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Radiative see-saw formula in nonsupersymmetric *SO*(10) with dark matter

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Abstract. In $SO(10)$, the type-I and type-II see-saw scales $\geq 10^{12}$ GeV are far away from being probed by direct experimental tests. In the absence of supersymmetry, we show how experimentally verifiable radiative see-saw formula of Ma type is realized in non-SUSY $SO(10)$ while fulfilling the twin objectives: precision gauge coupling unification and dark matter. This model is expected to have a dramatic impact on neutrino physics, dark matter and all fermion masses and mixings.

Keywords. Radiative see-saw; fermion masses; grand unification.

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1. Introduction

In $SO(10)$ grand unified theory which contains all standard fermions of one generation plus the right-handed (RH) neutrino in a single spinorial representation 16_{F}, the heavy RH neutrinos mediate type-I see-saw, while left-handed (LH) Higgs scalar triplet $\subset 126^{\dagger}_{\mathrm{H}}$ mediates type-II [1],

$$M_\nu^I = -M_{\mathrm{D}} M_{\mathrm{R}}^{-1} M_{\mathrm{D}}^{\mathrm{T}}, \quad M_\nu^{\mathrm{II}} = f v_{\mathrm{L}}. \tag{1}$$

Available data on light neutrino masses constrain these scales to be 10^{13}–10^{15} GeV and, as such, high-energy accelerators cannot directly test the underlying origin of neutrino masses. Further, the minimal non-SUSY $SO(10)$ can neither fulfil the very purpose of unifying the Standard Model (SM) gauge couplings for which it was designed, nor can it explain the dark matter (DM) phenomena. In this work, we show how the experimentally testable radiative see-saw formula [2] is embedded into non-SUSY $SO(10)$ while achieving precision unification and verifiable cold dark matter (CDM) and proton lifetime with the prospect of a dramatic impact on all fermion masses and mixings [3].

2. Precision unification

When the gauged $U(1)_{\mathrm{B-L}}$ subgroup of $SO(10)$ breaks spontaneously through the vacuum expectation value (VEV) of the RH Higgs triplet, Δ_{R}, carrying $(\mathrm{B}-\mathrm{L}) = -2$, the surviving matter parity $P_M = (-1)^{3(B-L)}$ as a discreet symmetry Z_2 can safeguard the stability of DM candidates once the latter are introduced into the model Lagrangian [4–6]. The $SO(10)$ representations 10, 45, 54, 120, 126 and 210 possess even matter parity, but the representations 16, 144, ... have odd matter parity. Consistently, the SM fermions (Higgs) carry odd (even) matter parity. Therefore, the general principle for prospective DM particles [7] is that they might be fermions (scalars) of even (odd) P_M. To achieve high-precision unification, we use all nonstandard particles of minimal supersymmetric Standard Model (MSSM) minus the superpartner scalars near the TeV scale,

$$\chi(2, 1/2, 1),\ F_\phi(2, 1/2, 1),\ F_\chi(2, -1/2, 1),\ F_\sigma(3, 0, 1),\ F_b(1, 0, 1),\ F_C(1, 0, 8). \tag{2}$$

In eq. (2) the second Higgs doublet $\chi \subset 16_H$ and the fermions $F_\phi, F_\chi \subset 10_F$ are analogues of two Higgsino doublets while $F_\sigma, F_b \subset 45_F$ and $F_C \subset 45_F$ are the analogues of wino, bino and gluino. All these fermions except the octet have been treated as potential CDM candidates in SUSY GUTs. Using the usual SM particle masses and $m_{F_\phi} \simeq m_{F_\chi} = 2$ TeV, $m_{F_\sigma} \simeq m_\chi \simeq 3$ TeV, $m_{F_C} \simeq 6$ TeV, the resulting precision unification of gauge couplings in the non-SUSY theory occurs close to the MSSM GUT scale with $M_U = 10^{15.96}$ GeV, $\alpha_G^{-1} = 35.3$ and threshold effects can be made negligible [8]. Our estimation of proton lifetime in this model and our improved evaluation in $SU(5) \times Z_2$ model [9] for $p \to e^+\pi^0$ gives,

$$\tau_p^{SO(10)} \simeq 4.28 \times 10^{35}\ \text{yr},\quad \tau_p^{SU(5)XZ_2} \simeq 6.26 \times 10^{33}\ \text{yr}$$

which are to be compared with the experimental lower limit, $(\tau_p)_{\text{expt.}} \geq 1.01 \times 10^{34}$ yr.

3. Radiative see-saw

Matter parity invariance forbids the VEV of 16_H which mediates the Yukawa interaction,

$$-L'_{\text{Yuk}} = Y 16_F 45_F 16_H^\dagger. \tag{3}$$

The part of $SO(10)$-invariant Higgs potential responsible for the radiative see-saw is found to be

$$\begin{aligned} V_{\text{Higgs}}^U &= m_{10}^2 10_H^2 + m_{16}^2 16_H^\dagger 16_H + \lambda_{10} 10_H^4 + \lambda_{16}(16_H^\dagger 16_H)^2 \\ &\quad + \lambda_m 16_H^\dagger 16_H 10_H 10_H + (\lambda_g/M_{Pl}) 16_H^\dagger 10_H \cdot 16_H^\dagger 10_H \cdot 126_H^\dagger. \end{aligned} \tag{4}$$

This leads to the low-scale Higgs potential

$$\begin{aligned} V &= m_\phi^2 \phi^\dagger\phi + m_\chi^2 \chi^\dagger\chi + \frac{1}{2}\lambda_\phi(\phi^\dagger\phi)^2 + \frac{1}{2}\lambda_\chi(\chi^\dagger\chi)^2 \\ &\quad + \lambda_1(\phi^\dagger\phi)(\chi^\dagger\chi) + \lambda_2(\phi^\dagger\chi)(\chi^\dagger\phi) + \frac{1}{2}\lambda_3[(\phi^\dagger\chi)^2 + \text{h.c.}], \end{aligned} \tag{5}$$

where $\lambda_3 = \lambda_g\langle\Delta_R\rangle/M_{Pl}$. Using $M_{Pl} = 1.2\times10^{19}$ GeV, $\langle\Delta_R\rangle \sim 10^{16}$ GeV and $\lambda_g \sim O(1)$ gives $\lambda_3 \simeq 10^{-5}$–10^{-3}, covering the assumed value in ref. [2].

With the replacement of RH neutrinos of ref. [2] by adjoint fermions of this model, $(N_1, N_2, N_3) \to (F_\sigma, F_b^1, F_b^2)$, the radiative see-saw mechanism emerges naturally,

$$M_\nu^{\rm rad} = \frac{\lambda_3}{8\pi^2}\left[m_a \frac{1}{M} m_a^T,\ \ \frac{m_a}{M_0} M \frac{m_a}{M_0}^{\rm T},\ \ m_a \frac{1}{\Lambda} m_a^{\rm T}\right], \tag{6}$$

where the first, second and the third entries hold for $M_i^2 \simeq M_0^2$, $M_i^2 \ll M_0^2$ and $M_i^2 \gg M_0^2$, respectively, and we have defined $m_a = y_a v$, $M = \text{diag}(M_1, M_2, M_3)$, $\Lambda_j = M_j[\ln(M_j^2/M_0^2) - 1]^{-1}$ and $\Lambda = \text{diag}(\Lambda_1, \Lambda_2, \Lambda_3)$ [3]. Thus, in this model, three types of contributions listed in eq. (1) and derived in eq. (6) are realized.

3.1 *Comparison and dominance*

The Yukawa coupling in radiative see-saw is not subject to quark–lepton unification constraint. Also the radiative see-saw has the natural design to dominate for mediating particle masses $\sim O(1\text{–}50)$ TeV. We note that even with Yukawa couplings $y_a \sim 10^{-2}(10^{-3})$, characteristic of charged leptons $\tau(\mu)$, the radiative see-saw dominates over the other two in eq. (1) thus relieving the Yukawas of $10_{\rm H}$ and $126_{\rm H}$ from the tension of fitting the neutrino data. As such they are expected to fit the GUT-scale fermion masses [10] more effectively compared to conventional $SO(10)$ models [11].

4. Summary and conclusion

While discrete symmetries are externally imposed on model extensions to maintain stability of incorporated dark matter, a minimal non-SUSY $SO(10)$ model naturally contains matter parity discrete symmetry, but it neither unifies gauge couplings nor predicts prospective DM candidates. Also, type-I and type-II see-saw scales are too large for direct experimental tests. We have successfully extended the model to achieve precision unification with experimentally testable proton lifetime and to predict inert scalar doublet and other potential fermionic DM candidates which may manifest at colliders or direct DM search experiments. It also predicts the verifiable low-scale radiative see-saw formula of Ma-type along with the natural emergence of its small quartic coupling. Moreover, the new contribution due to radiative see-saw dominates over those from type-I and type-II, making a major impact on neutrino physics and DM while opening up a high potential of $SO(10)$ as a theory of fermion masses in general.

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