PRAMANA
— journal of
physics

Vol. 79, No. 3
September 2012
pp. 417–426

The isospin mixing and the superallowed Fermi beta decay

A E ÇALIK[1,*], M GERÇEKLİOĞLU[2] and D I SALAMOV[3]
[1]Department of Physics, Faculty of Arts and Sciences, Dumlupınar University, 43100, Kütahya, Turkey
[2]Department of Physics, Faculty of Science, Ege University, 35100, Bornova, İzmir, Turkey
[3]Department of Physics, Faculty of Arts and Sciences, Muş Alparslan University, 49100, Muş, Turkey
*Corresponding author. E-mail: engincalik@yahoo.com

MS received 16 February 2012; accepted 18 April 2012

Abstract. In the present work, the isospin admixtures in the nuclear ground states of the parent nuclei and isospin structure of the isobar analog resonance (IAR) states have been investigated by studying the $0^+ \rightarrow 0^+$ superallowed Fermi β decays using Pyatov's restoration method. Within the random phase approximation (RPA), in this method, the effect of isospin breaking due to the Coulomb forces has been evaluated, taking into account the effect of pairing correlations between nucleons.

Keywords. Superallowed β decays; isospin mixing; isobar analog resonance.

PACS Nos 21.60.Cs; 23.40.Hc; 23.40.–s

1. Introduction

The superallowed Fermi β decay is one of the few processes to calculate the amplitude (V_{ud}) of up- and down-quark mixtures of Cabibbo–Kobayashi–Maskawa (CKM) matrix. Unitarity of the CKM matrix is an important problem in the Standard Model of particle physics. Estimation of the V_{ud} element of CKM matrix using the superallowed Fermi β decays has been the subject of many studies [1–21]. On the other hand, the isospin mixing of nuclei in the ground states and isospin structure of IAR states in the nuclei are also important for the investigation of superallowed Fermi β decays. The isospin mixing in nuclear ground states is related to the isospin symmetry breaking and has a significant place in nuclear physics. The theoretical and experimental studies have continued for many years. Isospin impurity plays an important role in the determination of vector coupling constant G_{V}, based on superallowed Fermi β decay.

The isospin admixture of $T_0 + 1$ state into the T_0 state is defined as isospin impurity, $T_0 = T$ ground state [22]. The mixing probability of the isospin $T_0 = 1$ admixed to the $T_0 = 0$ ground state was calculated in various models.

The isospin mixing probabilities in $N = Z$ nuclei between $A = 50$ and 100 were calculated by several studies [23–25]. Hamamoto and Sagawa [23] have computed these probabilities using the Hartree–Fock + Tamm–Dankoff (HF+TD) or HF+RPA approximation with the Skyrme-type interactions. Dobaczewski and Hamamoto [24] have evaluated them by the deformed HF solutions with Skyrme interaction. Colò *et al* [25] have estimated $T = T_0 + 1$ isospin mixing into the $T = T_0$ ground state of $N = Z$ nuclei with Hartree–Fock (HF) calculations using Skyrme forces. Suzuki *et al* [26] and Sagawa *et al* [27] have investigated the possible relations between the width of IAR states and isospin mixing probability using a microscopic model. This model is based on the Feschbach projection method [28,29]. Kicińska-Habior [30] defined isospin mixing probability and its dependence on the nuclear temperature. New results of the measured isospin mixing probabilities for S and Ar have been presented. Sagawa *et al* [11] have studied the effect of isospin impurity and isospin symmetry breaking correction (quenching factor) using the microscopic HF and RPA approximation (or TD approximation). The two new effects, the isospin impurity of the core and effect of the charge symmetry breaking (CSB) and charge independence breaking (CIB) forces, have been examined in this study. Another study, the isospin- and angular momentum-projected density functional theory (DFT) has been employed to calculate isospin mixing probability and isospin breaking corrections to ^{40}Ca, ^{42}Sc, ^{80}Zr and ^{100}Sn isotopes [31].

Auerbach [22] has investigated the Coulomb corrections to superallowed β decay in nuclei. In this study, isospin symmetry breaking correction was denoted as $\delta_c = 8(V_1/41\xi A^{2/3})P$, where ξ is a numerical factor which depends on the model used to describe the isovector monopole, $V_1 \approx 100$ MeV and P is the isospin admixture of the $T_0 + 1$ state into the T_0 state. Isospin impurity P has been calculated using four different models and the details of these four models are given in ref. [32]. One of them is the hydrodynamic model [33]. Estimation for the probability of isospin admixture P was given by Bohr and Mottelson based on a hydrodynamical description of isovector monopole state and denoted as

$$P = 5.5 \times 10^{-7} Z^{8/3}. \tag{1}$$

The isospin admixture probabilities are expected to be proportional to $Z^{8/3}$ in this model. The second model is based on the non-energy weighted sum rule (NEWSR):

$$P = \frac{1.8 \times 10^{-7} Z^3}{T + 1}. \tag{2}$$

The third model is based on the energy weighted sum rule (EWSR):

$$P = \frac{Z^2 A^{2/3}}{T + 1} \times 6.8 \times 10^{-7}. \tag{3}$$

In these three models, the isospin impurity P is evaluated in perturbation theory using the notion of isovector monopole resonance (IVMR).

The fourth model is the microscopic model which is based on microscopic RPA calculations of IVMR:

$$P = \frac{4.3 \times 10^{-6}}{T + 1} \frac{Z^3}{A^{2/3}}. \tag{4}$$

The properties of the isobaric analog states are discussed in refs [26,27,34–37]. In refs [34–37], the properties of IAR have been investigated using the finite Fermi system theory.

In all these studies, the residual interaction was not related to the shell model potential in the self-consistent way; in fact such a relation is necessary because the mean-field potential includes isovector term.

Here, the initial nuclear Hamiltonian (without electromagnetic interactions) should exactly obey the isospin invariance conditions.

In refs [38,39], the broken isospin symmetry for the nuclear part of the Hamiltonian has been restored by Pyatov method [40] self-consistently. In ref. [38], the isospin mixing of ground states of the nuclei has been computed without pairing interaction. On the other hand, in refs [39,41] the same calculations have been made by taking into account the pairing interaction.

The isospin mixing and structure of the isobar analog states were investigated using several studies based on Pyatov's method [41–43]. In ref. [41], the isospin admixtures in the ground states of even–even nuclei, energies of isobar analog states and effect of the pairing correlations between nucleons were investigated for $A = 208$ nuclei. In ref. [42], the isospin admixtures of the ground states and properties of IAR were examined for the medium and heavy mass nuclei. The isospin admixture probabilities in the ground states of $N = Z$ even–even nuclei in the mass region of 50–100 were calculated in ref. [43].

In our previous work [19], which depends on Pyatov's method, unitarity of the CKM matrix was investigated by studying the superallowed Fermi β decays.

In this work, the isospin mixing P in the ground states of the parent nuclei by considering the pairing interactions and isomultiplet structure of IAR states of the superallowed β decays have been investigated based on Pyatov's method. Since isospin symmetry breaking correction has been calculated for superallowed transitions in ref. [22], we have compared our calculated P values with the results of the hydrodynamical, NEWSR, EWSR and microscopic models.

The details of the Pyatov's method are given in ref. [41]. Here we shall only give formalism for isospin mixing in the ground state of the nucleus and the isospin structure of IAR.

2. Isospin mixing of ground state

Expanding the ground state wave function in terms of pure isospin components $|T, T_Z\rangle$, we obtain [38]:

$$|0\rangle = |N, Z\rangle = a\,|T_0, T_0\rangle + b\,|T_0 + 1, T_0\rangle\,,$$
$$a^2 + b^2 = 1. \tag{5}$$

The expectation value for the square of isospin in the ground state of the parent nucleus is related to the $M_\beta^{\pm}$ matrix elements as

$$\langle 0|\,\hat{T}^2\,|0\rangle = \sum_i \left|M^i_{\beta^+}\right|^2 + T_0(T_0 + 1). \tag{6}$$

The $M_\beta^\pm$ matrix elements are shown in refs [38,39,41]. On the other hand

$$\langle 0|\, \hat{T}^2\, |0\rangle = T_0(T_0+1) + 2b^2(T_0+1). \tag{7}$$

From these equations, it follows that the $T_0 + 1$ isospin admixture in the ground state of the parent nucleus is determined by the sum of squares for the β decay matrix elements from isobaric state of nucleus:

$$P(T_0+1) = b^2 = \frac{\sum_i |M^i_{\beta^+}|^2}{2(T_0+1)}. \tag{8}$$

3. Isospin structure of IAR

Using the operator $\hat{T}_-$, we can generate a collective analog state in the nucleus:

$$|A\rangle = |\langle 0|\, \hat{T}_+\hat{T}_-\, |0\rangle|^{-1/2}\, \hat{T}_-\, |0\rangle\, . \tag{9}$$

Generally, this state is not an eigenstate of the Hamiltonian $\hat{H} + \hat{h}$ (the residual interaction $\hat{h}$ is defined by Pyatov in ref. [38]). It is distributed over the spectrum of isobaric resonances $\hat{Q}^\dagger_{\mathrm{IAR}}|0\rangle$ which contain admixtures of states with isospin $T_0 - 1$, T_0, $T_0 + 1$ and $T_0 + 2$. Thus, for IAR:

$$\begin{aligned} \hat{Q}^\dagger_{\mathrm{IAR}}|0\rangle \;=\; & \gamma_{\mathrm{IAR}}|T_0-1, T_0-1\rangle + \alpha_{\mathrm{IAR}}|T_0, T_0-1\rangle \\ & + \beta_{\mathrm{IAR}}|T_0+1, T_0-1\rangle + \Delta_{\mathrm{IAR}}|T_0+2, T_0-1\rangle\, , \\ & \gamma^2_{\mathrm{IAR}} + \alpha^2_{\mathrm{IAR}} + \beta^2_{\mathrm{IAR}} + \Delta^2_{\mathrm{IAR}} = 1. \end{aligned} \tag{10}$$

Neglecting the small admixtures of isospin $T_0 + 2$ in IAR, we obtain [38]

$$\alpha_{\mathrm{IAR}} = \frac{a\sqrt{T_0}M^{\beta^-}_{\mathrm{IAR}}}{T_0 + b^2(T_0+1)}, \tag{11}$$

$$\beta_{\mathrm{IAR}} = \frac{b\sqrt{(2T_0+1)}M^{\beta^-}_{\mathrm{IAR}}}{T_0 + b^2(T_0+1)}, \tag{12}$$

$$\gamma^2_{\mathrm{IAR}} = 1 - (\alpha^2_{\mathrm{IAR}} + \beta^2_{\mathrm{IAR}}). \tag{13}$$

4. Result and discussion

In this section, the numerical calculations for isospin admixtures in the ground states of the parent nuclei and isospin structure of IAR states for the superallowed transitions are performed by considering pairing correlations between nucleons and including the effective Fermi interaction term in a self-consistent way.

In the calculations, the Woods–Saxon potential with the Chepurnov parametrization [44] was used and the pairing correlation function was chosen as $C_n = C_p \approx 12/\sqrt{A}$ for open shell nuclei.

Table 1. The calculated values of $b^2 = P(T_0 + 1)$ (%), α^2 (%), β^2 (%) and γ^2 (%) for superallowed $0^+ \rightarrow 0^+$ Fermi β decays.

Isobars	$b^2 = P(T_0 + 1)$ (%)		α^2 (%)		β^2 (%)		γ^2 (%)	
	Without pairig	With pairing	Without pairig	With pairig	Without pairig	With pairig	Without pairig	With pairig
${}^{10}_{6}C \rightarrow {}^{10}_{5}B$	0.008	0.008	98.561	98.561	0.024	0.024	1.415	1.415
${}^{14}_{8}O \rightarrow {}^{14}_{7}N$	0.039	0.039	99.232	99.232	0.114	0.114	0.654	0.654
${}^{26}_{13}Al \rightarrow {}^{26}_{12}Mg$	0.075	0.093	99.458	99.130	0.224	0.276	0.318	0.593
${}^{34}_{17}Cl \rightarrow {}^{34}_{16}S$	0.177	0.193	99.102	98.543	0.528	0.570	0.370	0.887
${}^{38}_{19}K \rightarrow {}^{38}_{18}Ar$	0.231	0.230	99.004	98.116	0.686	0.678	0.310	1.206
${}^{42}_{21}Sc \rightarrow {}^{42}_{20}Ca$	0.319	0.259	98.563	97.971	0.945	0.763	0.492	1.266
${}^{46}_{23}V \rightarrow {}^{46}_{22}Ti$	0.456	0.319	97.969	97.371	1.345	0.933	0.686	1.696
${}^{50}_{25}Mn \rightarrow {}^{50}_{24}Cr$	0.636	0.446	97.319	96.464	1.869	1.298	0.813	2.238
${}^{54}_{27}Co \rightarrow {}^{54}_{26}Fe$	0.880	0.433	95.957	95.799	2.555	1.250	1.488	2.951
${}^{62}_{31}Ga \rightarrow {}^{62}_{30}Zn$	0.962	0.569	94.676	91.793	2.744	1.577	2.580	6.630
${}^{66}_{33}As \rightarrow {}^{66}_{32}Ge$	1.083	0.603	94.367	90.134	3.084	1.641	2.548	8.226
${}^{74}_{37}Rb \rightarrow {}^{74}_{36}Kr$	1.533	0.545	92.219	90.264	4.306	1.484	3.475	8.252

The calculated values of $b^2 = P(T_0+1)$ (%), α^2 (%), β^2 (%) and γ^2 (%) are given for superallowed $0^+ \rightarrow 0^+$ Fermi β decays in table 1. All values tabulated for ^{10}C and ^{14}O are the same for both with and without pairing calculations. Because of the poor nucleon number, it is not possible to calculate the pairing effects of ^{10}C and ^{14}O.

In figure 1, the T_0+1 isospin admixtures in the ground states of the parent nuclei of the investigated superallowed decays calculated by the Pyatov method are compared with the results of the hydrodynamical, NEWSR, EWSR and microscopic models. In figure 1, the curves with filled and open circles correspond to our calculations without and with pairing correlations between nucleons, respectively. As seen from the figure, the admixture probability $b^2 = P(T_0+1)$ (%) increases with increasing proton number for all models. It is well known that the isospin admixture probability is proportional to $Z^{8/3}$ in hydrodynamical model [33], and it is given by eq. (1). In the other models, the admixture probability is proportional to the power of Z as can be seen in eqs (2)–(4). The calculations of hydrodynamic and EWSR models are overlapped. The values for microscopic model are smaller than for our calculations and hydrodynamic models. Values for NEWSR model are the smallest ones. In general, the values for hydrodynamic, NEWSR, ESWR, microscopic and our calculations with pairing are almost equivalent. Thus, it can be said that our calculated values depend on the power of Z as other models. According to our calculations, when proton number increases the difference between values without and with pairing correlations also increases. In the first five nuclei, there is no difference between values with and without pairing but after the fifth one, the pairing correlations between nucleons lead to a shift in the isospin admixture probability values to the results of the other model calculations. This result is natural. When the mass number increases, the pairing correlations are more effective because the isovector potential decreases. As a result, the isospin mixing probability values with pairing interactions are in good agreement with the other model results.

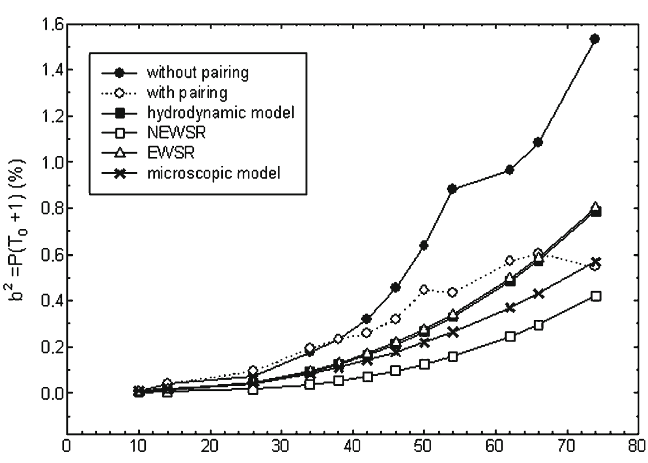

Figure 1. Amplitudes of isospin T_0+1 admixtures in the ground states of the parent nuclei of the investigated isobars.

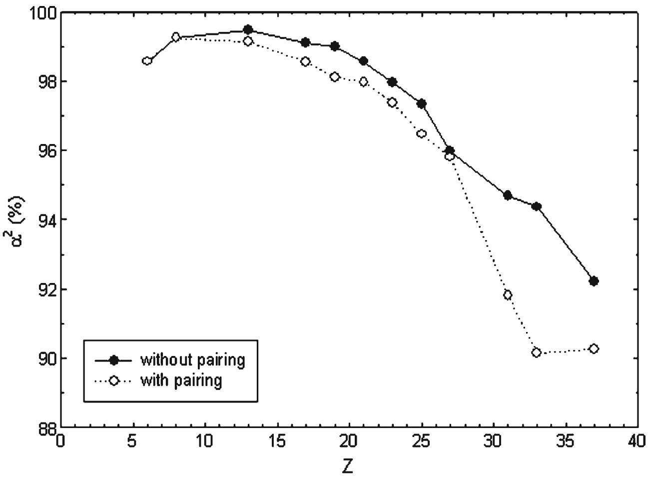

Figure 2. The variation of $\alpha^2(\%)$ (the contribution of the T_0 isospin state to the IAR) with respect to the proton number Z.

The calculated values for α^2 (%), β^2 (%), γ^2 (%) which characterize the T_0, $T_0 + 1$ and $T_0 - 1$ isospin components of IAR states are given in figures 2, 3 and 4, respectively. In all the figures, the solid lines correspond to no pairing while the dashed lines stand for pairing correlations between nucleons.

The contribution of T_0 isospin state to IAR state (α^2 (%)) is shown in figure 2. In IAR state of the investigated nuclei, the highest value is attributed to α^2 (%). The contribution varies between 90 and 99% in all the nuclei. As it is known from eq. (11), α^2 (%) is related to a and b. In eq. (5), a and b are defined as $a^2 + b^2 = 1$. It was already explained

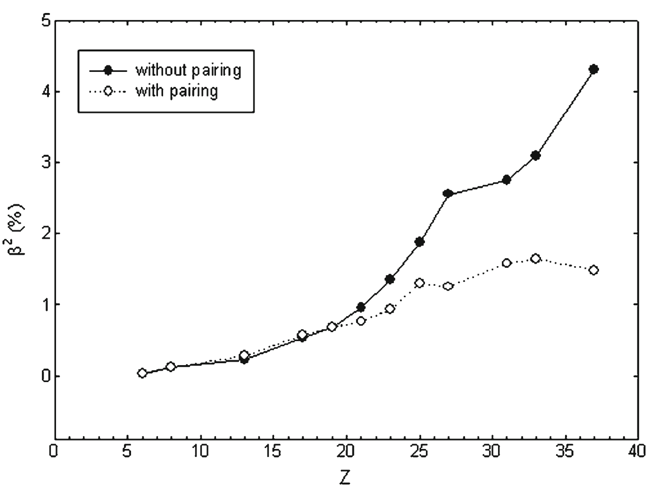

Figure 3. The variation of $\beta^2(\%)$ (the contribution of the $T_0 + 1$ isospin state to the IAR) with respect to the proton number Z.

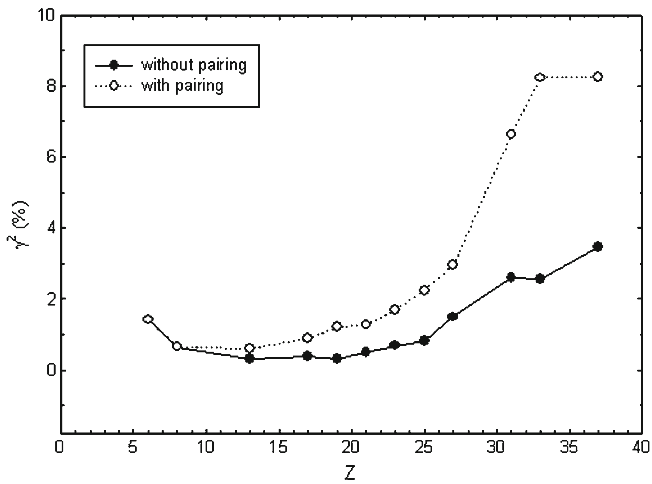

Figure 4. The variation of $\gamma^2(\%)$ (the contribution of the $T_0 - 1$ isospin state to the IAR) with respect to the proton number Z.

as to why the admixture probability $b^2 = P(T_0 + 1)$ (%) increases with increasing proton number. According to eq. (5), when b^2 increases, the a^2 values decrease. Consequently, α^2 (%) values decrease with increasing proton number for calculations without and with pairing. As seen in the figure, the α^2 (%) values with pairing are smaller than the values without pairing. This result can be explained as; when the pairing interaction is considered, densities of final states of the transition are increasing, and due to the strength of the transition re-distribution between these new states, the IAR values decrease.

It can be seen that, curves corresponding to our calculations are similar in figures 1 and 3. This is the natural consequence of proportionality of β^2 (%) to the $T_0 + 1$ isospin admixture $b^2 = P(T_0 + 1)$ (%) seen in eq. (12). Our results confirm this similarity. Therefore, the same physical arguments for the $b^2 = P(T_0 + 1)$ (%) values are given for the β^2 (%) values also. The β^2 (%) contribution changes to 0.1–4.5%. The β^2 (%) values are very small compared to α^2 (%) and γ^2 (%) values.

The contribution of $T_0 - 1$ isospin state to IAR state (γ^2 (%)) is plotted in figure 4. The γ^2 (%) values are 10–30 times lower than the α^2 (%) results. The contribution changes to 0.3–8% in all nuclei. In both calculations without and with pairing, it shows that γ^2 (%) values increase whereas proton number increases. It is an expected result from eq. (13).

5. Conclusion

In the microscopic theory of atomic nucleus, some essential symmetries of the nuclear Hamiltonian are broken by the model. Here, isotopic symmetry, i.e. charges independent of the nuclear forces, has been broken by Coulomb forces. However, the isovector term in the nuclear shell model Hamiltonian also breaks the isotopic invariance. The former breaking is not natural and it is necessary to compensate its effect in the wave functions and matrix elements. This point has not been stressed in other studies. The mentioned compensation has been performed using the Pyatov's restoration method. Here, restoration procedure is based on the shell model potential and on the assumption of seperability of the residual interaction. After the restoration, the model is free of any adjustable parameters. As a result of the present calculations, the following conclusions are drawn:

The effect of pairing correlations between nucleons on the isospin admixtures have been dominantly seen in the superallowed β transitions with big mass number. This implies that the isospin admixtures play a more important role in these isotopes. As a result, the calculated values of $P = b^2$ (%) with pairing by Pyatov method are the same order of those of other models.

The investigated isospin structure of IAR states of the transitions determines that when the mass number increases, the affect of pairing interaction between nucleons is important for the superallowed Fermi β transitions.

Acknowledgements

A E Çalık is grateful to Dr C Parlak for his help in editing.

References

[1] R J Blin-Stoyle, *Isospin in nuclear physics* edited by D H Wilkinson (North-Holland, Amsterdam, 1969) pp. 115–172
[2] I S Towner and J C Hardy, *Nucl. Phys.* **A205**, 33 (1973)
[3] D H Wilkinson, *Phys. Lett.* **65B**, 9 (1976)
[4] J C Hardy and I S Towner, *Nucl. Phys.* **A254**, 221 (1975)
[5] I S Towner, J C Hardy and M Harvey, *Nucl. Phys.* **A284**, 269 (1977)
[6] W E Ormand and B A Brown, *Phys. Rev. Lett.* **62**, 866 (1989)
[7] F C Barker, *Nucl. Phys.* **A537**, 134 (1992)
[8] F C Barker, *Nucl. Phys.* **A579**, 62 (1994)
[9] D H Wilkinson, *Nucl. Instrum. Methods* **A335**, 201 (1993)
[10] W E Ormand and B A Brown, *Phys. Rev.* **C52**, 2455 (1995)
[11] H Sagava, N Van Giai and T Suzuki, *Phys. Rev.* **C53**, 2163 (1996)
[12] P Navrátil, B R Barrett and W E Ormand, *Phys. Rev.* **C56**, 2542 (1997)
[13] D H Wilkinson, *Nucl. Instrum. Methods* **A488**, 654 (2002)
[14] I S Towner and J C Hardy, *Phys. Rev.* **C66**, 035501 (2002)
[15] J C Hardy and I S Towner, *Phys. Rev.* **C71**, 055501 (2005)
[16] I S Towner and J C Hardy, *Phys. Rev.* **C77**, 025501 (2008)
[17] H Liang, N Van Giai and J Meng, arXiv:0904.3673v2 [nucl-th]
[18] J C Hardy and I S Towner, *Phys. Rev.* **C79**, 055502 (2009)
[19] A E Çalık, M M Gerçeklioğlu and D I Salamov, *Z. Naturforsch.* **64a**, 865 (2009)
[20] J C Hardy and I S Towner, *Nucl. Phys.* **A844**, 138c (2010)
[21] G F Grinyer, C E Svensson and B A Brown, *Nucl. Instrum. Methods* **A622**, 236 (2010)
[22] N Auerbach, *Phys. Rev.* **C79**, 035502 (2009)
[23] I Hamamoto and H Sagawa, *Phys. Rev.* **C48**, 960 (1993)
[24] J Dobaczewski and I Hamamoto, *Phys. Lett.* **B345**, 181 (1995)
[25] G Colò, M A Nagarajan, P Van Isacker and A Vitturi, *Phys. Rev.* **C52**, R1175 (1995)
[26] T Suzuki, H Sagawa and G Colò, *Phys. Rev.* **C54**, 2954 (1996)
[27] H Sagawa, P F Bortignon and G Colò, *Phys. Lett.* **B444**, 1 (1998)
[28] H Feschbach, *Annu. Rev. Nucl. Sci.* **8**, 44 (1958)

[29] H Feschbach, *Ann. Phys.* **19**, 287 (1962)
[30] M Kicińska-Habior, *Acta Phys. Pol.* **B36**, 1133 (2005)
[31] W Satula *et al*, arXiv:1010.3099v1 [nucl-th]
[32] N Auerbach, *Phys. Rep.* **98**, 273 (1983)
[33] A Bohr and B R Mottelson, *Nuclear structure* (Benjamin, New York, 1969) Vol. 1
[34] D F Zaretskii and M G Urin, *Zh. Eksp. Teor. Fiz.* **53**, 324 (1967), [*Sov. Phys. JETP* **26**, 217 (1968)]
[35] Yu V Gaponov and Yu S Lyutostanskii, *Yad. Fiz.* **16**, 484 (1972), [*Sov. J. Nucl. Phys.* **16**, 270 (1973)]
[36] S A Fajans, *Phys. Lett.* **B37**, 155 (1971)
[37] B L Birbrair and V A Sadovnikova, *Yad. Fiz.* **20**, 645 (1974), [*Sov. J. Nucl. Phys.* **20**, 347 (1975)]
[38] N I Pyatov *et al*, *Sov. J. Nucl. Phys.* **29**, 10 (1979)
[39] T Babacan *et al*, *J. Phys.* **G30**, 759 (2004)
[40] N I Pyatov and D I Salamov, *Nukleonika* **22**, 127 (1977)
[41] A Kucukbursa, D I Salamov, T Babacan and H A Aygör, *Pramana – J. Phys.* **63**, 947 (2004)
[42] D I Salamov *et al*, *Pramana – J. Phys.* **66**, 1105 (2006)
[43] T Babacan, D I Salamov and A Kucukbursa, *Nucl. Phys.* **A788**, 279 (2007)
[44] V G Soloviev, *Theory of complex nuclei* (Pergamon, New York, 1976)