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Allowable irreducible representations of the point groups with five-fold rotational axes

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Abstract. Allowable irreducible representations of the point groups with five-fold rotations – that represent the symmetry of the quasicrystals in two and three dimensions – are derived by employing the little group technique in conjunction with the solvability property. The point groups $D_{5h}(\overline{10}m2)$ and $I_h(\frac{2}{m}\overline{3}\,\overline{5})$ are taken to illustrate the method.

Keywords. Allowable irreducible representations of little groups; solvability property; composition series; induced and engendered representations.

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1. Introduction

Character table for the 32 crystallographic point groups – that represent the point symmetry of the crystals and the symmetry of some molecules – as also for the nine non-crystallographic point groups with five-fold rotation that represent the symmetry of quasicrystals in two and three dimensions – along with the method for obtaining them are available in literature [1–3]. Out of the well-known methods employed to find the irreducible representations (IRs) of finite groups, the application of the little group method in conjunction with the solvability property [4] is considered as the most elegant and simple method. This classic method explained in Bradley [5] and Bradley and Cracknell [3] was applied to the plane group $P4g$ and to the space groups by Raghavacharyulu [6] and to the crystallographic point groups by Ramachandra Rao [7]. A slightly different approach to this little group method described by Bhagavantam and Venkatarayudu [8], was applied by Krishnamurthy *et al* [9] to obtain all the one-dimensional allowable irreducible representations (AIRs) of the appropriate little groups that induce the degenerate representations of the 32 crystallographic point groups.

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Among the finite groups with five-fold rotational symmetry, the icosahedral point group $I(235)$ is the most complex point group which has been the subject of interest for several group - theoretical physicists. The discovery of quasicrystals [10] has revived the interest of these researchers to study the seven pentagonal point groups $C_5(5)$, $S_{10}(\overline{5})$, $C_{5h}(\overline{10})$, $D_5(52)$, $D_{5h}(\overline{10}m2)$, $C_{5v}(5m)$, $D_{5d}(\overline{5}2m)$ and the two icosahedral point groups $I(235)$ and $I_h(\frac{2}{m}\overline{3}\,\overline{5})$ that exhibit five-fold rotational symmetry. The IRs of the icosahedral point groups have been studied by Backhouse and Gard [11]. Using a new approach to the group representation theory – which in turn paved way for eigenfunction method – Chen [12] obtained the characters, IRs and isoscalar factors for the icosahedral point groups which have been of interest in connection with the vibrational and electronic propagation [13,14]. Interest in the icosahedral point groups also stemmed from the icosahedral symmetry of biological macromolecules [15].

In this note, an attempt has been made to obtain/derive the allowable irreducible representations (AIRs) of the little groups that induce various IRs of the seven pentagonal point groups and the icosahedral point group $I_h(\frac{2}{m}\overline{3}\,\overline{5})$ by applying the little group method in conjunction with the solvability property. The application of this powerful technique is illustrated with the help of two composition series:

(i) $D_{5h}(\overline{10}m2) \supset D_5\,(52) \supset C_5\,(5) \supset C_1\,(1)$

(ii) $I_h(\frac{2}{m}\,\overline{35}) \supset I\,(235)$.

In §2, we shall first familiarize the reader with the necessary basic terminology of the method and illustrate each one of them with an example from the chosen series. To aid the discussion, we shall refer to the character table of the point groups $D_{5h}(\overline{10}m2)$, $D_5(52)$, $C_5(5)$, $I_h(\frac{2}{m}\,\overline{35})$ and $I(235)$ provided in §2 and 3 and to the factor groups involving these groups in the considered composition series. The generating elements and the defining relations are listed for each group in the considered series along with the character table and the multiplication tables for the elements of these groups are available in [16].

2. Some definitions and basic terminology

In the discussion that follows, G is a finite group and H is a normal subgroup of G. We say that G is solvable if the order of the factor groups H_i/H_{i+1} in the composition series $G = H_0 \supset \ldots \supset H_i \supset H_{i+1} \supset \ldots \supset C_1 = E$ are prime numbers. For example, the group D_{5h} is solvable since the composition indices 5, 2, 2 in the composition series $D_{5h} \supset D_5 \supset C_5 \supset C_1\,(= E)$ are prime numbers.

(i) *Conjugate representation* Δ^A: A representation Δ^A of H conjugate to Δ relative to G is defined by $\Delta^A \rightarrow D^{\Delta}(AHA^{-1})$ where $D^{\Delta}(AHA^{-1})$ is the matrix representing the element A in the representation Δ. If Δ and Δ^A are equivalent, then Δ is called self-conjugate.

In the group C_5 the total symmetric representation A is self-conjugate whereas the pair of 1-d complex representations E_a and E_b are conjugate but inequivalent to each other, relative to the group D_5.

(ii) *Little groups of the second kind* $L^{(2)}$ *and of the first kind* $L^{(1)}$: All the elements of a group G for which Δ is self-conjugate, i.e., $\Delta\cdot \equiv \cdot\Delta^{A}$ form the little group of the second kind (or little group) relative to (G, H, Δ) and is denoted by $L^{(2)}(G, H, \Delta)$. The quotient group $L^{(2)}/H$ is the corresponding little group of the first kind (or little co-group) and is denoted by $L^{(1)}(G, H, \Delta)$.

If $G = D_5$ and $H = C_5$, then for the IR A of C_5, $L^{(2)}(G, H, \Delta) = D_5$ and $L^{(1)}(G, H, \Delta) = D_5/C_5 \cong C_2$.

(iii) *Orbit (or Star)* θ: An orbit θ of the normal subgroup H of G is the set of all inequivalent IRs of H which are mutually conjugate relative to the elements of G. The number of IRs in θ gives the order of the orbit θ.

The group C_5 has three orbits: $\theta_1 = \{A\}$, $\theta_2 = \{E_{a_1}, E_{a_2}\}$, $\theta_3 = \{E_{b_1}, E_{b_2}\}$ with respect to D_5. Similarly, D_5 has four orbits: $\theta_1 = \{A_1\}$, $\theta_2 = \{A_2\}$, $\theta_3 = \{E_1\}$, $\theta_4 = \{E_2\}$ with respect to D_{5h}.

(iv) *Subduced representation* $(\Gamma^S = \Gamma \downarrow H)$: Let $D^{\Gamma}(A)$ and $D^{\Gamma}(B)$ be the matrices representing the elements A and B in the representation Γ of G. In the representation $\Gamma \to D^{\Gamma}(A)$ of G, the matrices $D^{\Gamma}(B)$ which are images of H, form a representation of H called the subduced representation Γ^S of H and is denoted by $\Gamma \downarrow H = \Gamma^S \to D^{\Gamma}(B)$. Here Γ^S will be of the same dimension as Γ and is in general reducible.

If $G = D_5$ and $H = C_5$, then $A_1 \downarrow C_5 = A\,(C_5)$, $A_2 \downarrow C_5 = A\,(C_5)$, $E_1 \downarrow C_5 = E_a\,(C_5)$ and $E_2 \downarrow C_5 = E_b\,(C_5)$. Here A_1, A_2, E_1, E_2 are the IRs of D_5 (table 1) and A, E_a, E_b are the IRs of C_5 (table 2).

Table 1. Character table for the point group D_5.

D_5	E	$2C_5$	$2C_5^2$	$5C_2$
A_1	1	1	1	1
A_2	1	1	1	1
E_1	2	2 cos 72°	2 cos 144°	0
E_2	2	2 cos 144°	2 cos 72°	0

Generating elements: C_5, C_2; Defining relations: $(C_5)^5 = (C_2)^2 = E$ and $C_2C_5 = C_2'$.

IR of D_5	C_5	C_2
E_1	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^4 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
E_2	$\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega^3 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Table 2. Character table for the point group C_5.

C_5		E	C_5	C_5^2	C_5^3	C_5^4
A		1	1	1	1	1
E_a	E_{a_1}	1	ω	ω^2	ω^3	ω^4
	E_{a_2}	1	ω^4	ω^3	ω^2	ω
E_b	E_{b_1}	1	ω^2	ω^4	ω	ω^3
	E_{b_2}	1	ω^3	ω	ω^4	ω^2

Generating element: C_5; Defining relations: $(C_5)^5 = E$.

(v) *Induced representation* ($\Gamma \equiv \Delta^{\alpha} = \Delta \uparrow G$): Let $|G| = g$, $|H| = h$ and $G = \sum_{i=1}^{g/h} A_i H$ be a left coset decomposition of G relative to H. Define $\sigma(A, B)$ as a matrix of dimension g/h having the elements

$$\sigma_{i,j}(A, B) = \begin{cases} 1, & \text{if } A_i B A_j^{-1} = A \\ 0, & \text{otherwise} \end{cases}.$$

If $D^{\Gamma}(A)$ is the matrix representing the element A in the IR Γ of G and $D^{\Delta}(B)$ is the matrix representing the element B in the IR Δ of H, then

$$D^{\Gamma}(A) = \sum_{B \in H} \sigma(A, B) \otimes D^{\Delta}(B).$$

Then $\Gamma \to D^{\Gamma}(A)$ is a representation of G of dimension $(g/h)d$, induced by the IR Δ of H and is denoted by $\Gamma = \Delta \uparrow G$.

Let $G = D_5$ and $H = C_5$, $D_5 = EC_5 \cup C_2 C_5$ and E_1 be an IR of D_5, E_{a_1} be an IR of C_5. Then

$$\begin{aligned} D^{E_1}(C_5) &= \sigma(C_5, E) \otimes D^{E_{a_1}}(E) + \sigma(C_5, C_5) \otimes D^{E_{a_1}}(C_5) + \sigma\left(C_5, C_5^2\right) \otimes D^{E_{a_1}}\left(C_5^2\right) \\ &\quad + \sigma\left(C_5, C_5^3\right) \otimes D^{E_{a_1}}\left(C_5^3\right) + \sigma\left(C_5, C_5^4\right) \otimes D^{E_{a_1}}\left(C_5^4\right) \\ &= \begin{pmatrix} \omega & 0 \\ 0 & \omega^4 \end{pmatrix}, \end{aligned}$$

where $\omega = \exp(2\pi i/5)$.

Similarly,

$$\begin{aligned} D^{E_1}(C_2) &= \sigma(C_2, E) \otimes D^{E_{a_1}}(E) + \sigma(C_2, C_5) \otimes D^{E_{a_1}}(C_5) + \sigma\left(C_2, C_5^2\right) \otimes D^{E_{a_1}}\left(C_5^2\right) \\ &\quad + \sigma\left(C_2, C_5^3\right) \otimes D^{E_{a_1}}\left(C_5^3\right) + \sigma\left(C_2, C_5^4\right) \otimes D^{E_{a_1}}\left(C_5^4\right) \\ &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \end{aligned}$$

Hence the IR E_1 of D_5 is induced by E_{a_1} of C_5, i.e. $E_{a_1} \uparrow D_5$. We observe here that the actual form of the induced IR depends on the choice of the coset representative A_i. However, an alternative choice of A_i may yield an equivalent IR.

(vi) *Engendered representation*: A representation of the group G obtained from a representation of G/H is called an engendered representation of G. If a representation of G/H is irreducible, then the engendered representation of G is also irreducible.

Consider $D_5/C_5 \cong C_2$. It can be seen that the total symmetric IR A_1 of C_2 engender the total symmetric IR A_1 of D_5 and the 1-d alternating IR of C_2 engender the alternating IR A_2 of D_5.

(vii) *Allowable irreducible representation (AIR)*: γ is said to be an AIR of the little group $L^{(2)}(G, H, \Delta)$ if γ subduces an integer multiple m of Δ on H, i.e. $\gamma \downarrow H = m\Delta$.

The 1-d IR A_1 and A_2 of D_5 are the AIRs of $L^{(2)}(D_5, C_5, A) = D_5$.

3. The little group method

The little group method of finding the IRs of a finite group G from those of the IRs of its maximal normal subgroup H in a chosen composition series in conjunction with solvability property involves the following basic steps:

(a) Express the group G in terms of the composition series $G = H_0 \supset H_1 \supset \cdots \supset H_i \supset H_{i+1} \supset \cdots \supset C_1 = E$. Since G is solvable, the quotient group G/H is cyclic and is of prime order $(g/h_i) = \alpha_i$ (say) for $H = H_i$ and the IRs for H are supposed to be known.
(b) Classify the IRs of H into orbits θ_i with respect to G. The order of an orbit θ_i is either α_i or 1.
(c) Choose an IR, say Δ_i of dimension d_i from each orbit θ_i. Then, (i) if the order of θ_i is α_i, $L^{(2)}(G, H, \Delta_i) = H$ and there is a unique IR of dimension $(g/h)d_i$ of G obtained by inducing with Δ_i. (ii) If the order of θ_i is 1 and Δ_i is non-degenerate, then α_i IRs of G are engendered by the IRs of G/H. If Δ_i is degenerate, then IRs of G are obtained from those of the IRs of G/H with the help of the defining relations for the generators of G.
(d) If γ is an AIR of $L^{(2)}(G, H, \Delta)$, then $\Gamma = \gamma \uparrow G$ is irreducible. If the AIRs of only one little group per orbit θ of H are used to induce the IRs of G, then each one of the IR of G occurs only once.

It can be seen (table 3) that composition series exist among the seven pentagonal point groups which are subgroups of either of the groups $D_{5h}(\overline{10}m2)$ or $D_{5d}(\overline{5}2m)$ and the icosahedral point group $I(235)$ is a normal subgroup of I_h $(\frac{2}{m}\,\overline{3}\,\overline{5})$. Furthermore, the point group $I(235)$ is not solvable, in that, it has no non-trivial normal subgroup. Taking the composition series for the group $D_{5h}(\overline{10}m2)$: $D_{5h} \supset D_5 \supset C_5 \supset C_1 (= E)$ we observe that $C_5/C_1 \equiv C_5$ is a cyclic group of order 5. The point group C_5 has three orbits $\theta_1 = \{A\}$, $\theta_2 = \{E_{a_1}, E_{a_2}\}$, $\theta_3 = \{E_{b_1}, E_{b_2}\}$ with respect to D_5 and $L^{(2)}(D_5, C_5, A) = D_5$. The IR A of C_5 engender the two IRs A_1 and A_2 of D_5.

To find the 2×2 matrices $\sigma(A, B)$, we consider

$$\sigma_{i,j}(A, B) = \begin{cases} 1, & \text{if } A_i B A_j^{-1} = A \\ 0, & \text{otherwise} \end{cases} .$$

For $A \in \{C_2, C_5\}$ and $B \in \{E, C_5, C_5^2, C_5^3, C_5^4\}$. Since $D^{\Gamma}(A) = \sum_{B \in H} \sigma(A, B) \otimes D^{\Delta}(B)$, for the IR E_1 of D_5 and for $A = C_5, A = C_2$, we get

$$D^{E_1}(C_5) = \begin{pmatrix} \omega & 0 \\ 0 & \omega^4 \end{pmatrix}, \quad D^{E_1}(C_2) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

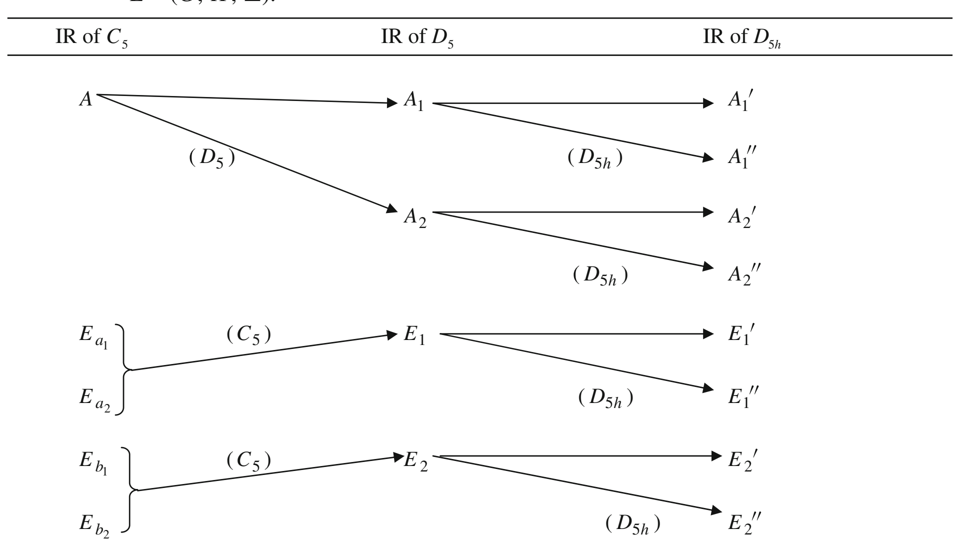
Similarly, for IR E_2 of D_5 and for $A = C_5, A = C_2$, we get respectively

$$D^{E_2}(C_5) = \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega^3 \end{pmatrix}, \quad D^{E_2}(C_2) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

The matrices for the remaining elements of D_5 in the considered IR can be obtained using the generating relations $(C_5)^5 = (C_2)^2 = E$ and $C_2 C_5 = C_2'$. The pair of 1-d complex IR E_{a_1}, E_{a_2} induce the IR E_1 and E_{b_1}, E_{b_2} induce the IR E_2 of D_5. The IRs A_1 and A_2 are two AIRs of $L^{(2)}(D_5, C_5, A) = D_5$. These results are shown in table 3.

To obtain the AIRs of D_5 that induce the IRs of D_{5h}, the IRs of D_5 are classified into four orbits $\theta_1 = \{A_1\}$, $\theta_2 = \{A_2\}$, $\theta_3 = \{E_1\}$, $\theta_4 = \{E_2\}$ relative to D_{5h}. Since

Table 3. Schematic diagram showing the AIRs of the little group that induce the IRs of the point groups in the composition series $D_{5h} \supset D_5 \supset C_5 \supset C_1 (= E)$. The point groups within the parenthesis represent the little groups of the second kind $L^{(2)}(G, H, \Delta)$.



$D_{5h}/D_5 \cong C_2$, from the little group method all the IRs of D_{5h} are engendered from those of the IRs of D_5: The IR A_1 of D_5 engenders A_1', A_1'', A_2 of D_5 engenders A_2', A_2'', E_1 of D_5 engenders E_1', E_1'', E_2 of D_5 engenders E_2', E_2'' of D_{5h}. The matrices representing the generating elements C_5, C_2, σ_h of D_{5h} for the IRs E_1', E_1'', E_2', E_2'' are provided in table 4. The generating matrices for the remaining elements of D_{5h} in the considered IR can be obtained using the defining relations $\sigma_h^2 = \sigma_\vartheta^2 = C_2^2 = C_5^5 = E$ and $\sigma_h C_2 = \sigma_\vartheta$. The results obtained for the considered composition series are shown in table 3.

Following a similar procedure, the AIRs of the point groups that induce/engender the IRs of the other groups can be obtained, by a proper choice of the composition series.

In respect of the icosahedral point groups I and I_h, since the group I is not solvable, we start the little group method from the matrix representation of the generating elements C_2 and C_5 of I corresponding to the IRs F_1, F_2, G and H as given by Matossi [17]. The IRs of I are classified into five orbits $\theta_1 = \{A\}$, $\theta_2 = \{F_1\}$, $\theta_3 = \{F_2\}$, $\theta_4 = \{G\}$ and $\theta_5 = \{H\}$, relative to I_h. Since $I_h/I \cong C_2$, which is of prime order, all the IRs of I_h are engendered from those of the IRs of I from the little group method, as shown in table 5.

Table 4. Character table for the point group D_{5h}.

D_{5h}	E	$2C_5$	$2C_5^2$	$5C_2$	σ_h	$2S_5$	$2S_5^3$	$5\sigma_\vartheta$
A_1'	1	1	1	1	1	1	1	1
A_2'	1	1	1	−1	1	1	1	−1
E_1'	2	2 cos 72°	2 cos 144°	0	2	2 cos 72°	2 cos 144°	0
E_2'	2	2 cos 144°	2 cos 72°	0	2	2 cos 144°	2 cos 72°	0
A_1''	1	1	1	1	−1	−1	−1	−1
A_2''	1	1	1	−1	−1	−1	−1	1
E_1''	2	2 cos 72°	2 cos 144°	0	−2	−2 cos 72°	−2 cos 144°	0
E_2''	2	2 cos 144°	2 cos 72°	0	−2	−2 cos 144°	−2 cos 72°	0

Generating elements: C_5, C_2, σ_h; Defining relations: $\sigma_h^2 = \sigma_\vartheta^2 = C_2^2 = C_5^5 = E$ and $\sigma_h C_2 = \sigma_\vartheta$.

IR of D_{5h}	C_5	C_2	σ_h
E_1'	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^4 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
E_2'	$\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega^3 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
E_1''	$\begin{pmatrix} \omega & 0 \\ 0 & \omega^4 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
E_2''	$\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega^3 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Table 5. Schematic diagram showing the IRs of the point group I_h engendered from the IRs of the point group I. The point groups within parenthesis represent the little groups of the second kind $L^{(2)}(G, H, \Delta)$.

IR of I	IR of I_h
A	$\rightarrow A_1'$
	$\rightarrow A_1''$ (I_h)
F_1	$\rightarrow F_1'$
	$\rightarrow F_1''$ (I_h)
F_2	$\rightarrow F_2'$
	$\rightarrow F_2''$ (I_h)
G	$\rightarrow G'$
	$\rightarrow G''$ (I_h)
H	$\rightarrow H'$
	$\rightarrow H''$ (I_h)

Defining relations of I_h: $i^2 = \sigma^2 = C_2^2 = C_5^5 = E,\ iC_5 = S_{10},\ iC_3 = S_6,\ iC_2 = \sigma.$

Matrix representation of the generating elements of the point group I:

IR of I	C_5	C_2
F_1	$\begin{pmatrix} \left(\sqrt{5}-1\right)/4 & -(\sqrt{\tau+2})/2 & 0 \\ (\sqrt{\tau+2})/2 & \left(\sqrt{5}-1\right)/4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1/\sqrt{5} & -2/\sqrt{5} \\ 0 & -2/\sqrt{5} & 1/\sqrt{5} \end{pmatrix}$
F_2	$\begin{pmatrix} -\left(\sqrt{5}+1\right)/4 & -(\sqrt{3-\tau})/2 & 0 \\ (\sqrt{3-\tau})/2 & -\left(\sqrt{5}+1\right)/4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1/\sqrt{5} & 2/\sqrt{5} \\ 0 & 2/\sqrt{5} & -1/\sqrt{5} \end{pmatrix}$
G	$\begin{pmatrix} \left(\sqrt{5}-1\right)/4 & 0 & 0 & -(\sqrt{\tau+2})/2 \\ 0 & -\left(\sqrt{5}+1\right)/4 & -(\sqrt{3-\tau})/2 & 0 \\ 0 & (\sqrt{3-\tau})/2 & -\left(\sqrt{5}+1\right)/4 & 0 \\ (\sqrt{\tau+2})/2 & 0 & 0 & \left(\sqrt{5}-1\right)/4 \end{pmatrix}$	$\begin{pmatrix} -2/\sqrt{5} & -1/\sqrt{5} & 0 & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$
$\boldsymbol{H}$	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \left(\sqrt{5}-1\right)/4 & 0 & 0 & -(\sqrt{\tau+2})/2 \\ 0 & 0 & -\left(\sqrt{5}+1\right)/4 & -(\sqrt{3-\tau})/2 & 0 \\ 0 & 0 & (\sqrt{3-\tau})/2 & -\left(\sqrt{5}+1\right)/4 & 0 \\ 0 & (\sqrt{\tau+2})/2 & 0 & 0 & \left(\sqrt{5}-1\right)/4 \end{pmatrix}$	$\begin{pmatrix} -1/5 & 0 & 0 & -\sqrt{12}/5 & -\sqrt{12}/5 \\ 0 & -1/\sqrt{5} & -2/\sqrt{5} & 0 & 0 \\ 0 & 2/\sqrt{5} & 1/\sqrt{5} & 0 & 0 \\ \sqrt{12}/5 & 0 & 0 & 3/5 & -2/5 \\ \sqrt{12}/5 & 0 & 0 & -2/5 & 3/5 \end{pmatrix}$

Here $\tau = (\sqrt{5}+1)/2$.

4. Conclusions

In this work, the AIRs of the appropriate little groups that induce/engender the IRs of the point groups with five-fold rotational symmetry are derived by exploring the little group method in conjunction with the solvability property. It has already been established that the constants required for specifying a physical or magnetic property occurring before the degenerate IR of a chosen point group can be obtained in an elegant and simple way from the AIR of the appropriate little groups [9]. In short, the AIRs of a little group give us every information that is expected of a degenerate IR, which is induced by it, of the point group.

After the discovery of the quasicrystals [10], quite a good deal of work has been carried out on the point groups with five-fold rotational axis, for example, calculation of Raman and hyper-Raman scattering tensors, selection rules for atomic transitions, computation of spherical harmonic base for point groups with five-fold rotation axes etc. for the past two decades. The authors believe that this work would be useful for group theoretical physicists working in the area of quasicrystals in their future endeavors.

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