBr interactions at 400 GeV/c. It is also of interest to note that μ_2' does not depend on whether the projectile is proton or pion.

It is evident from table 1 that the values of μ_2' for VENUS-generated data are compared very well to that obtained from the experimental data. This suggests that VENUS event generator is unable to reproduce the event-to-event fluctuations of spatial patterns of final states.

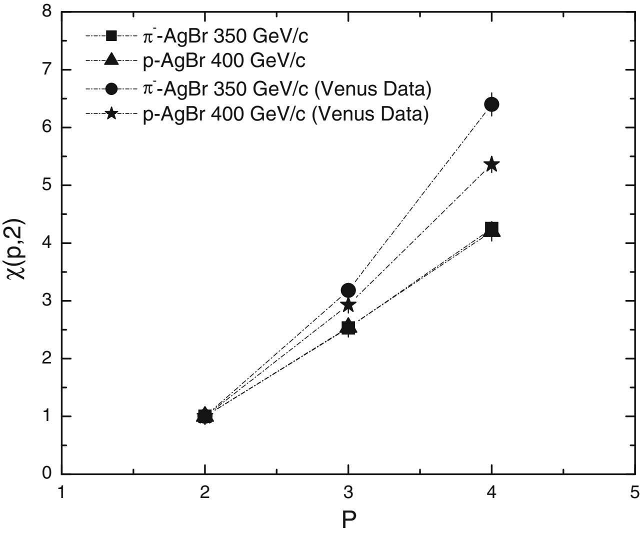

Figure 4. The dependence of $\chi(p,2)$ on p for both experimental and VENUS datasets.

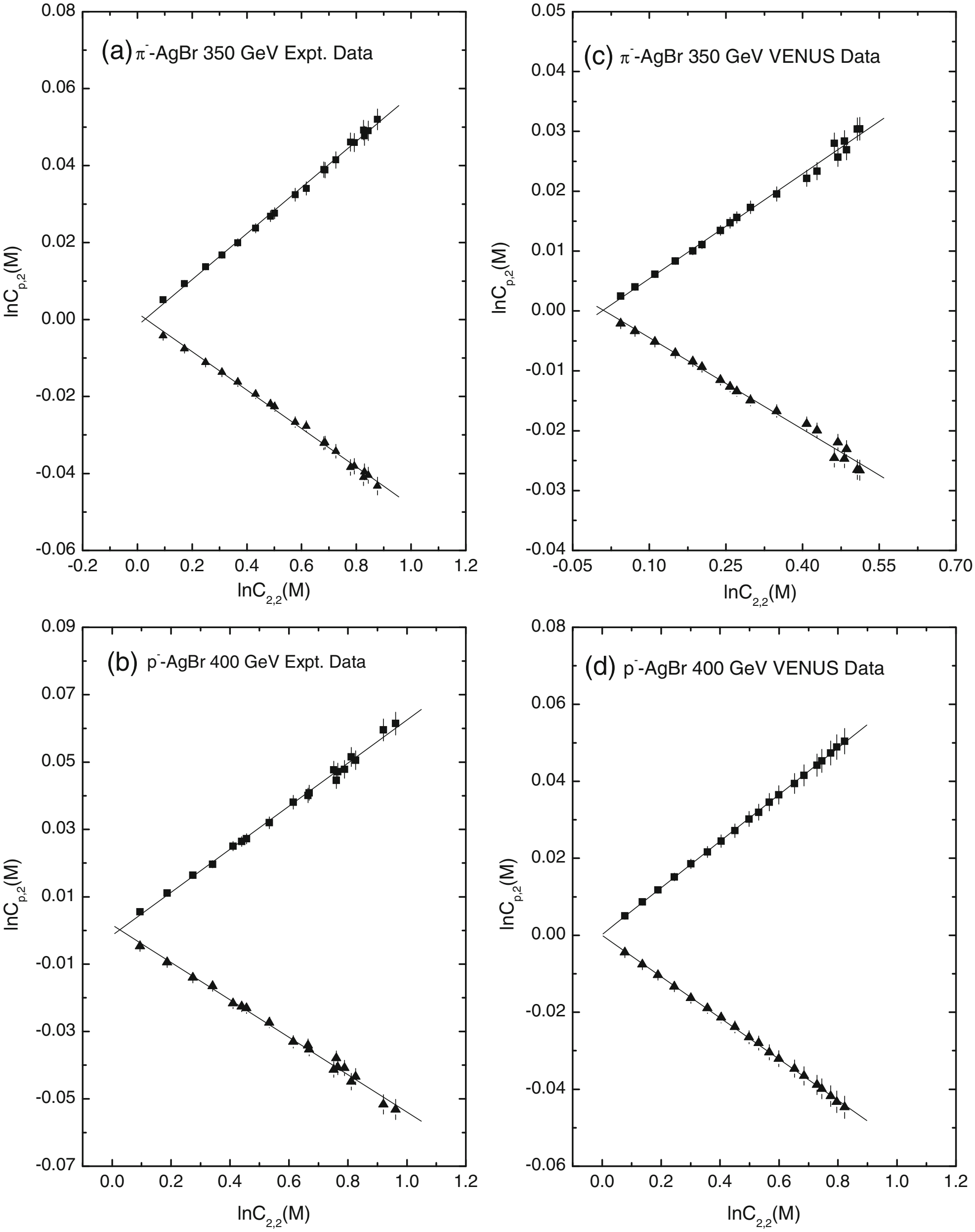

Figure 5. The plots of $\ln C_{p,2}(M)$ vs. $\ln C_{2,2}(M)$ for $p = 0.9$ and 1.1 for both experimental and VENUS datasets.

Wang *et al* [23] have used this method to analyse NA27 data of 400 GeV/c p–p and have found evidences for the predicted erraticity phenomenon. Their values of μ_2' are given in table 1. It is extremely interesting to observe that the values of μ_2' is

Table 1. The values of χ'_2, $\Psi'_2(2)$ and μ'_2 for both experimental and VENUS-generated data.

Data	Average multiplicity	χ'_2	$\Psi'_2(2)$	μ'_2
π^-–AgBr 350 GeV/c	11.7	0.55 ± 0.05	0.141 ± 0.006	0.078 ± 0.008
p–AgBr 400 GeV/c	10.1	0.6 ± 0.007	0.14 ± 0.01	0.084 ± 0.006
π^-–AgBr 350 GeV/c (VENUS)	13.3	0.545 ± 0.007	0.048 ± 0.005	0.026 ± 0.003
p–AgBr 400 GeV/c (VENUS)	11.3	0.575 ± 0.018	0.089 ± 0.002	0.051 ± 0.002
p–p 400 GeV/c [23]	10.27			0.049 ± 0.002

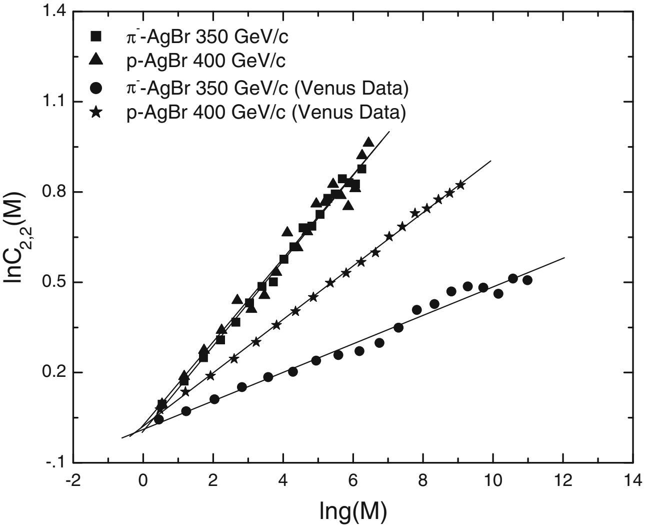

Figure 6. The plots of $\ln C_{2,2}(M)$ vs. $\ln g(M)$ for $p = 0.9$ and 1.1 for both experimental and VENUS datasets.

significantly greater in hadron–nucleus interactions (p–AgBr interactions at 400 GeV/c) than in hadron–hadron interactions (p–p interactions at 400 GeV/c [23]). This may be due to the fact that nucleus is composed of many nucleons and hadron–nucleus collision at a particular impact parameter involves a number of participants. The erraticity analysis is therefore sensitive to two types of fluctuations: one due to the particle production of an elementary collision between nucleons and the other due to the fluctuations in the sources. This analysis ensures quantitatively that the particle production process becomes more chaotic as target changes from hadron to nuclei but the chaoticity does not depend on the nature of the projectile.

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References

[1] JACEE Collaboration: T H Burnett *et al*, *Phys. Rev. Lett*. **50**, 2062 (1983)
[2] UA5 Collaboration: G J Alner *et al*, *Phys. Rep*. **154**, 247 (1987)
[3] NA22 Collaboration: M Adamus *et al*, *Phys. Lett*. **B185**, 200 (1987)
[4] EMU-01 Collaboration: M I Adamovich *et al*, *Phys. Lett*. **B201**, 397 (1988)
[5] P Pozek, M Ploszajczak and R Botet, *Phys. Rep*. **252**, 101 (1995)
[6] E A De Wolf, I M Dremin and W Kittel, *Phys. Rep*. **270**, 1 (1996)
[7] G Singh, M K Ghosh and A Mukhopadhyay, *Int. J. Mod. Phys*. **E17**, 802 (2008)
[8] M K Ghosh, A Mukhopadhyay and G Singh, *J. Phys*. **G34**, 177 (2007)
[9] M K Ghosh, A Mukhopadhyay and G Singh, *J. Phys*. **G32**, 2293 (2006)
[10] Z Cao and R C Hwa, *Phys. Rev. Lett*. **75**, 1268 (1995)
[11] Z Cao and R C Hwa, *Phys. Rev*. **D53**, 6608 (1996)
[12] R C Hwa, *Acta Phys. Pol*. **B27**, 1789 (1996)
[13] Z Cao and R C Hwa, *Phys. Rev*. **E56**, 326 (1997)
[14] Z Cao and R C Hwa, *Phys. Rev*. **D61**, 074011 (2000)
[15] S Wang *et al*, *Phys. Lett*. **B416**, 216 (1998)
[16] S Wang *et al*, *Phys. Rev*. **D57**, 3036 (1998)
[17] M R Atayan *et al*, *Phys. Lett*. **B558**, 22 (2003)
[18] D Ghosh, *Phys. Lett*. **B540**, 52 (2002)
[19] R C Hwa and Y Wu, *Phys. Rev*. **C60**, 054904 (1999)
[20] R C Hwa, *Nonlinear Phenomena in Complex Systems* **3**, 93 (2000)
[21] D Ghosh *et al*, *J. Phys*. **G35**, 125005 (2008)
[22] Z Cao and R C Hwa, *Phys. Rev*. **D61**, 074011 (2000)
[23] S Wang *et al*, *Phys. Lett*. **B458**, 505 (1999)
[24] D Ghosh *et al*, *Phys. Rev*. **D26**, 2983 (1982)
[25] D Ghosh *et al*, *Can. J. Phys*. **64**, 334 (1986)
[26] D Ghosh *et al*, *Phys. Rev*. **D46**, 3712 (1992)
[27] D Ghosh *et al*, *Phys. Rev*. **D47**, 1235 (1993)
[28] D Ghosh *et al*, *Phys. Rev*. **C65**, 067902 (2002)
[29] W Ochs, *Z. Phys*. **C50**, 339 (1991)