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Open flavour charmed mesons in a quantum chromodynamics potential model

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Abstract. We modify the mesonic wave function by using a short distance scale r_0 in analogy with hydrogen atom and estimate the values of masses and decay constants of the open flavour charm mesons D, D_s and B_c within the framework of a QCD potential model. We also calculate leptonic decay widths of these mesons to study branching ratios and lifetime. The results are in good agreement with experimental and other theoretical values.

Keywords. Heavy-light mesons; masses; decay constants; branching ratio.

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1. Introduction

Recently, we have reported a regularization procedure to avoid the singularity by introducing a flavour-dependent short distance scale at the origin to study the oscillation frequency of B and $\bar{B}$ mesons in a QCD potential model [1]. The purpose of this paper is to use the potential model to calculate the masses and decay constants of open flavour charmed mesons D, D_s and B_c and then to find the decay width and branching ratio of the same.

If the CKM element is well known from other measurements, then f_P can be measured well. If, on the other hand, the CKM element is not known or poorly measured, having theoretical input on f_P can allow a determination of the CKM element. These decay constants can be accessed both experimentally and through lattice quantum chromodynamics (lQCD) simulations. While for f_π, f_K, f_D, experimental measurements agree well with lattice QCD calculations, a discrepancy is seen for the value of f_{D_s}: The 2008 PDG average for f_{D_s} is 273 ± 10 MeV [2], about 3σ larger than the most precise $N_f = 2 + 1$ lQCD result from the HPQCD/UKQCD Collaboration [3], 241 ± 3 MeV. On the other hand, experiments and lQCD calculations agree very well with each other on the value of f_D, f_D(expt) $= 205.8 \pm 8.9$ MeV and f_D(lQCD) $= 207 \pm 4$ MeV. The discrepancy concerning f_{D_s} is quite puzzling because the systematic errors that have affected the lQCD calculation of f_D, should have affected the calculation of f_{D_s} [4] also.

However, the discrepancy is reduced to 2.4σ with the new (updated) data from CLEO [5,6] and Babar [7], together with the Belle measurement [8] and the latest PDG average is $f_{D_s} = 257.5 \pm 6.1$ MeV [9]. Lately, the HPQCD Collaboration has also updated its study of the D_s decay constant [10]. By including additional results at smaller lattice spacing along with improved determinations of the lattice spacing and improved tuning of the charm and strange quark masses, a new value for the D_s decay constant has been reported as $f_{D_s} = 248.0 \pm 2.5$ MeV. A better understanding of the decay properties of D and D_s in this regard is quite important.

The properties of the B_c meson are of special interest [11], since it is the only heavy meson consisting of two heavy quarks with different flavours. A peculiarity of B_c decays is that both the quarks may involve in its weak decays. From the experimental point of view, a study of weak decays of B_c meson is quite important for the determination of CKM elements. More detailed information about its decay properties are expected in near future at LHC and other experiments. In this paper, the decay constants of D, D_s and B_c are computed and are used to study the decay width and decay time of the same. Rest of the paper is organized as follows: In § 2, we discuss the formalism, while in § 3 we summarize the calculations and results. Section 4 contains summary and conclusion.

2. Formalism

2.1 *Wave function in the model*

The wave function in a specific potential model with its relativistic correction procedure is [1,12]:

$$\psi_{\text{rel+conf}}(r') = \frac{N'}{\sqrt{\pi a_0^3}} e^{-r'/a_0} \left(C' - \frac{\mu b a_0 (r')^2}{2}\right) \left(\frac{r'}{a_0}\right)^{-\epsilon}, \quad (1)$$

where

$$N' = \frac{2^{1/2}}{\sqrt{(2^{2\epsilon}\Gamma(3-2\epsilon)C'^2 - \frac{1}{4}\mu b a_0^3 \Gamma(5-2\epsilon)C' + \frac{1}{64}\mu^2 b^2 a_0^6 \Gamma(7-2\epsilon))}} \quad (2)$$

$$C' = 1 + cA_0\sqrt{\pi a_0^3} \quad (3)$$

$$\mu = \frac{m_i m_j}{m_i + m_j} \quad (4)$$

$$a_0 = \left(\frac{4}{3}\mu\alpha_s\right)^{-1} \tag{5}$$

$$\epsilon = 1 - \sqrt{1 - \left(\frac{4}{3}\alpha_s\right)^2} \tag{6}$$

$$r' = r + r_0 \tag{7}$$

and the QCD potential is taken as

$$V(r) = -\frac{4}{3r}\alpha_s + br + c. \tag{8}$$

2.2 *The mesonic wave function at the origin*

The term $\{(r + r_0)/a_0\}^{-\epsilon}$ in eq. (1) is used to incorporate relativistic effect in a free Dirac way [12,13]. The cut-off parameter is used to regularize the wave function at the origin such that the wave function remains approximately constant from r_0 down to $r = 0$. In analogy to QED, we use the cut-off parameter to regularize the wave function as [1]

$$r_0 \sim a_0 \mathrm{e}^{-1/\epsilon}. \tag{9}$$

Unlike QED, it is flavour dependent depending on the flavours of the quark masses.

The input parameters in the numerical calculations are: $m_u = 0.336$ GeV, $m_s = 0.483$ GeV, $m_c = 1.550$ GeV and $m_b = 4.950$ GeV, $b = 0.183\,\mathrm{GeV}^2$, $c = -0.5$ GeV, $\alpha_s = 0.39$ and $\alpha_s = 0.22$ at charm and bottom mass scale. The value of the undetermined factor A_0, which appears in the series solution [12] is so chosen that the consistency of the value of $cA_0 = 1\ \mathrm{GeV}^{3/2}$ is sustained with our previous works [1,14]. With these values, we compute the short distance scale r_0 for open flavour charmed mesons D, D_s and B_c and is shown in table 1.

Table 1 shows that as the masses of the mesons increase, r_0 is decreased. For heavy–heavy mesons like B_c it becomes as small as $10^{-10}\ \mathrm{GeV}^{-1}$ which presumably means that the relativistic effect is too small for this heavy meson.

Table 1. Values of cut-off parameter r_0 for different mesons.

Mesons	Reduced mass	Values of r_0 (GeV^{-1})
$D(c\bar{u}/c\bar{d})$	0.2761	0.0073
$D_s(c\bar{s})$	0.3682	0.0055
$B_c(\bar{b}c)$	1.1838	3.872×10^{-10}

3. Calculation and results

3.1 *Masses and decay constants of open flavoured charm mesons*

The decay constant with relativistic correction can be expressed through the meson wave function $\Phi_P(p)$ in the momentum space [15,16] as

$$f_P = \sqrt{\frac{12}{M_P}} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left(\frac{E_q + m_q}{2E_q}\right)^{1/2} \left(\frac{E_{\bar{q}} + m_{\bar{q}}}{2E_{\bar{q}}}\right)^{1/2} \times \left(1 + \lambda_P \frac{p^2}{[E_q + m_q][E_{\bar{q}} + m_{\bar{q}}]}\right) \Phi_P(p) \tag{10}$$

with $\lambda_P = -1$. In the nonrelativistic limit $(p^2/m^2) \ll 1.0$, this expression reduces to the well-known relation between f_P and the ground state wave function at the origin $\psi(0)$ the Van-Royen–Weisskopf formula [17].

$$f_P = \sqrt{\frac{12}{M_P} |\psi(0)|^2} \tag{11}$$

with QCD correction factor the decay constant can be written as [18]

$$f_P = \sqrt{\frac{12}{M_P} |\psi(0)|^2 \bar{C}^2}, \quad \text{where } \bar{C}^2 = 1 - \alpha_s/\pi \left[2 - \frac{m_Q - m_{\bar{Q}}}{m_Q + m_{\bar{Q}}} \ln \frac{m_Q}{m_{\bar{Q}}}\right], \tag{12}$$

where M_P is the mass of the pseudoscalar meson in the ground state and can be obtained as [19]

$$M_P = m_Q + m_{\bar{Q}} + \langle H \rangle, \tag{13}$$

where

$$\langle H \rangle = \left\langle \frac{p^2}{2\mu} \right\rangle + \langle V(r) \rangle. \tag{14}$$

In tables 2 and 3 we record the prediction of the model for masses and decay constants.

Table 2. The masses of heavy-light mesons in GeV.

Mesons	Masses	Experimental values
$D(c\bar{u}/c\bar{d})$	1.860	1.869 ± 0.0016 [9]
$D(c\bar{s})$	1.959	1.968 ± 0.0033 [9]
$B_c(\bar{b}c)$	6.507	6.277 ± 0.006 [9]

Table 3. The decay constants of heavy-light mesons with and without QCD correction in GeV.

Meson	Our work		Other work
	f_P	f_{Pc}	
$D(c\bar{u}/c\bar{d})$	0.240	0.225	$0.205 \pm 0.085 \pm 0.025$ [20] (Exp)
			0.206 [21]
			0.234 [22]
			0.208 [23]
			0.201 [24]
			0.235 [25]
$D(c\bar{s})$	0.291	0.266	0.254 ± 0.059 [20] (Exp)
			0.245 [21]
			0.268 [22]
			0.256 [23]
			0.249 [24]
			0.266 [25]
$B_c(\bar{b}c)$	0.435	0.413	0.433 [26] (Theory)
			0.470 [19]

f_P= decay constants without QCD correction.
f_{Pc}= decay constant with QCD correction.

3.2 *Leptonic decay width and branching ratio of D and D_s mesons*

Charged mesons formed from a quark and antiquark can decay to a charged lepton pair when these objects annihilate via a virtual $W^{\pm}$ boson. Quark–antiquark annihilations via a virtual W^{+} (W^{-}) to the $l^{+}\nu$ ($l^{-}\nu$) final states occur for the $\pi^{\pm}$, $K^{\pm}$, $D_s^{\pm}$ and $B^{\pm}$ mesons. There are several reasons for studying the purely leptonic decays of charged mesons. Such processes are rare but they have clear experimental signatures due to the presence of a highly energetic lepton in the final state. The theoretical predictions are very clean due to the absence of hadrons in the final state. The total leptonic decay width of D and D_s mesons are given by

$$\Gamma(D_q^{+} \to l^{+}\nu) = \frac{G_F^2 |V_{cq}|^2 f_{D_q}^2}{8\pi} m_l^2 \left(1 - \frac{m_l^2}{M_{D_q}^2}\right)^2 M_{D_q}, \quad q = d, s. \tag{15}$$

These transitions are helicity suppressed, i.e., the amplitude is proportional to m_l, the mass of the lepton l, in complete analogy to $\pi \to l^{+}\nu$.

The leptonic widths of the charged D and D_s mesons are obtained using eq. (12), by employing the predicted values of the pseudoscalar decay constants f_D and f_{D_s} along with the masses of M_D and M_{D_s} from our work. The leptonic widths for seperate lepton channel by choosing $m_{l=\tau,\mu,e}$ are computed. Branching ratio of D_q mesons are calculated using the relation

$$\mathrm{BR} = \Gamma \times \tau. \tag{16}$$

Table 4. The leptonic branching ratio of D and D_s mesons.

Mesons	$\mathrm{BR}_\tau \times 10^{-3}$	$\mathrm{BR}_\mu \times 10^{-4}$	$\mathrm{BR}_e \times 10^{-8}$
$D(c\bar{u}/c\bar{d})$	0.78(0.68)	4.4(3.89)	1.05(0.92)
Expt. [9]	<1.2	3.82	
B Patel *et al* [27]	0.9	6.6	1.5
	$\mathrm{BR}_\tau \times 10^{-2}$	$\mathrm{BR}_\mu \times 10^{-3}$	$\mathrm{BR}_e \times 10^{-7}$
$D(c\bar{s})$	6.4(5.3)	7.0(5.9)	1.67(1.40)
Expt. [9]	5.6 ± 0.4	5.8 ± 0.4	
B Patel *et al* [27]	8.4	7.7	1.8

Values within the bracket represent the branching ratio for f_P with QCD correction.

The lifetime of these mesons, $\tau_D = 1.04$ ps and $\tau_{D_s} = 0.5$ ps, are taken from the world average value reported by Particle Data Group (PDG-2010) [9]. The present results as tabulated in table 4 are in accordance with the available experimental values.

3.3 *Weak decay of B_c^+ meson*

Adopting the spectator model for the charm beauty mesons system [28], the total decay width of B_c^+ meson can be approximated as the sum of the widths of $\bar{b}$-quark decay keeping c-quark as spectator, the c-quark decay with $\bar{b}$ as spectator, and the annihilation channel $B_c^+ \rightarrow l^+\nu_l(c\bar{s}, u\bar{s}), l = e, \mu, \tau$ with no interference assumed between them. Accordingly, the total width is calculated as [28]

$$\Gamma(B_c \rightarrow X) = \Gamma(b \rightarrow X) + \Gamma(c \rightarrow X) + \Gamma(\text{anni})\,. \tag{17}$$

Neglecting the quark binding effects, the b and c inclusive widths in the spectator approximation are [28]

$$\Gamma(b \rightarrow X) = \frac{9G_F^2|V_{cb}|^2 m_b^5}{192\pi^3} \tag{18}$$

$$\Gamma(c \rightarrow X) = \frac{5G_F^2|V_{cs}|^2 m_c^5}{192\pi^3}. \tag{19}$$

Here we have used the model quark masses and the CKM matrix elements $|V_{cs}| = 0.957$, $|V_{cb}| = 0.039$ from the Particle Data Group.

Employing the computed mass and pseudoscaler decay constant from the present study, the width of the annihilation channel is computed using the expression given by [19,28],

$$\Gamma(\text{Anni}) = \frac{G_F^2|V_{bc}|^2 f_{B_c}^2 M_{B_C}}{8\pi} m_q^2 \left(1 - \frac{m_q^2}{M_{B_c}^2}\right)^2 C_q, \tag{20}$$

where $C_q = 1$ for the $\tau\nu_\tau$ channel and $C_q = 3|V_{cs}|^2$ for the $c\bar{s}$ channel, and m_q is the mass of the heaviest fermions. The computed results of the annihilation decay rate and

Table 5. Decay width (in 10^{-4} eV) and lifetime of B_c meson.

Meson	Γ(anni)	$\Gamma(B_c \to X)$	τ(inps)
$B_{c\bar{b}}$	1.17(1.06)	19.17(19.06)	0.344(0.346)
[9]			0.453 ± 0.041
[28]	1.40	14.00	0.47
[29]	0.67	8.8	0.75

total decay rate are tabulated in table 5. Our prediction for lifetime with these results lie well within the experimental value and is placed in table 5.

4. Conclusion

In the present work, we have computed the masses of heavy-light mesons in a specific potential model and use a short distance scale to regularize the wave function near the origin to compute the decay constants. The short distance scale as well as the decay parameters are very sensitive to the strong coupling constant α_s. It is to be noted that strong coupling constant used in ref. [1] cannot be used for D, D_s and B_c mesons since the results overshoots the experimental and theoretical values and hence the model is excluded to treat the D and B mesons together. However, we use the same α_s as used in our previous work [14,30,31] and with a different renormalization mass scale one can regain these strong coupling constants within the formula of ref.[1].

The analysis with QCD correction is found to be closer to experiments and other theoretical results. In our calculation we have found (f_{D_s}/f_D)=1.18 (with QCD correction) and $(f_{D_s}/f_D) = 1.21$ (without QCD correction), which is in accordance with the latest QCD sum rule result 1.193±0.025±0.007 [21], PDG average $(f_{D_s}/f_D) = 1.25\pm0.06$ [9], as well as with the recent lattice results $f_{D_s}/f_D = 1.164 \pm 0.011$ [3,10] and $f_{D_s}/f_D = 1.20 \pm 0.02$ [32].

Present study on the leptonic decay branching ratio of D and D_s mesons with QCD correction for τ and μ leptonic channels presented in table 4 are as per the available experimental limits. Large experimental uncertainty in the electron channel makes it difficult for any reasonable conclusion. The computed result within the framework of QCD potential model for annihilation decay width as well as the lifetime of B_c meson are also found to agree well with the available data presented in table 5. Probably, in future high-luminosity better statistics and high-confidence level data sets will be able to provide more light on the spectroscopy and decay properties of these open charm mesons.

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Appendix

Expression for the expectation value of the Hamiltonian ($\langle H \rangle$) is:

$$\langle H \rangle = \left\langle \frac{p^2}{2\mu} \right\rangle + \langle V(r) \rangle \tag{21}$$

$$\begin{aligned}\left\langle \frac{p^2}{2\mu} \right\rangle = -\frac{1}{2\mu}\Bigg(& \frac{2\pi C_1^2 \epsilon(\epsilon-1)}{(2g)^{1-2\epsilon}}\Gamma(1-2\epsilon) - \frac{2\pi h_1 C_1 \epsilon(\epsilon-1)}{(2g)^{3-2\epsilon}}\Gamma(3-2\epsilon) \\ & + \frac{4\pi g C_1^2(\epsilon+1)}{(2g)^{2-2\epsilon}}\Gamma(2-2\epsilon) - \frac{4\pi g h_1 C_1(\epsilon+1)}{(2g)^{4-2\epsilon}}\Gamma(4-2\epsilon) \\ & - \frac{2\pi C_1\big(g^2 C_1 + h_1(2-\epsilon)(3-\epsilon)\big)}{(2g)^{3-2\epsilon}}\Gamma(3-2\epsilon) \\ & + \frac{2\pi h_1\big(g^2 C_1 + h_1(2-\epsilon)(3-\epsilon)\big)}{(2g)^{5-2\epsilon}}\Gamma(5-2\epsilon) \\ & + \frac{4\pi g h_1 C_1}{(2g)^{4-2\epsilon}}\Gamma(4-2\epsilon) - \frac{4\pi g h_1^2}{(2g)^{6-2\epsilon}}\Gamma(6-2\epsilon) \\ & + \frac{2\pi C_1 g^2 h_1}{(2g)^{5-2\epsilon}}\Gamma(5-2\epsilon) - \frac{2\pi g^2 h_1^2}{(2g)^{7-2\epsilon}}\Gamma(7-2\epsilon)\Bigg)\end{aligned} \tag{22}$$

$$\begin{aligned}\left\langle \frac{-\alpha_c}{r} \right\rangle = & -\alpha_c 2\pi C_1^2 \frac{1}{(2g)^{2-2\epsilon}}\Gamma(2-2\epsilon) - \alpha_c 2\pi h_1^2 \frac{1}{(2g)^{6-2\epsilon}}\Gamma(6-2\epsilon) \\ & + \alpha_c 4\pi h_1 C_1 \frac{1}{(2g)^{4-2\epsilon}}\Gamma(4-2\epsilon)\end{aligned} \tag{23}$$

$$\begin{aligned}\langle br \rangle = & \; b2\pi C_1^2 \frac{1}{(2g)^{4-2\epsilon}}\Gamma(4-2\epsilon) + b2\pi h_1^2 \frac{1}{(2g)^{8-2\epsilon}}\Gamma(8-2\epsilon) \\ & - b4\pi h_1 C_1 \frac{1}{(2g)^{6-2\epsilon}}\Gamma(6-2\epsilon)\end{aligned} \tag{24}$$

where

$$\begin{aligned} h_1 &= hk_1, \quad h = \mu b a_0/2, \quad k_1 = k/l, \quad l = 1/a_0^{-\epsilon}, \quad k = N'/(\pi a_0)^{1/2}, \\ C_1 &= C' k_1, \quad g = 1/a_0 \quad \text{and} \quad \alpha_c = 4\alpha_s/3. \end{aligned} \tag{25}$$

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