PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 79, No. 4
October 2012
pp. 833–837

Non-singlet spin structure function $g_1^{\mathrm{NS}}(x, t)$ in the DGLAP approach

NEELAKSHI N K BORAH[1,*], D K CHOUDHURY[1] and P K SAHARIAH[2]
[1]Department of Physics, Gauhati University, Guwahati 781 014, India
[2]Department of Physics, Cotton College, Guwahati 781 001, India
*Corresponding author. E-mail: nishi_indr@yahoo.co.in

Abstract. An analytical solution of the non-singlet polarized parton distribution $\Delta q^{\mathrm{NS}}(x, Q^2) = (\Delta u(x, Q^2) - \Delta d(x, Q^2))$ is obtained by solving the DGLAP (Gribov and Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972); Lipatov, *ibid*, **20**, 94 (1975); Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1997); Altarelli and Parisi, *Nucl. Phys.* **B126**, 298 (1977)) equation by the method of characteristics. We then evaluate the non-singlet spin-dependent structure function g_1^{NS}. The result is compared with the data from HERMES (Airapetian *et al*, *Phys. Rev.* **D75**, 012007 (2007)).

Keywords. Deep inelastic scattering; structure function; DGLAP equations.

PACS Nos **12.38.−t; 12.38.Bx; 13.60.Hb; 02.30.Jr**

1. Introduction

1.1 *Spin structure function*

The spin physics received a tremendous boost both in the theoretical and experimental front since the publication of the EMC result in 1988 which reported a very small contribution from the quarks to the total spin of the proton. In the lowest order pQCD and single photon exchange, polarized inclusive DIS cross-section can be parametrized by two spin structure functions g_1 and g_2. In QCD-improved parton model and in the leading log approximation, g_1 has a probabilistic interpretation and can be decomposed into flavour singlet and non-singlet combinations for the proton as:

$$g_1^p(x, Q^2) = \frac{1}{2}\sum_q e_q^2[\Delta q^p(x, Q^2) + \Delta\bar{q}^p(x, Q^2)] \tag{1}$$

$$= \frac{1}{2}\langle e^2\rangle[\Delta q^S(x, Q^2) + \Delta q^{\mathrm{NS}}(x, Q^2)]. \tag{2}$$

Here e_q is the fractional charge of the quark, $\Delta q = (q^{\uparrow\uparrow} - q^{\uparrow\downarrow})$ is the polarized quark distribution which measures the quark density with spin parallel minus spin antiparallel to the spin direction of longitudinally polarized proton and

$$\langle e^2 \rangle = \frac{1}{n_f} \sum_{i=1}^{n_f} e_i^2. \tag{3}$$

The flavour singlet polarized quark distribution is

$$\Delta q^S(x, Q^2) = \sum_q [\Delta q^p(x, Q^2) + \Delta \bar{q}^p(x, Q^2)]. \tag{4}$$

This quantity measures the net helicity of the quarks to the total spin of the proton. The non-singlet polarized quark distribution is:

$$\begin{aligned} \Delta q^{\mathrm{NS}}(x, Q^2) &= \frac{1}{\langle e^2 \rangle} \sum_q e_q^2 [\Delta q(x, Q^2) + \Delta \bar{q}(x, Q^2)] - \Delta q^S(x, Q^2) \\ &= (\Delta u(x, Q^2) - \Delta d(x, Q^2)). \end{aligned} \tag{5}$$

The Q^2 evolution of the quark distribution and also of g_1 takes place according to the DGLAP evolution equations which are a set of integro-differential equations.

1.2 *Non-singlet DGLAP evolution equation*

In the LO the flavour non-singlet quark distribution does not mix with singlet quark and gluon and evolve independently. The non-singlet polarized DGLAP equation in LO [1]

$$\frac{\partial}{\partial t} \Delta q^{\mathrm{NS}}(x, t) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}z}{z} \Delta P_{qq}^{\mathrm{NS}}(z) \Delta q^{\mathrm{NS}} \left(\frac{x}{z}, t \right). \tag{6}$$

Here $t = \ln(Q^2/\Lambda^2)$ and $\Delta P_{qq}^{\mathrm{NS}}(z)$ is the polarized splitting function. Let $u = 1 - z$ such that

$$\frac{x}{z} = \frac{x}{1-u} \approx x(1+u). \tag{7}$$

Expanding $\Delta q^{\mathrm{NS}}((x/z), t)$ in a Taylor series and neglecting higher order terms [2]

$$\Delta q^{\mathrm{NS}} \left(\frac{x}{z}, t \right) \approx \Delta q^{\mathrm{NS}}(x, t) + xu \frac{\partial}{\partial x} \Delta q^{\mathrm{NS}}(x, t). \tag{8}$$

Using eq. (8) and the polarized splitting function [3], after some calculation, eq. (6) is reduced to a partial differential equation in two variables, Bjorken x and t:

$$Q(x, t) \frac{\partial \Delta q^{\mathrm{NS}}(x, t)}{\partial t} + P(x, t) \frac{\partial \Delta q^{\mathrm{NS}}(x, t)}{\partial x} = R(x, t) \Delta q^{\mathrm{NS}}(x, t), \tag{9}$$

where

$$\begin{aligned} Q(x, t) &= t, \\ P(x, t) &= \frac{2}{\beta_0} \left(\frac{4}{3} x \ln x - \frac{4x^2}{3} + \frac{4x}{3} \right) \end{aligned}$$

and

$$R(x, t) = -\frac{2}{\beta_0} \left(-\frac{4}{3} \ln \frac{1}{x} + \frac{2}{3} \right).$$

Beyond its traditional evolution in t, eq. (9) gives also x evolution at small x.

2. Solution by the method of characteristics

Being a differential equation in two variables, eq. (9) requires two boundary conditions for its solution. But usually we have only one boundary condition which is the non-perturbative x distribution of the function at some initial scale Q_0^2. So the solution obtained is not unique, it gives only a range of solutions. This limitation can be avoided by adopting the method of characteristics [4] in which the integration of the partial differential equations are reduced to a family of initial value problem for a system of ordinary differential equations. The original set of variables (x, t) is changed into a new set of variables (s, τ) such that the partial differential equation becomes an ordinary differential equation with respect to any one variable of this new set. The characteristic equations are:

$$\frac{\mathrm{d}x}{\mathrm{d}s} = P(x), \tag{10a}$$

$$\frac{\mathrm{d}t}{\mathrm{d}s} = Q(t). \tag{10b}$$

So, along the characteristic curve the PDE (eq. (9)) becomes an ODE:

$$\frac{\mathrm{d}\Delta q^{\rm NS}(s,\tau)}{\mathrm{d}s} + c(s,\tau)\Delta q^{\rm NS}(s,\tau) = 0. \tag{11}$$

Here,

$$c(s,\tau) = -\frac{2}{\beta_0}\left[\frac{4}{3}\left\{\ln\tau + \exp\left(\frac{8s}{3\beta_0}\right)\right\} + \frac{2}{3}\right]. \tag{12}$$

Integrating along the characteristic we get the solution in (s, τ) space as

$$\Delta q^{\rm NS}(s,\tau) = \Delta q^{\rm NS}(\tau)\exp\left[\frac{4}{3\beta_0}\left\{2s\ln\tau + 2s\exp\left(\frac{8s}{3\beta_0}\right)\right\}\right]. \tag{13}$$

The solution of the characteristic equations (eqs (10a) and (10b)) are

$$s = \ln\left(\frac{t}{t_0}\right) \tag{14a}$$

$$\tau = x\exp\left[-\left(\frac{t}{t_0}\right)^{8/3\beta_0}\right]. \tag{14b}$$

Using eqs (14) we go back to the (x, t) space. Then the solution in (x, t) space is

$$\Delta q^{\rm NS}(x,t) = \Delta q^{\rm NS}(\tau)\exp\left[\left(\frac{t}{t_0}\right)^{8/3\beta_0} - 1\right]\left(\frac{t}{t_0}\right)^{\frac{4}{3\beta_0}(1-2\ln\frac{1}{x})}. \tag{15}$$

Equation (15) is our solution which we use to calculate $g_1^{\rm NS}(x,t)$. We note that the input distribution $\Delta q^{\rm NS}(\tau)$ acquires also a $t(=\ln(Q^2/\Lambda^2))$ dependence.

3. Results and discussion

In figure 1 our calculated $g_1^{\rm NS}(x,t)$ is plotted as a function of x at $Q^2 = 5$ GeV^2 together with data from ref. [5]. The data are at the corresponding values of x and Q^2 as explained

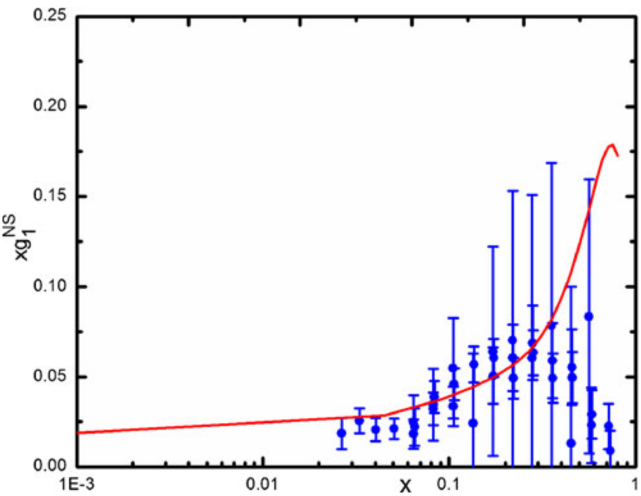

Figure 1. $g_1^{\mathrm{NS}}(x, t)$ as a function of x at fixed $Q^2 = 5\ \mathrm{GeV}^2$ together with data from HERMES [5]. The red solid line is our result. Only systematic errors are shown.

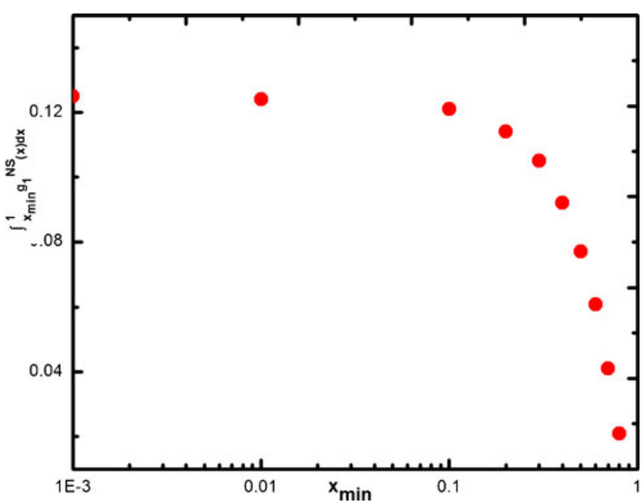

Figure 2. First moment of g_1^{NS} as a function of x_{min} at $Q^2 = 2\ \mathrm{GeV}^2$.

in [5]. For input distribution we use AAC03 [6]. Our prediction overshoots the data above $x = 0.3$. This is probably due to the fact that our approximation is valid only at low x. In figure 2 we show the first moment of g_1^{NS} calculated using our solution for $\Delta q^{\mathrm{NS}}(x, Q^2)$ for different values of x_{min} at $Q^2 = 2\ \mathrm{GeV}^2$. The first moment of g_1^{NS}, known as Bjorken sum rule [7], is a rigorous prediction of current algebra and QCD. Recent COMPASS measurement [8] gives its value to be $0.190 \pm 0.009 \pm 0.015$ at $Q^2 = 3\ \mathrm{GeV}^2$ in the range $0.004 \le x \le 0.7$. As can be seen from the graph, our result is much below this value.

4. Conclusion

An approximate analytical solution of polarized non-singlet DGLAP equation is obtained by the method of characteristics. The solution can explain the general trend of data for g_1^{NS}, but falls short off quantitatively. First moment of g_1^{NS} is much smaller than theoretical prediction.This might probably be improved by incorporating resummation and NLO effects in our calculation together with some higher order terms in eq. (8).

References

[1] V N Gribov and L N Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972)
L N Lipatov, *Sov. J. Nucl. Phys.* **20**, 94 (1975)
Yu L Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1977)
G Altarelli and G Parisi, *Nucl. Phys.* **B126**, 298 (1977)

[2] D K Choudhury and P K Sahariah, *Pramana – J. Phys.* **58**, 599 (2002)

[3] R D Ball, S Forte and G Ridolfi, *Nucl. Phys.* **B444**, 287 (1995)

[4] S J Farlow, *Partial differential equations for scientists and engineers* (John Wiley, NewYork, 1982)

[5] HERMES Collaboration: A Airapetian *et al*, *Phys. Rev.* **D75**, 012007 (2007)

[6] Asymmetry Analysis Collaboration: M Hirai, S Kumano and N Saito, *Phys. Rev.* **D69**, 054021 (2004)

[7] J D Bjorken, *Phys. Rev.* **D148**, 1467 (1966); *Phys. Rev.* **D1**, 1376 (1970)

[8] COMPASS Collaboration: M G Alekseev *et al*, arXiv:1001.4654 [hep-ex]