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Sensitivity in the trajectory of long-range α-particle

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Abstract. The factors influencing the trajectory of long-range α-particle in the cold ternary fission of ^{252}Cf are discussed. The trajectory of the α-particle is studied by considering the influence of the force on the α-particle due to Coulomb and proximity potentials and is found to have sensitive dependence on the initial position and initial energy of the α-particle. The sensitivity to initial conditions signifies the presence of deterministic chaos which is characterized by Lyapunov exponent (LE). The LE is calculated using Wolf's algorithm and found positive which implies that the objectives of trajectory calculations are restricted.

Keywords. Cold ternary fission; Lyapunov exponent.

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1. Introduction

The trajectory calculations in fission are used mainly (1) to predict the future of the emitted particle when initial scission configurations are known and (2) to understand the scission configuration of nucleus at scission, by solving the equation of motion backward in time with the experimentally measured asymptotic angle and final energy of the emitted α-particle at infinity. Determination of scission point configuration (SPC) is very important in the study of fission which is a controversial problem in the theory of fission dynamics [1]. The formation and emission of α-particle in ternary fission is closely connected with the dynamics from saddle to scission and the SPC is characterized by the relative separation, shapes, excitation energies of fragments etc. Determination of SPC and hence the trajectory calculation in cold ternary fission is easier than in normal fission. The fact that all the fragments emitted are in the ground state and are of compact shapes at scission makes the computation of scission configuration easy in cold ternary fission [2].

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In the past, many trajectory calculations were done for the α-emissions in hot (normal) ternary fission under various assumptions. Boneh *et al* [3] used the point charge approximations for three fragments without considering nuclear interactions. They have found that the scission point moves closer to the light fragments as the mass ratio increases. Using the initial conditions that follow from statistical theory [4], the trajectory calculations have been done for the α-emissions from ^{235}U and compared the energy of the α-particles with experimental values.

The large yields of odd–odd fragments compared to even–even fragments observed in cold fission [5] imply that superfluidity is not conserved in cold fission. This brings out the possibility of intrinsic excitation and dissipation. The fact that dissipation is possible in cold fission implies the contribution of nuclear friction. The role of nuclear friction in the dynamics of low-energy fission when they move against each other is now well established [6]. The range of nuclear friction is strongly correlated with the range of nuclear force. Nuclear friction is expected to arise from either one-body or two-body process. The two-body nuclear friction which depends on the relative velocity of fragments is included in the present calculation.

In the present work we intend to determine the trajectory of α-particle in the cold ternary fission of ^{252}Cf into different neutron-less fragmentations and the factors that sensitively affects the trajectory. We have considered the finite size of the fragments, since the same will affect the outcome of the trajectory. The trajectory of the α-particle is calculated by considering the influence of the force due to Coulomb potential and nuclear proximity potential. The nuclear friction experienced by the α-particle, which depends on the relative velocity of fragments, is also included in the present calculation.

The details of the methods adopted in the present calculation are described in §2, the results and discussions are presented in §3 and conclusions are given in §4.

2. Method

The ternary fission or the fission accompanied by light charged particle (LCP) is energetically possible only if the Q value of the reaction is positive. That is

$$Q = M - \sum_{i=1}^{3} m_i > 0. \tag{1}$$

Here M and m_i are the mass excess of the parent nuclei and the fragments respectively. The potential energy barrier for cold ternary fission consists of Coulomb potential and nuclear proximity potential [7,8]. Here the trajectory of the α-particle is computed by taking the total force acting on the α-particle as

$$F = F_{\text{Coul}} + F_{\text{nucl}} + F_{\text{fri}}. \tag{2}$$

Here F_{Coul}, F_{nucl} and F_{fri} represent the Coulomb force, nuclear force and frictional force respectively with

$$F_{\text{Coul}} = \sum_{j=\text{H,L}} \frac{e^2 Z_\alpha Z_j}{r_{\alpha j}^2}, \tag{3}$$

The nuclear force F_{nucl} is taken as the negative gradient of the proximity potential, V_{p}, proposed by Blocki *et al* [7] as

$$V_{\text{p}}(z) = 4\pi\gamma b\left[\frac{C_1C_2}{(C_1+C_2)}\right]\Phi\left(\frac{z}{b}\right) \tag{4}$$

with nuclear surface tension coefficient given by Lysekil mass formula as

$$\gamma = 0.9517\left[1-1.7826\frac{(N-Z)^2}{A^2}\right]. \tag{5}$$

Here N, Z and A refer to neutron, proton and mass numbers respectively of the parent. $\Phi\,(\varepsilon)$ is the universal proximity potential given as [8]

$$\Phi\,(\varepsilon) = -4.41\mathrm{e}^{-\varepsilon/0.7176}, \quad \text{for}\ \varepsilon \geq 1.9475, \tag{6}$$

$$\Phi\,(\varepsilon) = -1.7817 + 0.9270\varepsilon + 0.0169\varepsilon^2 - 0.05148\varepsilon^3, \quad \text{for}\ 0 \leq \varepsilon \leq 1.9475. \tag{7}$$

Here $\varepsilon = z/b$ where z is the tip distance and $b \approx 1$ is the width of diffuseness of the nuclear surface. Frictional force, which is proportional to the relative velocity, is also expected to affect the trajectory and final energy of the light charged particle (LCP). The nuclear frictional force according to the formalism described in [9] is given as

$$F_r = K_r(r)v \quad \text{and} \quad F_\varphi = K_{\varphi(r)}r\varphi \tag{8}$$

with $K_r(r) = C_r g(r)$ and $K_{\varphi(r)} = C_\varphi g(r)$, where the form factor $g(r) = [\text{grad}\, V_N(r)]^2$. Here, $C_{\text{r}} = 4 \times 10^{-23}$ s MeV and $C_\varphi = 0.01 \times 10^{-23}$ s MeV.

Misicu *et al* [10] suggested a method for computing initial energy and the location of light charged particles (LCP) and interfragment distance at scission for cold ternary fission using deformed cluster model. The initial energy of the α-particle can be calculated by assuming the potential well corresponding to the light charged particle V_{LCP} with a harmonic potential in the y-direction and centred at the saddle point. The initial kinetic energy corresponds to the zero point energy in the harmonic potential well whose value is given by the formula

$$E_{\text{LCP}} = \frac{1}{2}\hbar\sqrt{\frac{C}{m_{\text{LCP}}}}, \tag{9}$$

where m_{LCP} is the mass of the α-particle and C is the stiffness constant which can be computed using the following formula:

$$C = \sum_{i=\text{L,H}}\frac{1}{R^0_{\alpha i}}\sum_{\lambda\geq 0}\left(\left.\frac{\partial V_{\lambda 0\lambda}(R_{\alpha i})}{\partial R_{\alpha i}}\right|_0 - \frac{\lambda\,(\lambda+1)}{2}\frac{V_{\lambda 0\lambda}(R_{\alpha i})}{R_{\alpha i}}\right), \tag{10}$$

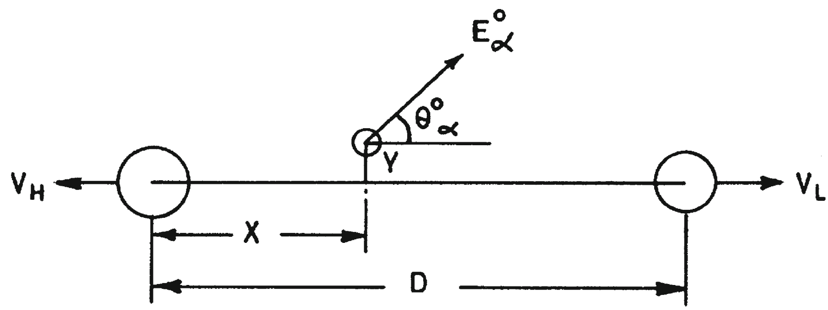

Figure 1. Schematics of the initial set of parameters in ternary fission.

where λ represents multipolarity. The distance of the light and heavy fragments $R^0_{\alpha\mathrm{L}}$ and $R^0_{\alpha\mathrm{H}}$ respectively to the LCP is given by

$$R^0_{\alpha\mathrm{L}} = \frac{D}{1+\sqrt{Z_\mathrm{L}/Z_\mathrm{H}}} \tag{11}$$

$$R^0_{\alpha\mathrm{H}} = \frac{D}{1+\sqrt{Z_\mathrm{H}/Z_\mathrm{L}}}. \tag{12}$$

Here Z_L and Z_H are the atomic numbers of the light and heavy fragments respectively and D is the interfragment distance. Schematics of the initial set of parameters in ternary fission are given in figure 1. The approximate initial tip distance D of the heavy fragments corresponds to the disappearance of the LCP pocket and this tip distance calculated lies around 7 fm which is much lower than that in hot ternary fission.

3. Results and discussion

The potential barrier for cold ternary splitting of ^{252}Cf is computed taking the barrier as the sum of Coulomb potential and nuclear proximity potential. The force experienced by the α-particle is taken as the sum of Coulomb force, nuclear force and the frictional force. The proximity potential has been extensively used by one of us (KPS) in cluster radioactivity studies [11,12] and in heavy ion induced fusion [13]. Initial energy and the location of the α-particle and interfragment distance at scission for cold ternary fission of ^{252}Cf are computed using the prescription proposed by Misicu *et al* [10].

The fact that the light charged particles, usually α-particles, emitted in cold ternary fission are focussed mainly onto the equatorial plane perpendicular to the fission axis indicates that most of the ternary clusters originate from the region of the neck between the two heavier fragments and emitted nearly perpendicular to the fission axis [14]. The trajectory of the α-particle for a tip distance of 7 fm and for an initial energy of 1.72 MeV, for the ^{92}Kr+^{156}Nd +^{4}He splitting in the presence and absence of nuclear force is shown in figure 2 and found deflected towards the polar axis in the presence of nuclear force. The motion of the α-particle is influenced by the state of scission and the trajectory is likely to be affected by the initial position and initial energy of the α-particle. The trajectories with slightly different initial position of α-particle are found not to follow the same path

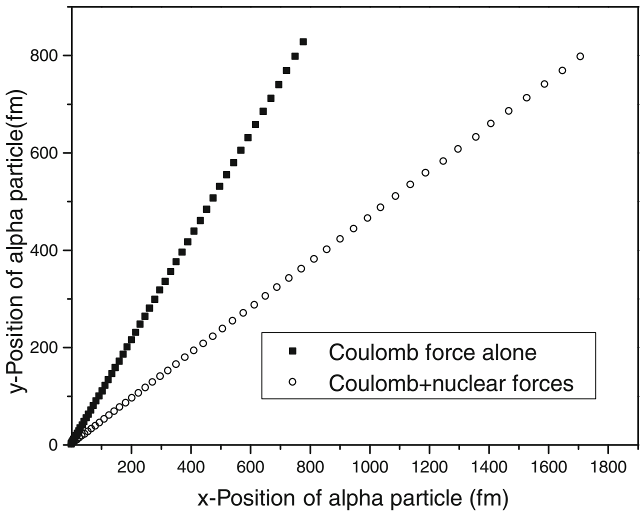

Figure 2. Plot of α-particle trajectory in the absence and presence of nuclear force for a tip distance of 7 fm for the ^{92}Kr+^{156}Nd +^{4}He splitting.

and shows divergence. This implies that the α trajectory shows sensitivity to the initial position of the α-particle. A plot of the trajectory for different initial positions x and $x+0.01$ fm are plotted in figure 3. The sensitivity in position is due to the fact that the position of α-particle appears as inverse function of the Coulomb force and as exponent and powers in the expression of the proximity force.

The sensitivity of the initial energy of the α-particle to the trajectory is another feature observed. The plot of trajectories that are of nearly identical values of initial energies shows divergence and is shown in figure 4. The motion of the two heavy fragments is

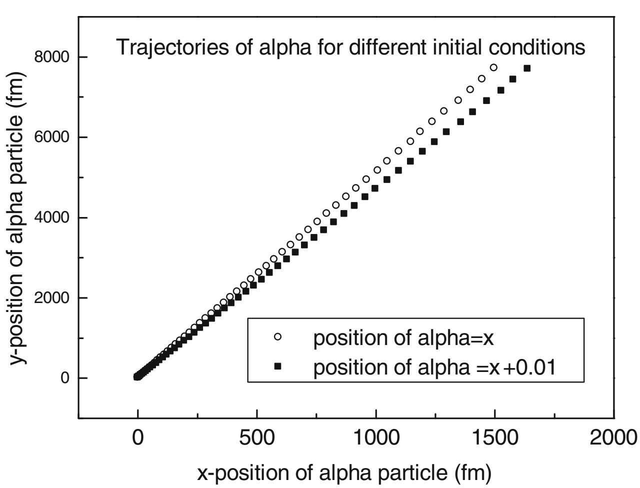

Figure 3. Deviations in α-trajectory for a slight change in the initial position of the α-particle.

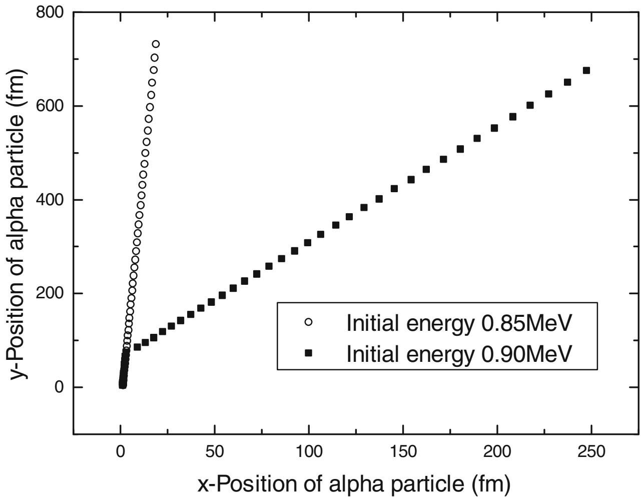

Figure 4. Change in α-trajectory for a slight change in the initial energy of the α-particle.

responsible for this strong dependence of the final position and energy of the α-particle on its initial values. Due to the motion of the heavy fragments, the force acting on the α-particle depends on its position. Therefore at any position, as the initial velocity changes the α-particle's acceleration also changes. Sine V_{LCP}, the potential experienced by the α-particle, is approximated to a harmonic oscillator potential, in the estimation of initial α-particle energy as a zero point energy approximation, an uncertainty is involved. Hence the trajectory is also influenced by the uncertainty of the initial energy of the α-particle. Another factor that affects the trajectory is the location of the α-particle along the y-axis and its estimated uncertainty is nearly 3 fm [15] which again follows from the uncertainty principle. This uncertainty changes the final energy by 0.8 MeV but the asymptotic angle remains nearly unchanged.

Normally, it is expected that with higher initial energy the trajectory will have higher slope of y vs. x, but figure 4 shows a reverse trend. This is because the system under consideration is a dissipative dynamical system which depends on the relative velocity of α-particle and fragments. We have used the realistic value for tip distance at scission, i.e., nearly 7 fm. So, initial values of the variable represent a realistic situation. By using this value we have estimated the initial energy and searched for sensitivity on the values of initial energy. We would like to mention that the kinetic energy of the α-particle coincides with the experimentally measured value of 16 MeV [16] at a tip distance of 7.2 fm. However, no data were known to compare the asymptotic angle.

The motion of the α-particle in the presence of two heavy fragments can be considered as a three-body problem and the equation that represents its motion is a nonlinear differential equation. These two factors, namely, the sensitivity to the initial position and initial energy along with the nonlinearity can be considered as a signature of deterministic chaos. The term 'deterministic chaos' implies the unpredictability of the trajectory when the initial conditions of a deterministic system are slightly changed. For a dynamical

Table 1. The computed Q value, initial energy, electrostatic saddle points $Z_\alpha(D)$ and Lyapunov exponents L_1 and L_2. For computing Q value, masses are taken from ref. [27].

Splitting	Q value (MeV)	$Z_\alpha(D)$ (fm)	Initial energy (MeV)	L_1	L_2
$^{92}Kr+^{156}Nd+^{4}He$	198.70	4.40	1.72	0.85	−0.80
$^{104}Mo+^{144}Xe+^{4}He$	209.11	4.14	1.74	0.84	−0.82
$^{116}Pd+^{132}Sn+^{4}He$	226.87	4.07	1.84	0.88	−0.81

system, sensitivity to initial conditions is quantified by the term 'Lyapunov exponent' (LE). It is the average exponential rates of convergence or divergence of nearby orbits in phase space, which is formally defined as

$$L = \frac{\lim(\delta x(t))}{t \to \infty\, (\delta x(t_0))}. \tag{13}$$

A positive LE signifies exponential divergence of trajectories in the given direction and is associated with chaotic dynamics, while negative LEs are associated with stable motion, when nearby trajectories converge [17]. Even though chaotic motion is often present in systems with many degrees of freedom, there are evidences that simple systems with few degrees of freedom show chaotic features [18]. Detecting the presence of chaos in a dynamical system is an important problem that is solved by measuring the largest Lyapunov exponent. Even though there are several tools to determine the presence of chaos, LE is considered as the most powerful diagnostic tool for chaotic system. For a space with N dimensions, there are N LEs and a system containing at least one positive LE is considered to be chaotic. The present system can be considered as a dynamical system consisting of α-particle moving in the presence of Coulomb and nuclear field with friction and hence can be considered as a dissipative system. There are different methods to compute Lyapunov exponent [19,20]. The most popular method to quantify chaos is the Grassberger–Procaccia algorithm (GPA) because of its simplicity [21]. However, since the GPA is sensitive to variations in its parameters, embedding dimension, reconstruction delay, it is usually unreliable except for long, noise-free time series [22]. Here we use the algorithm proposed by Wolf *et al* [23] which determines LEs from a time series and reorthonormalization technique is used. The computed Lyapunov exponents L_1 and L_2 are tabulated in table 1 along with initial energy and electrostatic saddle points $Z_\alpha(D)$. The values of Lyapunov exponents confirm that the system exhibits deterministic chaos. In any well-behaved dissipative dynamical system, one of the LEs must be strictly negative [24]. Since the second LE is negative, the system can be considered as a dissipative dynamic system exhibiting deterministic chaos. The magnitude of the LE reflects the time-scale in which the system dynamics become unpredictable [25].

4. Conclusions

The trajectory of the α-particle moving in the presence of Coulomb force, nuclear force and frictional force shows sensitivity to several parameters, mainly the initial position and the initial energy of the α-particle. The sensitivity to initial condition (SIC) tempted us to calculate the LE which is found positive implying unpredictability in the trajectory. Therefore, we have to conclude that in the cold ternary fission, the system in which α-particle emission occurs exhibits deterministic chaos. The presence of chaos is a drawback as far as the conventional trajectory calculation is concerned. There are cases in the literature where trajectory calculations have given improper scission configuration [26]. Hence we have to conclude that the future of the emitted particle and the actual scission configuration cannot be determined unambiguously in trajectory calculations to an extent determined by the magnitude of LE.

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