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Gauge boson production at colliders – Predictions for precision studies

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Abstract. The status of today's theoretical description of Standard Model (SM) processes involving gauge bosons at the Tevatron and the LHC, and of the tools that are used in their phenomenological studies are reviewed. A few recent ideas to further improve on the way technically challenging calculations can be performed also are discussed.

Keywords. Next-to-leading order; quantum chromodynamics; gauge bosons.

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1. Introduction

The physics programme involving gauge bosons is very rich. The road towards precision measurements and searches start from the measurement of inclusive W and Z cross-sections, which, from a theoretical point of view, are the most precisely known processes at hadron colliders. Beyond the measurements of purely inclusive W/Z production cross-sections, it is possible to study the production of W or Z bosons in association with one or more jets, and ratio of cross-sections. Interesting ratios are, for instance, $\sigma(V+(n+1)\,\text{jets})/\sigma(V+n\,\text{jets})$, with $V = W, Z$, that start at $\mathcal{O}(\alpha_s)$ in the perturbative expansion in the coupling constant, or ratios of $\sigma(W^{\pm}+n\,\text{jets})/\sigma(Z+n\,\text{jets})$, that are of order $\mathcal{O}(\alpha_s^0)$. In both cases, one expects many experimental and theoretical uncertainties (e.g. those related to the choice of renormalization or factorization scale, or uncertainties in the parton distribution functions) to cancel to a large extent. A further extension of simple ratios are asymmetry distributions, for instance, the W or lepton charge asymmetry. These distributions provide strong constraints on parton distribution functions and are useful probes of new physics. Other interesting observables measure gauge bosons produced in association with heavy quarks (charm or bottom quarks). These processes are particularly interesting because of the discrepancies between theoretical predictions and Tevatron data [1]. However, the perturbative calculation of these processes and of the related theoretical uncertainty is challenging. Finally, diboson cross-sections are sensitive to any type of new physics that would modify the trilinear gauge couplings, and would give rise to the so-called anomalous gauge couplings. These measurements are complementary to the ongoing direct searches for beyond the SM physics, and are able to probe new-physics scales that are not directly accessible.

It is instructive to first recall a recent measurement that was the cause of considerable excitement. In April 2011, CDF reported the observation of a peak in the m_{jj} distribution in $W +$ dijet events [2] (see figure 1). The first measurement had a 3.2σ significance, and was based on 4.3 fb^{-1}. Subsequently, more data (7.3 fb^{-1}) have been analysed, leading to a significance of more than 4σ [3]. Since then, a large number of tentative new-physics explanations appeared on the arXiv, along with a few SM analyses that addressed the question of whether this effect can be attributed to a mismodelling of one of the SM backgrounds (in particular single top) [4–6]. The excitement was curbed before this conference, when D0 announced that it did not confirm the excess seen by CDF [7]. It is yet unclear what the reasons for the discrepancy between CDF and D0 findings are, if any. However, this example demonstrates that even in the case where one identifies a mass peak in the tail of a distribution (a scenario that was considered 'an easy discovery') a robust control of SM backgrounds remains mandatory, in particular when the shape of the backgrounds is one of the issues. It is also interesting to note that it is in a process involving a gauge boson that this excess has been claimed. Indeed it is in processes that involve gauge bosons that one might look for smoking gun signatures at the LHC, both, because of the relatively clear experimental signature, compared to processes involving only jets, and because of the enhanced sensitivity to new physics.

The important question becomes then what the tools at our disposal are to make precise predictions of these and related processes, and whether we have solid control of signals and backgrounds that is needed in order to claim discoveries. In this write-up I shall discuss the status of our theoretical knowledge of gauge boson production processes, the tools at our disposal to describe these processes, and the impact of QCD higher orders in

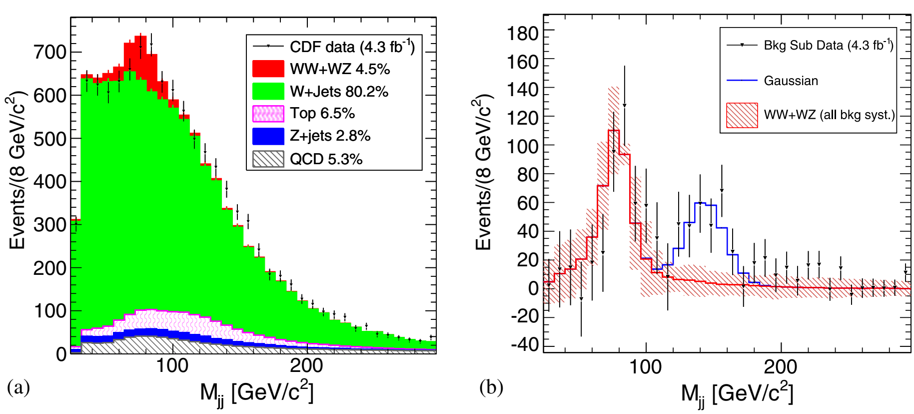

Figure 1. The dijet invariant mass distribution for $W +$ dijet events as measured by CDF: (**a**) shows the fits for known processes only and (**b**) shows, by subtraction, the resonant contribution to m_{jj} including WW and WZ production and a hypothesized narrow Gaussian contribution (figures taken from [2]).

view of recent Tevatron/LHC results. Finally, I shall discuss a few recent ideas to further improve the way we perform technically challenging calculations.

2. Gauge boson cross-sections

Figure 2 gives the cross-section for main processes involving gauge bosons at the LHC. The figure illustrates that with 1 fb^{-1} (the luminosity collected and partially analysed by the time of this conference) ATLAS and CMS could collect $\mathcal{O}(10^6)$ and $\mathcal{O}(10^5)$ W and Z events per experiment and per lepton channel. One also sees that, including all lepton channels, 1 fb^{-1} of data contain about 100 WW and 10 ZZ events. This means that even with the data available at the time of this conference, a number of interesting analyses can be performed.

2.1 *Drell–Yan*

The most important gauge boson production process is Drell–Yan. This is the best-known process at the LHC: it has been computed through next-to-next-to-leading order (NNLO) in QCD, fully differential in lepton momenta including spin-correlations, electroweak (EW) corrections, finite-width effects, and γ^*/Z interference. State-of the art codes are described in [9,10]. Calculations to all-orders also exist, for instance, the next-to-next-to-leading logarithms (NNLL), transverse momentum resummation [11] and soft gluon resummation have been computed [12]. These accurate perturbative calculations have been available for some time, and now that precise LHC data have been compared to those predictions, one cannot but praise the impressive agreement between NNLO theory and experiment (see e.g. figure 3 and ref. [13]). One thing to note is that in these comparisons of theory with experiment, the dominant error is the theoretical one. However, this is mainly dominated by the luminosity uncertainty (of the order of 4%).

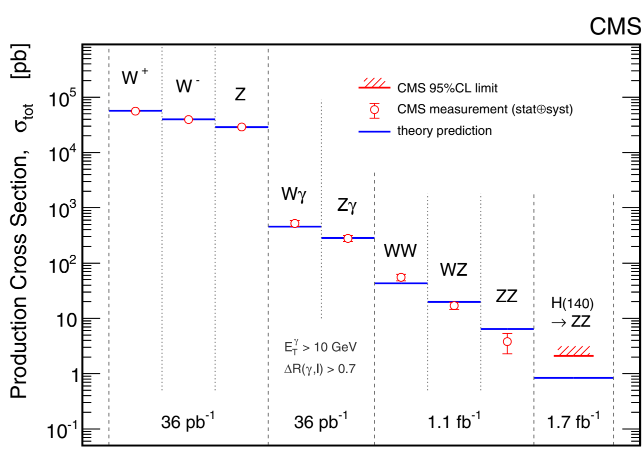

Figure 2. Production cross-sections for processes involving gauge bosons at the LHC at $\sqrt{s} = 7$ TeV (figure taken from [8]).

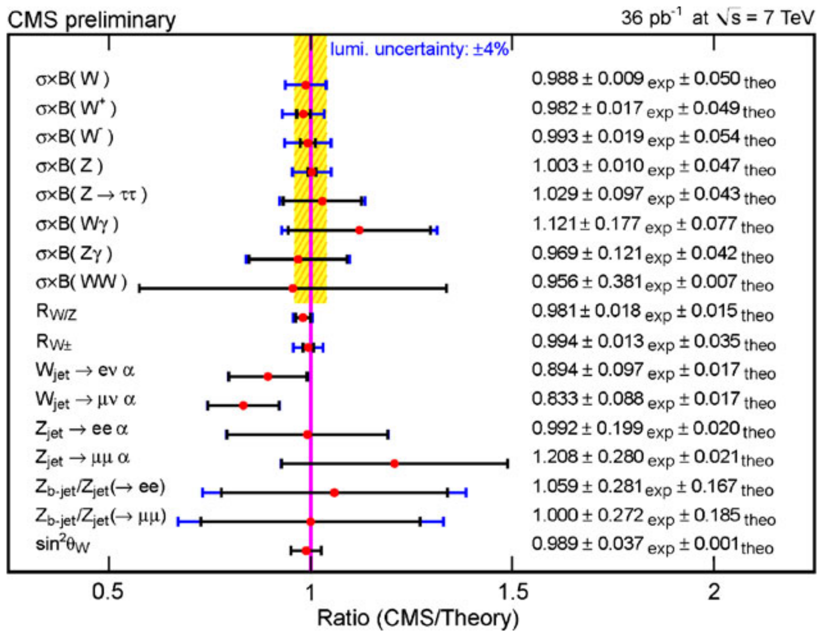

Figure 3. Comparison of NNLO theory and CMS data for Drell–Yan observables (figure taken from ref. [14]).

At this level of precision, it is legitimate to start worrying about mixed QCD and EW corrections. QCD+EW interference is formally a higher-order effect, and exact results are not known currently. However, the dominant effects come from emissions in the soft/collinear regions where an approximate factorization holds. It turns out that, in general, mixed corrections are small, $\mathcal{O}(0.5)\%$, but they are enhanced in the region of large transverse momenta, where they can reach up to around 5% [15].

2.2 *Charge asymmetry*

The natural extension of the inclusive cross-section is the $R_W = W^+/W^-$ ratio. One can then study R_W as a function of kinematical variables, e.g. one can look at the charge asymmetry as a function of lepton rapidity η

$$A(\eta) = \frac{R_W(\eta) - 1}{R_W(\eta) + 1}\,. \tag{1}$$

This measurement is very sensitive to parton distribution functions (PDFs) since in the ratio asymmetric properties of PDFs are enhanced, while many uncertainties cancel. Figure 4 shows the relatively good agreement of theoretical predictions that use various PDFs with ATLAS data. It also illustrates how the shape of the theoretical prediction is sensitive to the PDFs chosen. Indeed, ATLAS and CMS measurements of this distribution have been already used by the Neural Network (NN) Collaboration to constrain parton distributions. In particular, a reduction of uncertainty of the order of (10–30)% in

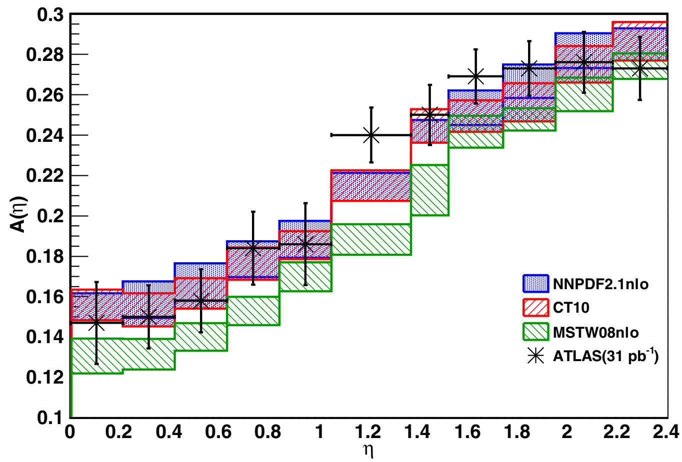

Figure 4. Predictions for the W lepton asymmetry at NLO, obtained with DYNNLO [9] using the CT10 [16], MSTW08 [17] and NNPDF2.1 [18] parton sets, compared to measurements for the muon charge asymmetry from ATLAS [19] (7 TeV) (figure taken from ref. [20]).

the range $x = [10^3, 10^1]$ was obtained for the valence- and sea-quark distributions. It is interesting to observe that LHCb data at larger rapidities probe larger and smaller values of x that have fewer experimental constraints. They will therefore soon have a larger impact in PDF determination than ATLAS and CMS have.

2.3 *Production of W/Z boson in association with jets*

At the LHC, because of the large energy, the production of W and Z bosons in association with jets is possible. This is illustrated in figure 5 which shows the differential distribution for H_{T}, the total transverse energy of the event, for various jet multiplicities. Since the cross-sections with an additional jet is rescaled by a factor 10^{-1} compared to the cross-section with one less jet, it is evident that at high H_{T} (a region of particular interest for various new-physics searches), all jet multiplicities contribute similar amounts [21a]. Because of this, it becomes very important to have a good perturbative control of processes involving the production of W/Z bosons together with many jets. The perturbative calculation of processes involving a large number of jets is quite difficult beyond leading order (LO). However, recent years have seen a revolution in the techniques used for next-to-leading order (NLO) calculations (see [22] and also [23] for a pedagogical review on unitarity methods).

In the last five years, these novel techniques allowed the calculation of a large number of processes involving gauge bosons and jets. In particular, while $V + 1$ and $V + 2$ jets have been described to NLO in QCD in 1983 [24,25] and 2002 [26–28], in recent years the NLO corrections to a variety of other processes involving gauge boson have been computed. To quote some examples, we know now at NLO $VV + 1$ jet [29–32], $W + 3$ jets [33,34], $Z + 3$ jets [35], $W^+W^+ +$ dijets [36] , $W^+W^- +$ dijets [37], W^+W^-bb [38,

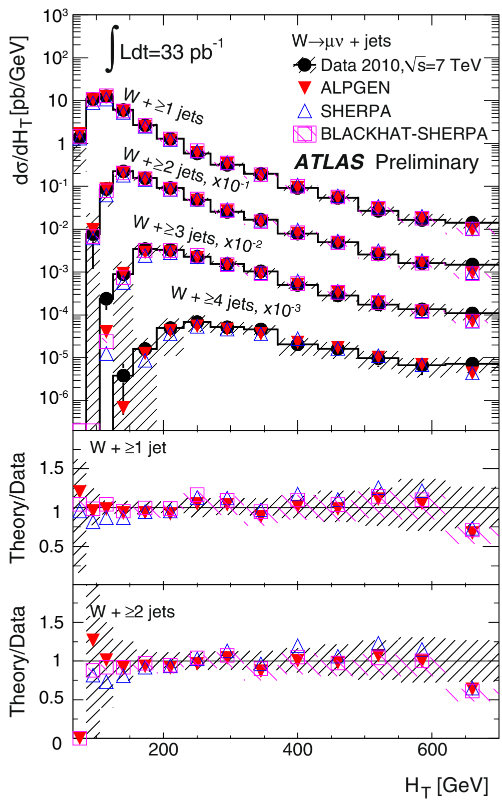

Figure 5. Cross-section for $W \to e\nu$ and jets at the LHC (7 TeV) as a function of the total transverse momentum of the event H_{T} (figure taken from [21]).

39], $W+4$ jets [40] and $Z+4$ jets [41]. Furthermore, various vector boson fusion (VBF) induced gauge boson production processes have been computed and are available in the public code VBFNLO [42].

Figure 6 shows the transverse momentum distribution for the four p_{t} ordered jets in $Z+4$ jets production. The middle pane illustrates the typical reduction of the dependence of the cross-section on renormalization and factorization scale at NLO, compared to LO. It is however also evident that while for some distributions (e.g. $p_{\mathrm{t},j4}$) the ratio of LO/NLO is flat, in other cases the shape of the NLO distribution is different from the LO one, so that LO/NLO has a non-flat slope. It is also interesting to look at the ratio of Z/W^+ and Z/W^- displayed in the lower pane. The agreement of LO and NLO predictions for these ratios illustrates the excellent perturbative control that one can achieve at NLO on these ratios. Furthermore, the fact that Z/W^+ is flat, while Z/W^- rises with p_{t} can be attributed to the fact that both W^+ and Z production are dominated by the u-quark distribution, while W^- is mainly produced from d quarks in the protons, and therefore the Z/W^- ratio displays the slope of the u/d-quark distributions as a function of the jet transverse momenta.

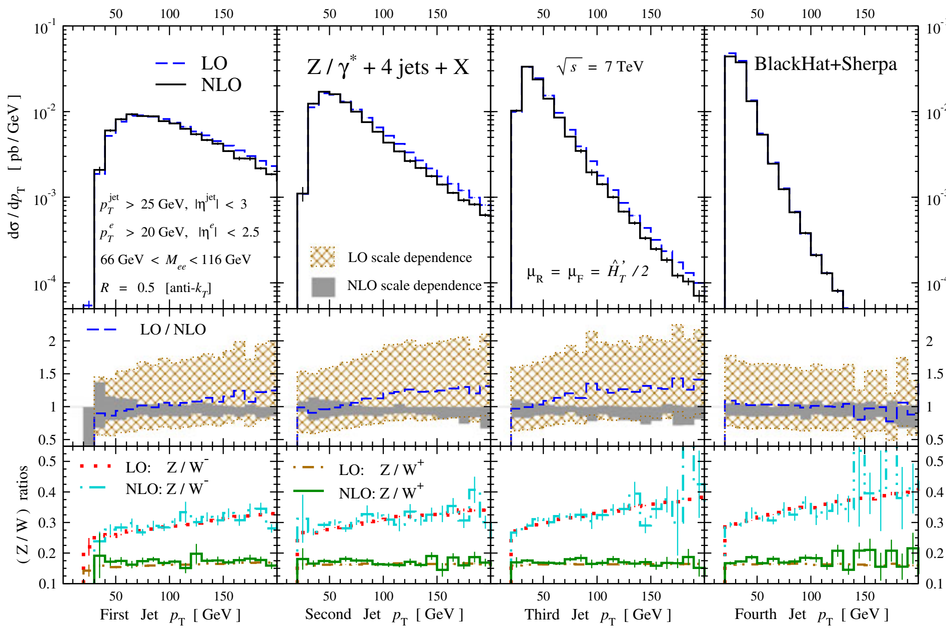

Figure 6. Transverse momentum distributions of the leading four jets in $Z + 4$ jets production at the LHC (7 TeV) (figure taken from [41]).

2.4 *Merging NLO and parton showers*

While NLO predictions provide relatively accurate results for inclusive cross-sections, they do not furnish an exclusive description of the final state that can be compared with actual particles in the detectors, as Monte Carlo (MC) programmes do. It is therefore useful to combine the best features of both approaches. Two public frameworks exist for this purpose, namely `MC@NLO` [43] and `POWHEG` [44]. These tools are almost 10 years old by now, and since their conception a long list of processes has been implemented in both frameworks.

In particular, recently the `POWHEG BOX` was released [45], which is a general framework for implementing NLO calculations in shower MC programmes according to the `POWHEG` method. The user only needs to provide a simple set of routines (Born, colour-correlated Born, virtual, real and phase space) that are parts of any NLO calculation.

A simple set of processes that have been implemented recently in the `POWHEG-BOX` are ZZ, WW, WZ production. The calculation includes γ^*/Z interference, single resonant contributions, interference effects for identical fermions and, for WW and WZ the effect of anomalous couplings. A pure NLO calculation, as implemented earlier in MCFM [46,47], reveals that for these processes the conventional scale variation of the LO result is very modest but underestimates completely the size of the NLO corrections. E.g. for $pp \to W^+W^- \to e^+\nu_e\mu^-\nu_\mu$ production at the 7 TeV LHC without any cuts, the LO cross-section using NNPDf2.1 [18] is $375.2^{+1.6}_{-3.8}$ fb, while the NLO cross-section is 499.8^{+12}_{-10} fb [48]. Here the error denotes the scale uncertainty. Similar results (with smaller cross-sections) are obtained for other processes. The reason for the large NLO corrections (not caught by the LO scale variation) is that new partonic channels open up at NLO. It is therefore clear that only a NLO calculation can provide a reliable estimate of the cross-section and of its error. These diboson production processes are particularly interesting since they are important backgrounds to Higgs searches. Furthermore, they are sensitive to new physics at high scales through the measurement of anomalous trilinear couplings (ATGCs). Indeed, while the LHC will probe new physics at the TeV scale directly, ATGCs indirectly probe physics in the multi-TeV range, since they arise when high-energy degrees of freedom are integrated out. Both the Tevatron [49–51] and LEP [52] were able to place quite stringent bounds on ATGCs. However, since their effects are enhanced at high energies, one expects even better bounds from the LHC. Indeed, CMS already presented bounds on the anomalous couplings appearing in an effective Lagrangian with the parametrization of ref. [53] without form factors [54].

Following refs [53,55,56], one can parametrize the most general terms for the WWV vertex ($V = \gamma, Z$) in a Lagrangian that conserves C and P as

$$\mathcal{L}_{\rm eff} = i g_{\rm WWV} \Bigg(g_1^{\rm V} (W^*_{\mu\nu} W^{\mu} V^{\nu} - W_{\mu\nu} W^{*\mu} V^{\nu}) + \kappa^{\rm V} W^*_{\mu} W_{\nu} V^{\mu\nu} + \frac{\lambda^{\rm V}}{M_W^2} W^*_{\mu\nu} W^{\nu}_{\rho} V^{\rho\mu} \Bigg), \tag{2}$$

where $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $g_{\rm WWZ} = -e \cot\theta_{\rm W}$ and $g_{\rm WW\gamma} = -e$. In the SM, $g_1^{\rm V} = \kappa_1^{\rm V} = 1$ and $\lambda^{\rm V} = 0$. Any departure from these values ($\Delta g_1^{\rm V} = g_1^{\rm V} - 1$ etc.) would be a sign of new physics. In the `POWHEG` generator, all six parameters can be set independently.

In the presence of anomalous couplings, the effective Lagrangian of eq. (2) gives rise to interactions that violate unitarity at high energy. Thus, in order to achieve a more realistic parametrization, the couplings are multiplied by form factors, that embody the effects arising from integrating out the new degrees of freedom. The precise details of the form factors therefore depend on the particular model considered. Paralleling the discussion of ref. [57], in the `POWHEG BOX` it is assumed that all anomalous couplings Δg are modified as

$$\Delta g \to \frac{\Delta g}{(1 + M_{\rm VV}^2/\Lambda^2)^2}, \tag{3}$$

where $M_{\rm VV}$ is the invariant mass of the vector boson pair and Λ is the scale of new physics.

Figure 7 sets the anomalous coupling to the maximum deviation from the SM allowed by LEP bounds and displays the sensitivity of the transverse momentum of the leading jet in W^+W^- events to anomalous couplings, for a form factor $\Lambda = 2$ TeV, $\Lambda = 5$ TeV, and $\Lambda = \infty$. The plot illustrates the great potential of the LHC to improve on existing bounds (even more so, when the machine will run at yet higher energy).

For the production of four charged leptons, a similar NLO study (again including all off-shell, spin-correlation, virtual-photon-exchange, and interference effects) can be performed with the `aMC@NLO` generator [58]. `aMC@NLO` is a novel approach to a complete event generation at NLO. The parton showering can be done either with `HERWIG` or (for the first time in this context) with `Pythia 6`. The $O(\alpha_s^2)$ contribution of the gg channel is also included directly here, as obtained from MadLoop [59]. In ref. [58] several key

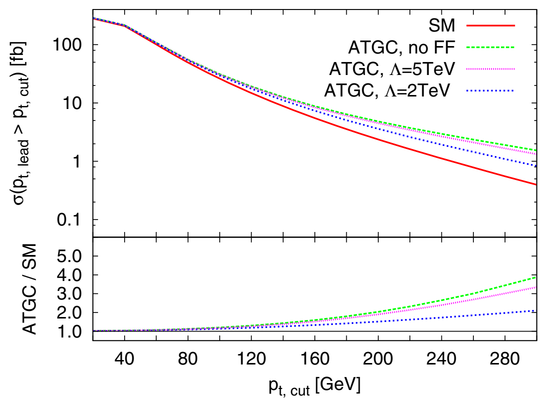

Figure 7. The integrated cross-section for W^+W^- production at the LHC (7 TeV) as a function of the cut on the transverse momentum of the leading lepton (figure taken from [48]).

distributions together with the corresponding theoretical uncertainties are presented. A further theoretical improvement is that the scale and PDF uncertainties are computed at essentially no extra CPU-time using re-weighting techniques.

A lot of effort has been devoted recently also to the implementation of higher multiplicity processes in the `POWHEG BOX` or in `aMC@NLO`. The first $2 \to 4$ process that has been implemented in the `POWHEG BOX` is $pp \to W^+W^+ + 2$ jets including both the QCD-induced part [60] as well as the VBF contributions [61]. This is a relatively simple $2 \to 4$ process since the cross-section is finite without any cut on the jets. As expected, for inclusive observables there are only minor differences between pure NLO and `POWHEG`+parton shower (PS), but for exclusive observables, depending on the details of the observable definition, there can be important differences. This is shown in figure 8 for two different definitions of $H_{\mathrm{T,TOT}} = \sum_j p_{\mathrm{t},j}$, the transverse energy of the event. From the figure it is clear that if only the three hardest jets are included in the definition of H_{T}, the corrections from the PS are very moderate (figure 8b), but if all soft jets present in the event are included, then additional radiation from the PS can alter the distribution substantially (figure 8a).

`aMC@NLO` has been used recently for the calculation $W/Zb\bar{b}$ [62] and W+dijet production [63]. Figure 9 shows an application to Higgs searches of the $W/Zb\bar{b}$ calculation: the invariant mass of the pair of the two leading b-jets, for the processes Wbb, Zbb, WH, and ZH. The figure illustrates a case where signals and irreducible backgrounds are computed with the same accuracy.

Figure 10 illustrates predictions from `aMC@NLO` for the invariant mass for the dijet system in Wjj (the observable mentioned in the introduction where CDF observed a large discrepancy from the SM). CDF and D0 estimate Wjj using a leading-order Monte Carlo (LO+PS) re-weighted to the NLO cross-section or to data. With `aMC@NLO` instead it is possible to directly compute the Wjj cross-section at the NLO+PS level. It was therefore particularly interesting to check whether there is any shape difference between LO+PS and NLO+PS in the M_{jj} distribution. The study of ref. [63] shows that there is no sizeable shape difference.

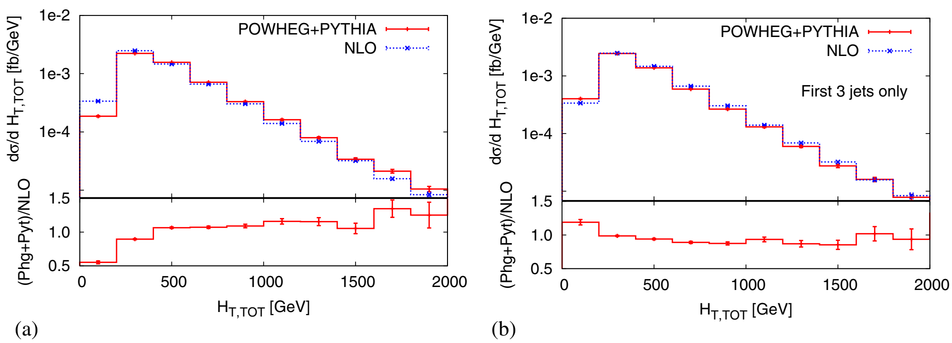

Figure 8. Comparison of NLO and POWHEG+PYTHIA results for the $H_{T,TOT}$ distribution in the process W^+W^++ 2 jets at the LHC (7 TeV), when all jets are included in the definition of $H_{T,TOT}$ (**a**), and when only the three hardest jets are included (**b**) (figure taken from ref. [60]).

2.5 MENLOPS *and* LoopSim

MENLOPS [64,65] is a method to further improve the NLO+PS predictions with matrix elements involving more partons in the final state. For example, for W production it includes, as in MC@NLO or POWHEG, W production at NLO, the PS, but also W + $1, 2, 3, \ldots$ jets using exact matrix elements. Roughly speaking, it uses a jet algorithm to define two different regimes, and then corrects the 1-jet fraction using exact matrix elements and the 2-jet fraction using the NLO K-factor. This achieves NLO quality accuracy for inclusive quantities but an improved sensitivity to hard radiation and multiparton kinematic features.

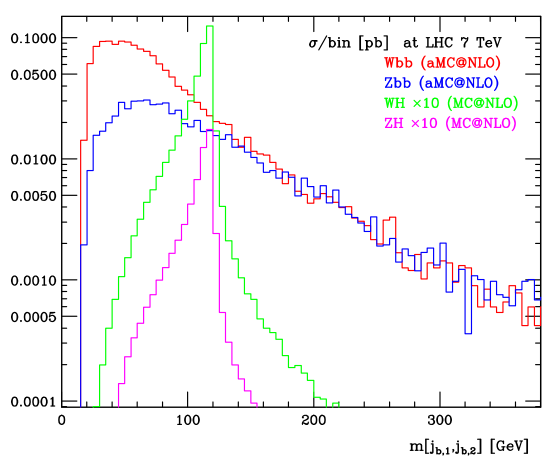

Figure 9. Invariant mass of the pair of the two leading b-jets for $Wb\bar{b}$, $Zb\bar{b}$, $WH(\to \ell\nu b\bar{b})$, $ZH(\to \ell^+\ell^- b\bar{b})$ at the LHC (7 TeV), the latter two are rescaled by a factor of ten (figure taken from ref. [62]).

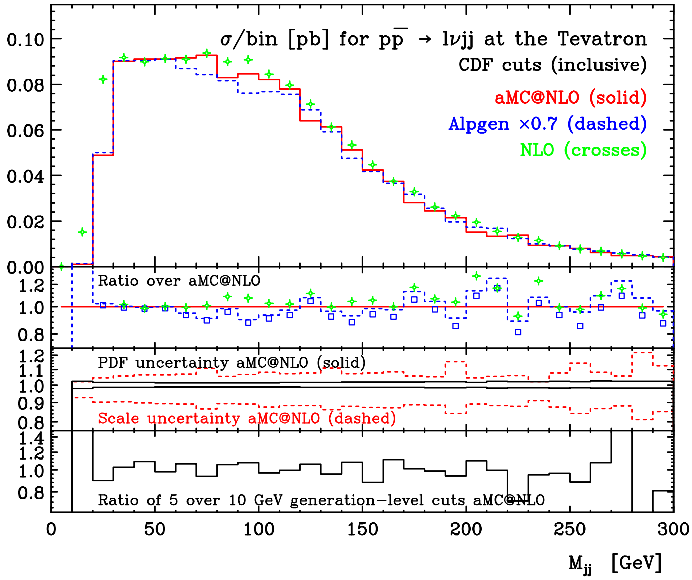

Figure 10. Invariant mass of the pair of two hardest jets, with CDF/D0 inclusive cuts (figure taken from ref. [63]).

Another recent theoretical development is `LoopSim`. If one considers the process $W+1$ jet, the three observables $p_{\mathrm{t,Z}}$, $p_{\mathrm{t},j}$, and $H_{\mathrm{T,jets}} = \sum_j p_{\mathrm{t,j}}$ are identical at LO. However, as illustrated in figure 11, at NLO $p_{\mathrm{t,Z}}$ has a moderate K-factor ($\lesssim 2$), $p_{\mathrm{t},j}$ has a large K-factor (~ 5) and $H_{\mathrm{T,jets}}$ has a giant K-factor (~ 50). The very large K-factors in the last two observables are due to the fact that the NLO result is dominated by configurations

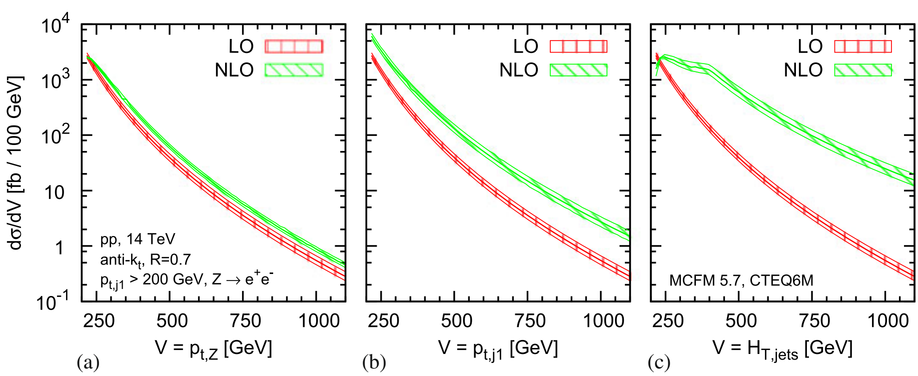

Figure 11. The LO and NLO distributions for three observables in Z+jet production that are identical at LO: (**a**) the Z transverse momentum, (**b**) the p_{t} of the hardest jet, and (**c**) the scalar sum of the transverse momenta of all the jets, $H_{\mathrm{T,jets}}$. The bands correspond to the uncertainty from a simultaneous variation of $\mu_{\mathrm{R}} = \mu_{\mathrm{F}}$ by a factor of two on either side of a default $\mu = \sqrt{p_{\mathrm{t},j1}^2 + m_{\mathrm{Z}}^2}$ (figure taken from ref. [66]).

where there are two hard jets and a soft W (these are enhanced by EW logarithms). Additionally, there is an important enhancement from incoming qq channels. `LoopSim` [66] is a procedure that uses a sequential algorithm, close to the Cambridge/Aachen one, to determine the branching history, 'loops' over soft particles (i.e. they are removed from the event and the residual event is adjusted), and it uses a unitary operator to cancel divergences. In essence, this is a way to extend a calculation that is exact at a given order in perturbation theory, in an approximate way to higher orders. The procedure is expected to be more accurate if the corresponding K-factor is larger. One might expect other extensions of the MLM/CKKW matching procedure along the same lines as `MENLOPS` and `LoopSim` in the near future.

3. Conclusions

The physics programme involving gauge bosons is very rich: it spans from most precise measurements (e.g. for Drell–Yan) to searches with the highest reach for new physics (through the potential presence of anomalous couplings). This experimental programme at the LHC is supplemented by robust theoretical predictions that include NLO QCD corrections, mixed NLO-QCD+EW corrections, NNLO and resummation of logarithmically enhanced contributions. Furthermore, different calculations are merged using clever matching procedures that catch the best features of different calculations. From the theoretical community, there is a clear and successful effort to produce predictions and public codes that have the flexibility required for today's sophisticated experimental analysis (including parton shower, decays with spin correlations, massive quark effect etc.).

Impressive results have already come out of the LHC, but this is certainly only the tip of the iceberg. After just one year of running at the LHC, W/Z physics starts being dominated by theoretical and parton density errors. It will therefore be a real challenge for theorists to keep up with the high experimental precision.

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