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Theoretical aspects of neutrino mass and lepton flavour violation

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Abstract. We consider lepton flavour violation (LFV) in the charged lepton sector both from the bottom-up effective Lagrangian approach and from the top-down approach via various case studies that have been analysed. The implications for LFV studies at the LHC is briefly discussed. Finally the nature of LFV in the neutrino sector is considered, paying particular regard to the implications of the recent measurements of θ_{13}.

Keywords. Neutrino mass; lepton flavour violation; lepton mixing angles.

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1. Introduction

The study of neutrino oscillations has established that there is lepton flavour violation analogous to the quark flavour violation described by the CKM matrix. To date there is no evidence for overall lepton number violation (LNV) although it provides a natural connection to new physics at a high scale from grand unified theories via the see-saw mechanism. In this case the neutrino masses are described by the lepton number violating Weinberg operator

$$O_{\mathrm{WB}}^{\mathrm{dim}=5} = \frac{g_{\nu}^{ij}}{\Lambda_{\mathrm{LNV}}} \left(\bar{L}^i H\right) \left(H^{\dagger} L^j\right)^c,$$

where

$$\Lambda_{\mathrm{LNV}} = O(10^{15}\,\mathrm{GeV}).$$

In the Type I see-saw the scale of new physics Λ_{LNV} is associated with the exchange of right-handed neutrinos and this opens the possibility of baryogenesis through leptogenesis. However, as we shall discuss, this scale is too large to generate observable LFV signals in the charged lepton sector, such effects requiring a lower scale of new physics, Λ_{LFV}.

Lepton flavour violation in the charged lepton sector has not yet been observed but is the subject of intense study (for recent reviews of the theoretical and experimental status,

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Table 1. A sample of charged lepton flavour violating reactions. Data from current experimental bounds, expected improvements from existing or funded experiments, and possible long-term advances (based on the review of Marciano *et al* [4], incorporating recent updates – see these proceedings [3]).

Reaction	Current bound	Expected	Possible
$B\left(\mu^+ \to e^+\gamma\right)$	$< 2.4 \times 10^{-12}$	2×10^{-13}	2×10^{-14}
$B\left(\mu^\pm \to e^\pm e^+ e^-\right)$	$< 1.0 \times 10^{-12}$	–	10^{-14}
$B\left(\mu^\pm \to e^\pm \gamma\gamma\right)$	$< 7.2 \times 10^{-11}$	–	–
$R\left(\mu^- \text{Au} \to e^- \text{Au}\right)$	$< 7 \times 10^{-13}$	–	–
$R\left(\mu^- \text{Al} \to e^- \text{Al}\right)$	–	10^{-16}	10^{-18}
$B\left(\tau^\pm \to \mu^\pm \gamma\right)$	$< 4.4 \times 10^{-8}$	–	$0(10^{-9})$
$B\left(\tau^\pm \to e^\pm \gamma\right)$	$< 3.3 \times 10^{-8}$	–	$0(10^{-9})$
$B\left(\tau^\pm \to \mu^\pm \mu^+ \mu^-\right)$	$< 2.0 \times 10^{-8}$	–	$0(10^{-10})$
$B\left(\tau^\pm \to e^\pm e^+ e^-\right)$	$< 2.6 \times 10^{-8}$	–	$0(10^{-10})$
$Z^0 \to e^\pm \mu^\mp$	$< 1.7 \times 10^{-6}$	–	–
$Z^0 \to e^\pm \tau^\mp$	$< 1.2 \times 10^{-5}$	–	–
$Z^0 \to \mu^\pm \tau^\mp$	$< 9.8 \times 10^{-6}$	–	–
$K_L^0 \to e^\pm \mu^\mp$	$< 4.7 \times 10^{-12}$	–	10^{-13}
$D^0 \to e^\pm \mu^\mp$	$< 8.1 \times 10^{-12}$	–	10^{-8}
$B^0 \to e^\pm \mu^\mp$	$< 9.2 \times 10^{-8}$	–	10^{-9}

see [1–3]). A sample of the current experimental limits is shown in table 1. If and when LFV is observed, a test of the underlying theory of LFV will be the correlation between the rates for these and other LFV processes. In this talk I shall consider the most likely correlations in some detail.

2. Theories of lepton flavour violation in charged lepton decays

2.1 *Bottom-up effective field theory description*

In an effective field theory description of new physics corresponding to a high scale one integrates out the heavy states and writes the Lagrangian in terms of operators involving only the light states. For the case of LNV and LFV processes the leading gauge invariant terms give an effective Lagrangian of the form

$$L_{\text{eff}} = L_{\text{SM}} + \frac{g_\nu^{ij}}{\Lambda_{\text{LNV}}}\left(\bar{L}^i H\right)\left(H^\dagger L^j\right)^c + \frac{1}{\Lambda_{\text{LFV}}{}^2} O^{\text{dim}=6} + \cdots, \tag{1}$$

where the second term is the dimension-5 LNV Weinberg operator and the third term is the leading dimension-6 terms responsible for LFV.

As mentioned above, for observable LFV effects a lower scale of new physics is needed than the GUT scale. For example, the operator $(1/\Lambda_{\text{LFV}}^2)\bar{L}^i\sigma^{\mu\nu}H\, e_R^j\, F_{\mu\nu}$ contributes to the process $\mu \to e\gamma$ and the current limits only require $\Lambda_{\text{LFV}} > 10^5$ GeV. To be

Table 2. The leading operators contribution to LFV processes.

Name	Operator	Coefficient
$O_{\rm LL}^{1}$	$\left(\bar{L}_i\gamma^{\mu}L^j\right)H^{\dagger}iD_{\mu}H$	Δ_j^{1i}
$O_{\rm LL}^{2}$	$\left(\bar{L}_i\gamma^{\mu}\tau^a L^j\right)H^{\dagger}\tau^a iD_{\mu}H$	Δ_j^{2i}
$O_{\rm LL}^{3}$	$\left(\bar{L}_i\gamma^{\mu}L^j\right)\left(\bar{Q}_{\rm L}\gamma^{\mu}Q_{\rm L}\right)$	Δ_j^{3i}
$O_{\rm LL}^{4d}$	$\left(\bar{L}_i\gamma^{\mu}L^j\right)\left(\bar{D}_{\rm R}\gamma^{\mu}D_{\rm R}\right)$	Δ_j^{4i}
$O_{\rm LL}^{4u}$	$\left(\bar{L}_i\gamma^{\mu}L^j\right)\left(\bar{U}_{\rm R}\gamma^{\mu}U_{\rm R}\right)$	Δ_j^{5i}
$O_{\rm LL}^{5}$	$\left(\bar{L}_i\gamma^{\mu}\tau^a L^j\right)\left(\bar{Q}_{\rm L}\gamma^{\mu}\tau^a Q_{\rm L}\right)$	Δ_j^{6i}
$O_{\rm RL}^{1}$	$g'H^{\dagger}\left(\bar{E}_{{\rm R}i}\sigma^{\mu\nu}L^j\right)B_{\mu\nu}$	Ω_j^{1i}
$O_{\rm RL}^{2}$	$gH^{\dagger}\left(\bar{E}_{{\rm R}i}\sigma^{\mu\nu}\tau^a L^j\right){W^a}_{\mu\nu}$	Ω_j^{2i}
$O_{\rm RL}^{3}$	$\left(D_{\mu}H^{\dagger}\right)\left(E_{{\rm R}i}D^{\mu}L^j\right)$	Ω_j^{3i}
$O_{\rm RL}^{4}$	$\left(\bar{E}_{{\rm R}i}L^j\right)\left(\bar{Q}_{\rm L}Y_{\rm D}D_{\rm R}\right)$	Ω_j^{4i}
$O_{\rm RL}^{5}$	$\left(\bar{E}_{{\rm R}i}\sigma^{\mu\nu}L^j\right)\left(\bar{Q}_{\rm L}\sigma_{\mu\nu}Y_{\rm D}D_{\rm R}\right)$	Ω_j^{5i}
$O_{\rm RL}^{6}$	$\left(\bar{E}_{{\rm R}i}L^j\right)\left(\bar{U}_{\rm R}{Y_{\rm U}}^{\dagger}i\tau^2 Q_{\rm L}\right)$	Ω_j^{6i}
$O_{\rm RL}^{7}$	$\left(\bar{E}_{{\rm R}i}\sigma^{\mu\nu}L^j\right)\left(\bar{U}_{\rm R}\sigma_{\mu\nu}{Y_{\rm U}}^{\dagger}i\tau^2 Q_{\rm L}\right)$	Ω_j^{7i}
$O_{\rm LLRR}^{4}$	$\left(\bar{L}_i\gamma^{\mu}L^j\right)\left(\bar{E}_{\rm R}\gamma_{\mu}E_{\rm R}\right)$	$\Delta_j^{\prime 1i}$
$O_{\rm LLLL}^{3}$	$\left(\bar{L}_i\gamma^{\mu}L^j\right)\left(\bar{L}_k\gamma_{\mu}L^l\right)$	Θ_j^{1i}
$O_{\rm LLLL}^{5}$	$\left(\bar{L}_i\gamma^{\mu}\tau^a L^j\right)\left(\bar{L}_k\gamma_{\mu}\tau^a L^l\right)$	Θ_j^{2i}

observable in future the relevant scale should not be much below this. However, there is good reason to expect this to be the case. The hierarchy problem, namely the difficulty of separating the electroweak breaking scale from the GUT scale or the Planck scale, suggests that there should be new physics beyond the Standard Model at a scale $\leq O(10^3\,{\rm GeV})$. This suggests that there may be already some tension between the limit on the LFV scale and this scale of new physics. For example, in supersymmetric extensions of the Standard Model, there are one-loop contributions to $\mu \rightarrow e\gamma$ involving sleptons and charginos as intermediate states. If the family violating couplings involved in these graphs are of $O(1)$ then we expect $\Lambda_{\rm LFV} = O(10^3\,{\rm GeV}/\alpha)$. From this one sees that it is likely that LFV is close to the present limits. However, the family structure of the new interactions is crucial in determining the expectation for LFV processes. Returning to eq. (1), the relevant operators are listed in table 2 [5]. As may be seen there are a large number of operators and if their coefficients are independent, correlations between LFV processes will be very difficult to find. However, in most theories of LFV these coefficients are related and I turn now to a discussion of the most likely such relationships.

2.2 *Symmetry structure of LFV interactions*

In the absence of Yukawa interactions, the leptonic sector of the Standard Model is symmetric under the $SU(3)_{\rm L} \times SU(3)_{e_{\rm R}} \times U(1)_{\rm L} \times U(1)_{e_{\rm r}}$ group, the symmetry of its kinetic terms. Here $SU(3)_{\mathrm{L}}$ is the symmetry acting on the three families of left-handed lepton doublets etc. It follows that the LFV processes only occur when this symmetry is broken via the terms

$$L = Y_{\mathrm{E}}^{ij}\left(\bar{L}^i H E_{\mathrm{R}}^j\right) + \frac{g_\nu^{ij}}{\Lambda_{\mathrm{LNV}}}\left(\bar{L}^i H\right)\left(H^\dagger L^j\right)^c + \frac{1}{{\Lambda_{\mathrm{LFV}}}^2} O^{\mathrm{dim}=6} + \cdots . \tag{2}$$

The first term is the Yukawa coupling matrix that, after spontaneous EW breaking, generates the charged lepton masses and mixing while the second term, the Weinberg operator, generates the neutrino masses and mixing. On the basis in which the charged lepton mass matrix is diagonal the coefficients have the form

$$\begin{aligned} g_\nu &\sim (6,1)_2 = \frac{\Lambda_{\mathrm{LNV}}}{v^2} U^* \mathrm{Diag}[m_\nu] U, \\ Y_{\mathrm{E}}^{ij} &\sim (3,\bar{3})_0 = \frac{1}{v}\mathrm{Diag}[m_l], \end{aligned} \tag{3}$$

where v is the Higgs vacuum expectation value, $M_{\nu,l}$ are the neutrino and charged lepton masses and U is the PMNS mixing matrix. The determination of the third term in eq. (3) requires a theory of symmetry breaking structure of the dimension 6 terms. Under $SU(3)_{\mathrm{L}} \times SU(3)_{e_{\mathrm{R}}} \times U(1)_{\mathrm{L}}$ they transform as

$$\begin{aligned} &\Delta^{I=1..5} \equiv \Delta \sim (8,1)_0, \\ &\Omega^{I=1..7} \equiv \Omega \sim (\bar{3},3)_0, \\ &\Theta^{I=1,2} \equiv \Theta \sim ((8+1)\times(8+1),1)_0, \\ &\Delta' \sim ((8+1),(8+1))_0. \end{aligned} \tag{4}$$

If, as is plausible, the symmetry breaking in a given representation is dominated by a single term, this structure implies that there are symmetry relations between the operators $\Delta^{I=1..5}$ and between $\Omega^{I=1..7}$. There will also be a symmetry relation between the operators $\Theta^{I=1,2}$, provided a single representation dominates.

A further plausible assumption is that the origin of the spontaneous symmetry breaking is due to the vacuum expectation of familons (spurions). These should generate both the mass terms and the dimension-6 terms of eq. (3) and if there are only a small number of such familons there may be relations between the lepton masses and mixings and the coefficients of the LFV operators. The most studied case is that of 'minimal flavour violation (MFV) that I shall now discuss.

2.3 *Minimal flavour violation*

In MFV one assumes that all the symmetry breaking originates in spurions (ϕ, θ familon VEVs) in the same representations as the charged lepton and neutrino mass matrices:

$$g_\nu = \langle\phi\rangle \sim (6,1)_2, \quad Y_{\mathrm{E}} = \langle\theta\rangle \sim \left(3,\bar{3}\right)_0. \tag{5}$$

This assumption is very predictive but theoretically questionable as the familon structure gives no indication of the fermion mass structure. Attempts to explain this structure usually introduce more fundamental familons in different representations to generate the hierarchical structure of the charged leptons encoded in $Y_{\rm E}$ by expressing individual matrix elements of $Y_{\rm E}$ as powers of these familons (examples appear in later sections). Ignoring this possibility, MFV is minimal in the sense that it assumes that $Y_{\rm E}$ is fundamental and that all family symmetry breaking can be expressed in terms of it (and g_ν). Even so the implementation of MFV is not unique as it depends on whether the Weinberg operator generating neutrino mass is considered fundamental or it results from an underlying see-saw.

2.3.1 *Non-see-saw version.* Treating g_ν as fundamental and using eq. (5), the coefficients of eq. (4) may be expressed in terms of g_ν and $Y_{\rm E}$ as

$$\Delta^I = \Delta = \left(g_\nu^\dagger g_\nu\right)^i_j - \frac{1}{3}\delta^i_j, \quad \Omega^I = \Omega = Y_{\rm E}^\dagger \Delta,$$
$$\Theta^I = \Theta = \Delta^i_j \delta^k_l + (g_\nu)^i_j\,(g_\nu)^k_l\,, \quad \Delta' = \Delta. \qquad (6)$$

Using eq. (3) the coefficients are then expressed in terms of masses and mixing angles; we shall refer to this case as MLFV0 [5].

2.3.2 *See-saw version.* As noted in the introduction, a very plausible origin for the Weinberg operator is the see-saw mechanism in which the operator is generated by the exchange of very heavy states associated with a stage of grand unification. For Type I see-saw the starting Lagrangian is

$$L = Y_{\rm E}^{ij}\left(\bar{L}^i H E_{\rm R}^j\right) + Y_\nu^{ij}\left(\bar{L}^i H^* \tau_2\, \nu_{\rm R}^{cj}\right) - \frac{1}{2} M_{\rm M}^{ij} \bar{\nu}_{\rm R}^{ci} \nu_{\rm R}^j + \frac{1}{{\Lambda_{\rm LFV}}^2} O^{\rm dim=6} + \cdots, \qquad (7)$$

where under the $SU(3)_{\rm L} \times SU(3)_{E_{\rm R}} \times SU(3)_{\nu_{\rm R}} \times U(1)_{\rm L}$ family symmetry the transformation properties of the couplings (spurions) are given by

$$Y_{\rm E} \sim (3, \bar{3}, 1)_0, \quad Y_\nu \sim (3, 1, \bar{3})_0, \quad M_{\rm M} \sim (1, 1, \bar{6})_0. \qquad (8)$$

The neutrino mass matrix is given by

$$m_\nu^\dagger = \frac{v^2}{\Lambda_{\rm LNV}} Y_\nu \frac{1}{M_{\rm M}} Y_\nu^T = U m_\nu^D U^T$$

so that $g_\nu = (\Lambda_{\rm LNV}/v^2) m_\nu$ is no longer fundamental. In this case the coefficients of table 2 are given in terms of the spurions by

$$\Delta^I,\ \Delta'^I = Y_\nu Y_\nu^\dagger, Y_\nu M_{\rm M}^\dagger M Y_\nu^\dagger$$
$$\Delta_6 \equiv Y_\nu M_{\rm M}^\dagger Y_\nu^T$$
$$\Theta^{3,5ik}_{jl} = \Delta^i_j \Delta^k_l, \Delta^i_{6l} \Delta^k_{6j}$$
$$\Omega = Y_{\rm E}. \qquad (9)$$

Due to the unknown parameters involved in the see-saw some assumptions are needed to determine the LFV structure. Here I review the implication following from two choices of simplifying assumptions. For other possibilities, see [6–9].

MLFV1. The first case assumes that the RH neutrino mass matrix is diagonal and the CP violating phases in the neutrino sector are absent

$$M_{\mathrm{M}} = \mathrm{Diag}(M, M, M), \qquad Y_\nu^\dagger = Y_\nu^T \ (\mathrm{CP}). \tag{10}$$

From this, one has [5]

$$\Delta_6 = \Delta = Y_\nu Y_\nu^T = \frac{\Lambda_{\mathrm{LNV}}}{v^2} U m_\nu U^T. \tag{11}$$

MLFV2. The second case assumes that the family symmetry is the same acting on the left- and right-handed components, i.e. $SU(3)_{E_{\mathrm{R}}} \equiv SU(3)_{\nu_{\mathrm{R}}}$. This would be true, for example, when there is an underlying $SO(10)$ GUT. Then one finds [10]

$$\Delta_6 = \frac{v^2}{\Lambda_{\mathrm{LNV}}} U \frac{1}{m_\nu} U^T, \qquad \Delta = \frac{v^4}{\Lambda_{\mathrm{LNV}}^2} U \frac{1}{m_\nu^2} U^\dagger. \tag{12}$$

Using the results of eqs (6), (9) and (12) one can relate the rates for $\tau \to \mu\gamma$ to $\tau \to \mu\gamma$ for the three cases considered. These are shown in figure 1.

Clearly from these figures there is a wide range of possibilities but one common feature remains. If $\sin^2\theta_{13} \geq 0.1$, the current experimental limits on $\mu \to e\gamma$ imply that $\mu \to \mu\gamma$ is unmeasurably small. Given the recent measurement that suggests θ_{13} may be in this range, this is a particularly interesting result. However, given the strong assumptions that go into the MLFV hypothesis, it is important to take this *cum grano salis*. To test the generality of the MLFV results, it is useful to consider the expectations for LFV in 'top down' models and I turn to this now.

3. Top down case studies – LFV in specific models

In this section, I shall consider models based on SUSY, little Higgs, fourth lepton and extra dimensions.

3.1 *SUSY*

There are wide varieties of SUSY models that have been considered. I shall discuss a model based on a SUSY see-saw mechanism for neutrino mass and a model with a family symmetry capable of generating the lepton mass hierarchy and neutrino mixing.

3.1.1 *SUSY see-saw.* There has been considerable effort to determine the implications for LFV when the neutrino masses are driven by the see-saw mechanism (for a recent review and references therein, see [11]). The parameters involved in the see-saw are constrained by the need to generate the observed neutrino mass squared differences and the

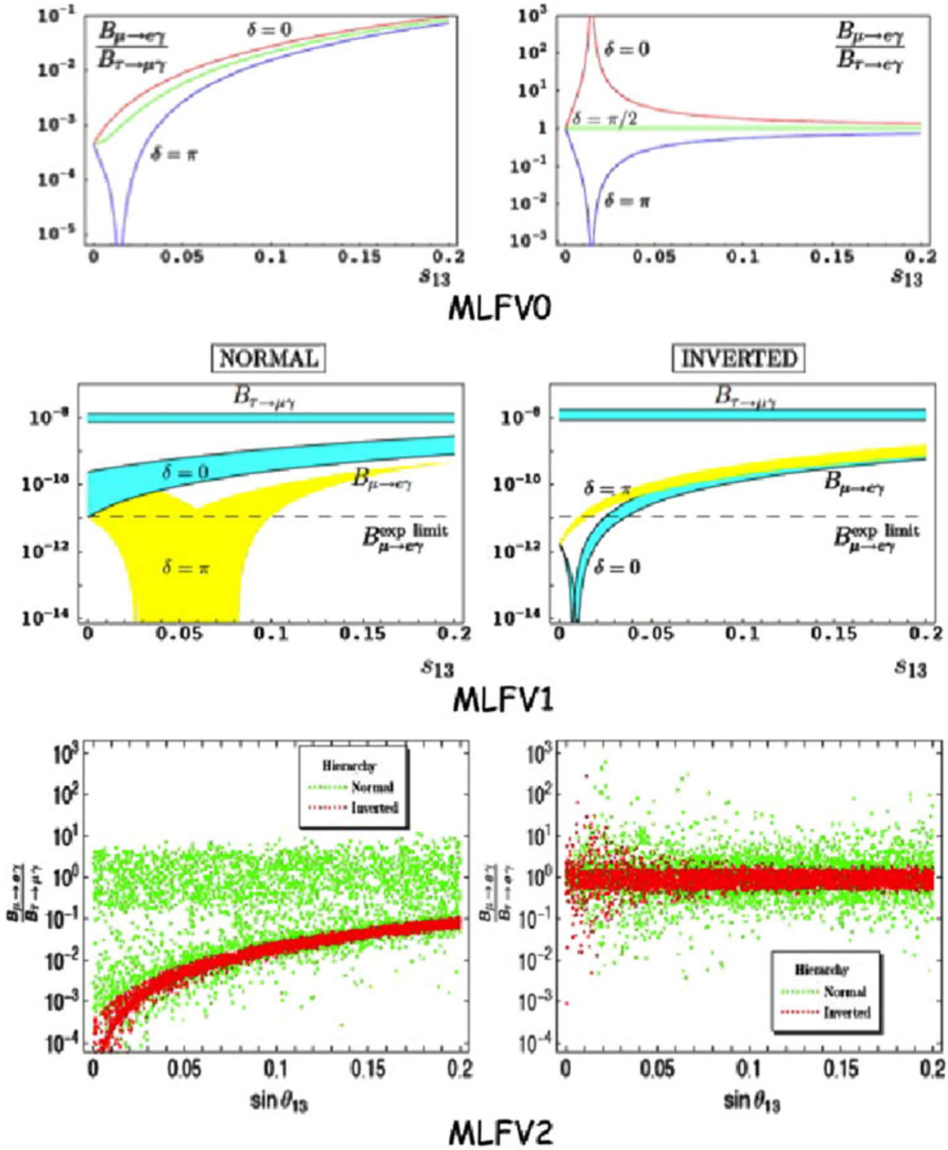

Figure 1. The prediction for BR($\mu \to e\gamma$), BR($\tau \to \mu\gamma$) and BR($\tau \to e\gamma$) in various implementations of MLV. The parameter δ refers to the (Dirac) phase in the PMNS matrix. For MLFV0, the results shown are for the normal hierarchy only, while for MLFV1 and MLFV2 the results for both normal and inverted hierarchies are shown.

mixing angles. However, as mentioned above, this leaves several undetermined parameters. These are conveniently described by a parametrization proposed in [12] in which the unknown parameters are described by a 3×3 orthogonal complex matrix, R, and

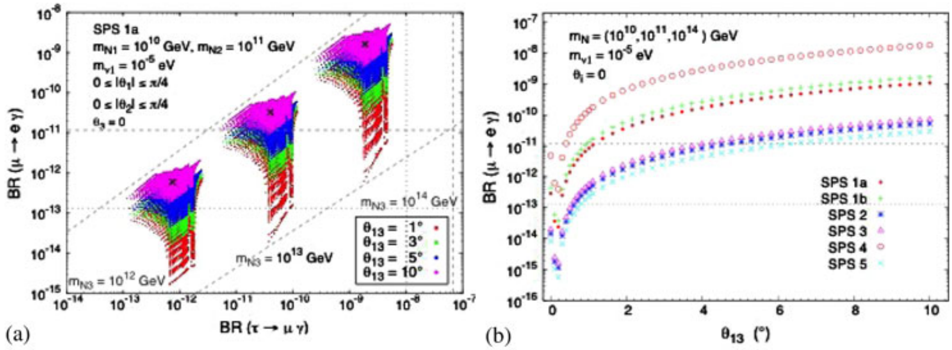

Figure 2. (**a**) Correlation between BR($\mu \to e\gamma$) and BR($\tau \to \mu\gamma$) as a function of M_{N_3} for the SUSY benchmark point SPS 1a. Horizontal and vertical dashed (dotted) lines denote the experimental bounds (future sensitivities). (**b**) BR($\mu \to e\gamma$) as a function of θ_{13} (figure taken from [14]).

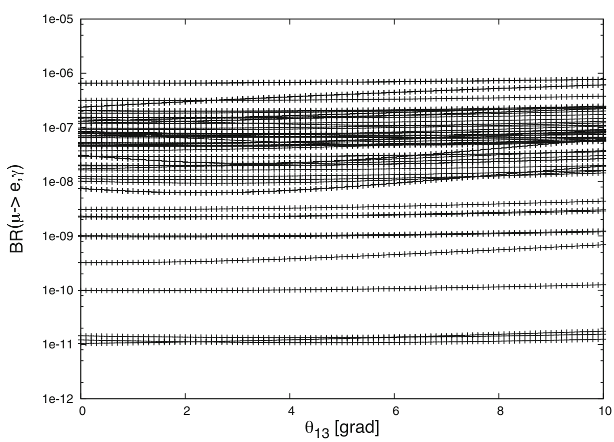

Figure 3. The dependence of BR($\mu \to e\gamma$) on θ_{13} in a SUSY see-saw model using an extended sampling technique.

the masses of the singlet right-handed (FH) neutrinos. By choosing the latter large one can have large neutrino Yukawa couplings, Y^{ν}_{ij}, and these in turn drive LFV processes proportional to $Y^{\dagger}Y$ which are calculated by solving the renormalization group equations [13,14]. For hierarchical RH neutrinos, $Y^{\dagger}Y \propto (M_{N_3}\ \log(M_{N_3})^2$. Sampling R the implications for LFV processes are shown in figure 2.

From this figure it seems that for $\theta_{13} > 5°$, $\tau \to \mu\gamma$ is too small to measure, which is in agreement with the conclusion from MLFV. However, the dependence on θ_{13} shown in this figure has recently been challenged. Casas *et al* [15] have shown that the structure is due to a sampling artifact of the unknown parameters and that the distribution is essentially flat as shown in figure 3. This implies that the large angle limit of figure 2b applies over the whole angular range.

The implication of this is that over all θ_{13} most phase space has $\tau \to \mu\gamma$ too small to measure. Moreover, the maximum rate for $\mu \to e\gamma$ is seen to be increased by a factor of 10 although, if one insists on baryogenesis via leptogenesis, this increase is cancelled by a reduction by a factor of 10.

3.1.2 *Family symmetry.* As mentioned above, models capable of explaining the pattern of fermion masses and mixings often break the family symmetry via familon VEVs that do not correspond to the spurions assumed in the MLFV analyses. One may expect that this leads to significant differences and to illustrate this we consider a model that has been analysed in detail based on an A_4 discrete family symmetry [16,17]. In this model the symmetry breaking parameter u is approximately $\sin\theta_{13}$ and the LFV depends on the soft SUSY breaking masses, the common scalar mass, M_{SUSY}, and the common gaugino mass, $m_{1/2}$, at the grand unification scale. In figure 4 we show the expectation for $\mu \to e\gamma$. If the expansion parameter is large quite significant areas of parameter space are already excluded by the current experimental bound forcing one to larger values of the soft SUSY breaking parameters. It is interesting to note that the limits on the SUSY parameters are comparable to those obtained from direct SUSY searches at the LHC, demonstrating that

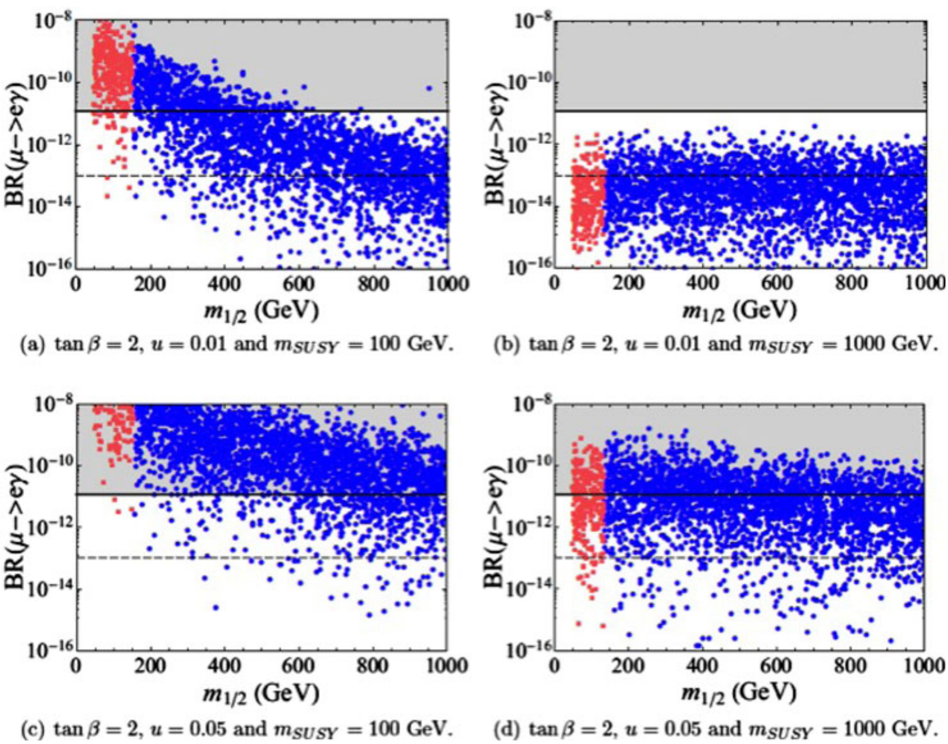

Figure 4. Prediction for BR($\mu \to e\gamma$) in an A_4 model of charged lepton and neutrino masses and mixings for various choices of the parameters. The shaded area is excluded by experimental bounds (figure taken from [15]).

SUSY models typically generate large LFV at the level of current bounds. Of particular interest is the prediction for the relative LFV processes. In this case the A_4 symmetry implies $\mathrm{BR}(\mu \to e\gamma) \approx \mathrm{BR}(\tau \to \mu\gamma) \approx \mathrm{BR}(\tau \to e\gamma)$. So again it is unlikely that the LFV τ decays will be observable. However, one should note that the results are very sensitive to the vacuum alignment method and to the familon A terms [18].

3.2 *Little Higgs models*

In little Higgs models the Higgs is identified with a pseudo-Goldstone boson associated with a spontaneous symmetry breaking of an approximate global symmetry at the scale f (for reviews, see [19,20]). They are characterized by having new heavy gauge bosons and/or heavy leptons and loop diagrams involving these states generate LFV processes. To avoid unacceptably large contributions to precision electroweak observables for low f scales, one can demand the theory satisfies T-parity under which the new massive particels are T-odd [21]. In figure 5, I show the implication for the relative value of $\mathrm{BR}(\mu \to e\gamma)$ to $\mathrm{BR}(\tau \to \mu\gamma)$ for the littlest Higgs model with T-parity [22,23]. In contrast with the previous analyses, the LFV in the τ sector can be large and observable. Interestingly, as shown in figure 6 the model introduces strong correlations between different LFV processes and so is distinguishable from SUSY [24].

However, the present bounds on LFV are already putting little Higgs models under considerable pressure. For little Higgs with T-parity, one needs to choose $f \geq 10\,\mathrm{TeV}$ or $\sin(2\theta) < 0.01$ where θ is the family mixing angle in the model. For the simplest little Higgs model one requires $f \geq 14\,\mathrm{TeV}$ or $\sin(2\theta) < 0.005$. Such large values for f reintroduce a significant little hierarchy problem that little Higgs models were designed to reduce.

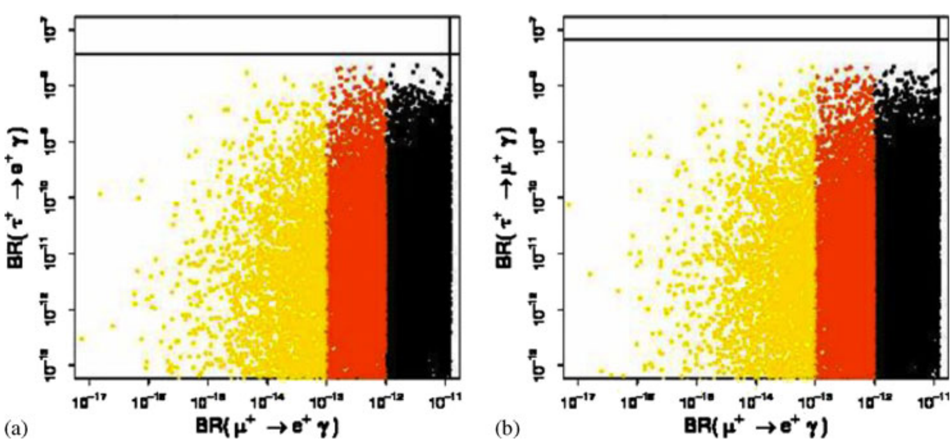

Figure 5. Correlations among branching ratios of $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ in the littlest Higgs model with T-parity. The horizontal and the vertical lines are experimental upper bounds. The colour of each dot represents the value of $\mathrm{BR}(\mu \to e\gamma)$. Black, red and yellow correspond to $10^{-12} < \mathrm{BR} < 1.2 \times 10^{-11}$, $10^{-13} < \mathrm{BR} < 10^{-12}$ and $\mathrm{BR} \leq 10^{-13}$ respectively (figure taken from [21]).

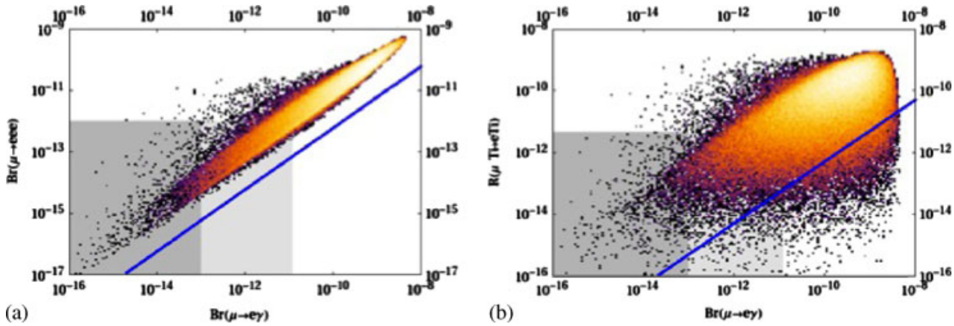

Figure 6. Correlation between $\mu \to e\gamma$ and $\mu^- \to e^-e^+e^-$ in the littlest Higgs model with T-parity as obtained from a general scan over the parameters. The shaded area represents the present (light) and future (darker) experimental constraints. The solid blue line represents the dipole contribution to BR($\mu^- \to e^-e^+e^-$) (figure taken from [25]).

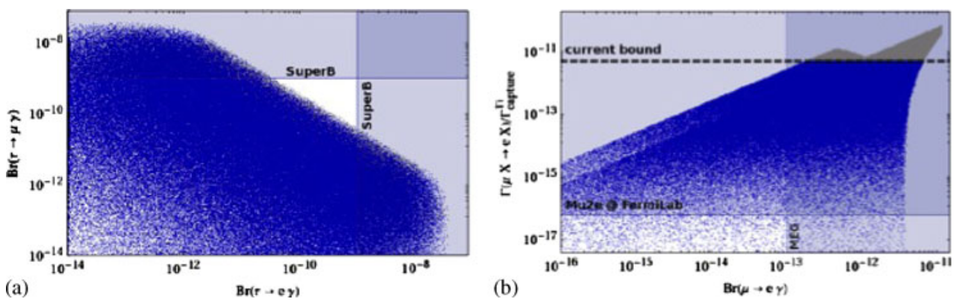

Figure 7. **(a)** Correlation between BR($\tau \to e\gamma$) and BR($\tau \to \mu\gamma$). **(b)** Correlation between BR($\mu \to e\gamma$) and R(μTi $\to \varepsilon$Ti) for a fourth lepton family. The shaded areas indicate the expected future experimental bounds.

3.3 *Fourth lepton family*

A fourth lepton family introduces new mixing angles U_{i4}, where U is a 4×4 mixing matrix, and these induce LFV processes via radiative corrections. One finds [25,26]

$$\frac{\mathrm{BR}(\tau \to \mu\gamma)}{\mathrm{BR}(\mu \to e\gamma)} = \left|\frac{U_{\tau 4}}{U_{e4}}\right|^2 \mathrm{BR}\left(\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu\right),$$
$$\frac{\mathrm{BR}(\tau \to \mu\gamma)}{\mathrm{BR}(\tau \to e\gamma)} = \left|\frac{U_{\mu 4}}{U_{e4}}\right|^2 \frac{\mathrm{BR}\left(\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu\right)}{\mathrm{BR}\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \simeq \left|\frac{U_{\mu 4}}{U_{e4}}\right|^2,$$
$$\frac{\mathrm{BR}(\tau \to e\gamma)}{\mathrm{BR}(\mu \to e\gamma)} = \left|\frac{U_{\tau 4}}{U_{\mu 4}}\right|^2 \mathrm{BR}\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right). \qquad (13)$$

Also the $\mu - e$ conversion rate is proportional to $\left|U_{e4}U_{\mu 4}\right|^2$. Unitarity gives strong mass independent limits on U_{i4} [27]. The resulting prediction for the magnitudes of BR($\tau \to e\gamma$) and BR($\tau \to \mu\gamma$) is shown in figure 7. As may be seen in this case it may be possible to observe LFV τ decays at the SuperB factory, although there are strong

Table 3. Comparison of various ratios of branching ratios in the LHT model [24], the MSSM without [28,29] and with significant Higgs contributions [30,31] and the SM4 [27].

Ratio	LHT	MSSM (dipole)	MSSM (Higgs)	SM4
$\frac{BR(\mu^-\to e^-e^+e^-)}{BR(\mu\to e\gamma)}$	$0.02\dots 1$	$\sim 6\cdot 10^{-3}$	$\sim 6\cdot 10^{-3}$	$0.06\dots 2.2$
$\frac{BR(\tau^-\to e^-e^+e^-)}{BR(\tau\to e\gamma)}$	$0.04\dots 0.4$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.07\dots 2.2$
$\frac{BR(\tau^-\to \mu^-\mu^+\mu^-)}{BR(\tau\to \mu\gamma)}$	$0.04\dots 0.4$	$\sim 2\cdot 10^{-3}$	$0.06\dots 0.1$	$0.06\dots 2.2$
$\frac{BR(\tau^-\to e^-\mu^+\mu^-)}{BR(\tau\to e\gamma)}$	$0.04\dots 0.3$	$\sim 2\cdot 10^{-3}$	$0.02\dots 0.04$	$0.03\dots 1.3$
$\frac{BR(\tau^-\to \mu^-e^+e^-)}{BR(\tau\to \mu\gamma)}$	$0.04\dots 0.3$	$\sim 1\cdot 10^{-2}$	$\sim 1\cdot 10^{-2}$	$0.04\dots 1.4$
$\frac{BR(\tau^-\to e^-e^+e^-)}{BR(\tau^-\to e^-\mu^+\mu^-)}$	$0.8\dots 2$	~ 5	$0.3\dots 0.5$	$1.5\dots 2.3$
$\frac{BR(\tau^-\to \mu^-\mu^+\mu^-)}{BR(\tau^-\to \mu^-e^+e^-)}$	$0.7\dots 1.6$	~ 0.2	$5\dots 10$	$1.4\dots 1.7$
$\frac{R(\mu Ti\to e Ti)}{BR(\mu\to e\gamma)}$	$10^{-3}\dots 10^{2}$	$\sim 5\cdot 10^{-3}$	$0.08\dots 0.15$	$10^{-12}\dots 26$

correlations between the decay modes and so only one of the decay modes will be visible. The prediction for the magnitudes of $BR(\mu\to e\gamma)$ and $R(\mu Ti\to \varepsilon Ti)$ also shows strong correlation and most of the available parameter space should be probed by future experiments.

3.4 *Distinguishing models*

As we have seen there are differences in the predictions of various models for the LFV decays. A convenient comparison of these predictions for a selection of models has been compiled in [25] and is shown in table 3.

4. Neutrino masses and mixing

The T2K experiment has recently announced a measurement that favours a non-zero value of the lepton mixing angle θ_{13} [32]. A recent analysis of all the present data [33] gives $\sin^2\theta_{13} = 0.021(0.025)\pm 0.007$ (1σ), the central value depending on reactor neutrino flux systematics. This corresponds to $\theta_{13} = (8(9)\pm 1.5)^\circ$. What are the implications if θ_{13} is close to the central value? There have already been more than 35 theoretical papers written on the subject but will it really change our ideas about the origin of fermion mass structure? To address this, let me start with the tri-bi-maximal mixing matrix, U_{TB}, that provides a good approximation to the observed mixing found in neutrino oscillations [34].

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} P, \tag{14}$$

where P is a diagonal phase matrix. This gives $\sin^2\theta_{23} = 0.5,\ \sin^2\theta_{12} = 0.333,\ \theta_{13} = 0$. Such a form of mixing can be obtained from a non-Abelian family symmetry (for a general review, see [35]). For example, from a discrete subgroup (A_4, $\Delta(27)\cdots\subset SU(3)$) of an $SU(3)$ family symmetry acting on the three generations. The question is whether a non-zero value for θ_{13} negates the family symmetry explanations of the observed near tri-bi-maximal mixing. To address this point I shall use a specific example based on $\Delta(27)\subset SU(3)_{\mathrm{L}}\equiv SU(3)_{\mathrm{E}}\equiv SU(3)_\nu$ to illustrate the possibilities [36]. In the $\Delta(27)$ model, U_{TB} is identified with the neutrino mixing matrix $V_\nu^\dagger$. The charged lepton sector also contributes to the PNMS matrix $U_{\mathrm{PMNS}} = V_\nu^\dagger U_l$ and its contribution to θ_{13} gives $\theta_{13}=\theta_{13}^l=(1/3\sqrt{2})\theta_C\simeq 3^\circ$ [37], still too small to explain the T2K central value.

However, the discrepancy is in an angle that is anomalously small and so one might expect it to be sensitive to small corrections. To quantify this we start with the effective Lagrangian responsible for the neutrino mass. It is driven by the see-saw mechanism and integrating out the heavy RH neutrinos with Majorana masses M_1, M_2, M_3 gives

$$L_{\mathrm{effective}}=\frac{1}{M_1}\psi_i\theta_{23}^i\psi_j\theta_{23}^j+\frac{1}{M_2}\psi_i\theta_{123}^i\psi_j\theta_{123}^j+\frac{1}{M_3}\psi_i\theta_3^i\psi_j\theta_3^j, \tag{15}$$

where i is the family index and θ_3, θ_{23}, θ_{123} are familon fields which acquire the vacuum expectation values (VEVs) given by

$$\frac{\phi_3}{M}=\begin{pmatrix}0\\0\\1\end{pmatrix},\qquad \frac{\phi_{23}}{M}=\begin{pmatrix}0\\1\\-1\end{pmatrix}\varepsilon,\qquad \frac{\phi_{123}}{M}=\begin{pmatrix}1\\1\\1\end{pmatrix}\varepsilon^2, \tag{16}$$

where $\varepsilon\approx 0.15$ is an expansion parameter and the structure of these VEVs is found minimizing the familon potential that is constrained by $\Delta(27)$ and the other symmetries of the model. The θ_3 familon is introduced to generate the large third-generation quark and charge lepton masses but is unwanted in the neutrino sector. However, due to the see-saw mechanism its contribution can be suppressed if $M_1\approx M_2\ll M_3$ and the symmetries incorporated in the model ensure that this is the case. Then the third term can be made negligible and, with $M_1=M_2=M$, the remaining terms give two massive neutrinos with mass $m_@$ and $m_\odot$

$$L_{\mathrm{effective}}\simeq m_@\psi_i\theta_{23}^i\psi_j\theta_{23}^j+m_\odot\psi_i\theta_{123}^i\psi_j\theta_{123}^j. \tag{17}$$

The mixing matrix following from this Lagrangian is U_{TB}. However, the structure of eq. (16) applies only in the leading order in the expansion parameter and one expects the VEVs to deviate slightly from this form. Due to its small value, such changes will affect θ_{13} more than the other angles. To illustrate this, consider the effect of an $O(\epsilon)$ correction to θ_{23} VEV in its small (zero) entry giving the form $(\phi_{23}/M)=(\varepsilon,1,-1)^T\,\varepsilon$. This gives

$$\sin^2\theta_{23}=0.509,\quad \sin^2\theta_{12}=0.295,\quad \theta_{13}=9^\circ\ \left(\theta_{13}^\nu=6^\circ,\theta_{13}^e=3^\circ\right) \tag{18}$$

which is close to the tri-bi-maximal values for θ_{23} and θ_{12} and is in good agreement with the experimental values

$$\sin^2\theta_{23}=0.42^{+0.08}_{-0.03},\quad \sin^2\theta_{12}=0.312^{+0.017}_{-0.016},\quad \theta_{13}=(8(9)\pm1.5)^\circ\,. \tag{19}$$

In this case the departure from tri-bi-maximal mixing does not invalidate the approach using an underlying non-Abelian family symmetry generating tri-bi-maximal mixing in leading order. For other suggestions giving near tri-bi-maximal mixing and large θ_{13} see [38]. Of course the generation of significant θ_{13} is not possible in all cases and the measurement of θ_{13} remains an important discriminant between models.

5. Summary

LFV is well established in the neutrino sector through neutrino oscillations. In the Standard Model LFV is tiny in the charged lepton sector but can be large, within future experimental reach, in models going beyond the Standard Model if they involve new physics at the TeV scale. The need to explain why the electroweak breaking scale is much less than the Grand Unified or Planck scales (the hierarchy problem) suggests that indeed there should be new physics at this scale giving encouragement for the prospect of success in LFV searches in the charged lepton sector.

Given our poor understanding of the origin of fermion masses and mixings, it is difficult to predict the family structure of charged LFV processes. A promising approach is to relate various LFV processes through their family symmetry properties making the plausible assumption that each independent representation is dominated by a single spurion. To go further requires that only a subset of spurions contribute. The most studied scheme, minimal lepton flavour violation, does make definite predictions although, if neutrino masses arise from the see-saw, it is necessary to make guesses about unknown parameters associated with the heavy lepton sector. Alternatively one can start with a 'top-down' model and many such schemes have been studied. These studies demonstrate that, if LFV is established in the charged lepton sector, correlations between LFV observables should be able to distinguish between models.

In the neutrino sector, the structure observed in neutrino masses and mixings may indicate an underlying (discrete) non-Abelian symmetry. The recent indications of a significant θ_{13} can help distinguish between models but can readily be made consistent with near tri-bi-maximal mixing.

References

[1] T Feldmann, PoSBEAUTY **2011** (2011) 017, arXiv:1105.2139 [hep-ph].

[2] Tau lepton physics, *Proceedings of the 11th International Workshop, TAU 2010* (Manchester, UK) G Lafferty and S Soldner-Rembold (eds), *Nucl. Phys. B, Proc. Suppl.* **218** (2011)

[3] Andreas Hoecker, Physics of Lepton Flavour, *Lepton Photon*, Mumbai, 2011

[4] William J Marciano, Toshinori Mori and J Michael Roney, *Ann. Rev. Nucl. Part. Sci.* **58**, 315 (2008)

[5] Vincenzo Cirigliano, Benjamin Grinstein, Gino Isidori and Mark B Wise, *Nucl. Phys.* **B728**, 121 (2005); *Nucl. Phys.* **B752**, 18 (2006)

[6] S Davidson and F Palorini, *Phys. Lett.* **B642**, 72 (2006), hep-ph/0607329

[7] G C Branco, A J Buras, S Jager, S Uhlig and A Weiler, *J. High Energy Phys.* **0709**, 004 (2007), hep-ph/0609067

[8] M B Gavela, T Hambye, D Hernandez and P Hernandez, *J. High Energy Phys.* **0909**, 038 (2009), arXiv:0906.1461 [hep-ph]

[9] F J Botella, G C Branco, M Nebot and M N Rebelo, arXiv:1102.0520 [hep-ph]

[10] R Alonso, G Isidori, L Merlo, L A Munoz and E Nardi, *J. High Energy Phys.* **1106**, 037 (2011), arXiv:1103.5461 [hep-ph]
[11] J C Romao, arXiv:1105.5608 [hep-ph]
[12] J A Casas and A Ibarra, *Nucl. Phys.* **B618**, 171 (2001), hep-ph/0103065
[13] L Calibbi, A Faccia, A Masiero and S K Vempati, *J. High Energy Phys.* **0707**, 012, (2007), hep-ph/0610241
[14] S Antusch, E Arganda, M J Herrero and A M Teixeira, *Nucl. Phys. Proc. Suppl.* **169**, 155 (2007),hep-ph/0610439]; *J. High Energy Phys.* **0611**, 090 (2006), hep-ph/0607263
[15] J A Casas, J M Moreno, N Rius, R Ruiz de Austri and B Zaldivar, *J. High Energy Phys.* **1103** (2011) 034, arXiv:1010.5751 [hep-ph]
[16] F Feruglio, C Hagedorn, Y Lin and L Merlo, *Nucl. Phys.* **B832**, 251 (2010), arXiv:0911.3874 [hep-ph]
[17] C Hagedorn, E Molinaro and S T Petcov, *J. High Energy Phys.* **1002**, 047 (2010), arXiv:0911.3605 [hep-ph]
[18] S Antusch, S F King, M Malinsky and G G Ross, *Phys. Lett.* **B670**, 383 (2009), arXiv:0807.5047 [hep-ph]
S Antusch, S F King and M Malinsky, *J. High Energy Phys.* **0806**, 068 (2008), arXiv:0708.1282 [hep-ph]
G G Ross and O Vives, *Phys. Rev.* **D67**, 095013 (2003), hep-ph/0211279
[19] M Schmaltz and D Tucker-Smith, *Ann. Rev. Nucl. Part. Sci.* **55**, 229 (2005), hep-ph/0502182
[20] M Perelstein, *Prog. Part. Nucl. Phys.* **58**, 247 (2007), hep-ph/0512128
[21] J Wudka, hep-ph/0307339
H-C Cheng and I Low, *J. High Energy Phys.* **0309**, 051 (2003), hep-ph/0308199
[22] T Goto, Y Okada and Y Yamamoto, *Phys. Rev.* **D83**, 053011 (2011), arXiv:1012.4385 [hep-ph]
[23] F del Aguila, J I Illana and M D Jenkins, *J. High Energy Phys.* **1103**, 080 (2011), arXiv:1101.2936 [hep-ph]; *J. High Energy Phys.* **1009**, 040 (2010), arXiv:1006.5914 [hep-ph]; *J. High Energy Phys.* **0901**, 080 (2009), arXiv:0811.2891 [hep-ph]
[24] M Blanke, A J Buras, B Duling, S Recksiegel and C Tarantino, *Acta Phys. Polon.* **B41**, 657 (2010), arXiv:0906.5454 [hep-ph]
[25] A J Buras, B Duling, T Feldmann, T Heidsieck and C Promberger, *J. High Energy Phys.* **1009**, 104 (2010), arXiv:1006.5356 [hep-ph]
[26] W-j Huo and T-F Feng, hep-ph/0301153
[27] H Lacker and A Menzel, *J. High Energy Phys.* **1007**, 006 (2010), arXiv:1003.4532 [hep-ph]
[28] J R Ellis, J Hisano, M Raidal and Y Shimizu, *Phys. Rev.* **D66**, 115013 (2002), hep-ph/0206110
[29] A Brignole and A Rossi, *Nucl. Phys.* **B701**, 3 (2004), hep-ph/0404211
[30] G Isidori and P Paradisi, *Phys. Rev.* **D73**, 055017 (2006), hep-ph/0601094
[31] P Paradisi, *J. High Energy Phys.* **0602**, 050 (2006), hep-ph/0508054
[32] T2K Collaboration: K Abe *et al*, *Phys. Rev. Lett.* **107**, 041801 (2011), arXiv:1106.2822 [hep-ex]
[33] G L Fogli, E Lisi, A Marrone, A Palazzo and A M Rotunno, *Phys. Rev.* **D84**, 053007 (2011), arXiv:1106.6028 [hep-ph]
[34] P F Harrison, D H Perkins and W G Scott, *Phys. Lett.* **B530**, 167 (2002), hep-ph/0202074; *Phys. Lett.* **B535**, 163 (2002), hep-ph/0203209; *Phys. Lett.* **B557**, 76 (2003), hep-ph/0302025
Z-z Xing, H Zhang and S Zhou, *Phys. Lett.* **B641**, 189 (2006), hep-ph/0607091
[35] G Altarelli and F Feruglio, *Rev. Mod. Phys.* **82**, 2701 (2010), arXiv:1002.0211 [hep-ph]
[36] I de Medeiros Varzielas, S F King and G G Ross, *Phys. Lett.* **B648**, 201 (2007), hep-ph/0607045; *Phys. Lett.* **B644**, 153 (2007), hep-ph/0512313; *Nucl. Phys.* **B733**, 31 (2006), hep-ph/0507176

[37] S Antusch and S F King, *Phys. Lett.* **B631**, 42 (2005), hep-ph/0508044
[38] S F King, *Phys. Lett.* **B675**, 347 (2009), arXiv:0903.3199 [hep-ph]
R d A Toorop, F Feruglio and C Hagedorn, *Phys. Lett.* **B703**, 447 (2011), arXiv:1107.3486 [hep-ph]
Y Shimizu, M Tanimoto and A Watanabe, *Prog. Theor. Phys.* **126**, 81 (2011), arXiv:1105.2929 [hep-ph]
T Araki, *Phys. Rev.* **D84**, 037301 (2011), arXiv:1106.5211 [hep-ph]
E Ma and D Wegman, *Phys. Rev. Lett.* **107**, 061803 (2011), arXiv:1106.4269 [hep-ph]
Z-z Xing, *Phys. Lett.* **B696**, 232 (2011), arXiv:1011.2954 [hep-ph]
Y Lin, *Nucl. Phys.* **B824**, 95 (2010), arXiv:0905.3534 [hep-ph]