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On wave characteristics of piezoelectromagnetics

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Abstract. This report gives a discussion of a new wave characteristic as a material parameter for a composite with the magnetoelectric effect. The new parameter depends on the material constants of a piezoelectromagnetic composite. It can be implemented on: (A) mechanically free, electrically and magnetically open surface and (B) mechanically free, electrically and magnetically closed surface. These theoretical investigations are useful for researchers in the fields of acousto-optics, photonics and opto-acousto-electronics. Some sample calculations are carried out for $BaTiO_3$–$CoFe_2O_4$ and PZT-5H–Terfenol-D composites of class 6 mm. Also, the first and second derivatives of the new parameter with respect to the electromagnetic constant α are graphically shown.

Keywords. Piezoelectromagnetics; PZT-5H and Terfenol-D; magnetoelectric effect; surface SH waves.

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1. Introduction

According to ref. [1], all materials exhibiting magnetoelectric (ME) effect can be classified into single-phase materials and composite materials. The single-phase materials have an ordered structure and require a ferroelectric/ferrielectric/antiferroelectric state. In composite materials, the ME effect is realized by using the idea of average product properties through various connectivities including any combination of p-q, p and q running from 0 to 3 and representing the dimension of either phase [2–8]. For instance, the ME effect in PZT-Terfenol-D 2–2 laminate composites is described in refs. [4,6]. This 2–2 laminate composite is of particular interest for this study. Most ferromagnetic materials show magnetostrictive effect. In these materials, a magnetic field causes deformation which is quadratically dependent on the magnetic field strength. This is completely different from the single-phase materials where the ME effect shows a linear dependence on the magnetic or electric field. Also, the ME effect in these composites exhibits a hysteretic behaviour. As a result, it is difficult for such composites to be used in various linear devices. Therefore, the non-linear ME effect of composite systems is the case for bias magnetic field application to the system. A linear behaviour is usually observed by

AC magnetic field application to the ME composite. The latest review [7] by Srinivasan discusses recent advances in the physics of ME interactions in layered composites and nanostructures and potential device applications. Magnetoelectric response of composites is a product property of individual ferromagnetic and ferroelectric phases.

The ME response can be limited by the following relation [8]:

$$\alpha^2 < \varepsilon\mu, \tag{1}$$

where α is the ME constant, ε is the dielectric permittivity coefficient and μ is the magnetic permeability coefficient. The main purpose of many experimental investigations on the ME effect is to observe a possible maximum value of α for a composite. From eq. (1), α^2 for one material can be significantly smaller than that for another material, but closer to $\varepsilon\mu$ for a very small value of α^2 due to the very small value of $\varepsilon\mu$. The importance of comparison of α^2 with $\varepsilon\mu$ is briefly discussed in the following section. Also, negative data for the magnetic permeability coefficient μ have been experimentally measured or analytically predicted [9,10]. Also, negative and positive magnetic permeability coefficients [11], i.e. $\mu < 0$ and $\mu > 0$, can contradict with each other. In certain cases, a negative magnetic permeability results in a negative internal energy. Therefore, for a piezoelectromagnetic (PEM) composite, it is reasonable to study the material average properties of the piezoelectrics (PEs) and the PMs.

For a piezoelectromagnetic medium, the electromagnetic wave velocity V_{EM} is $V_{\mathrm{EM}}^2 = 1/(\varepsilon\mu)$. In a free space, the above speed reduces to the speed of light in vacuum: $C_{\mathrm{L}}^2 = 1/(\varepsilon_0\mu_0)$ where ε_0 and μ_0 are respectively the dielectric permittivity and magnetic permeability coefficients in vacuum. These constants are the fundamental characteristics used in optics, photonics, optoelectronics and acousto-optics. The purpose of this short report is to continue the theoretical investigations carried out in ref. [12]. The following section acquaints the readers with a new wave characteristic used for evaluating the ME effect. The new characteristic naturally depends on both shear-horizontal surface and bulk acoustic waves (SH-SAWs and SH-BAWs) and reveals some coupling of bulk and surface wave properties in piezoelectromagnetic composites. It is clear that BAWs represent average wave characteristics for a piezoelectromagnetic composite (for instance, a multi-layered structure) because each microscopic part of the composite structure participates in such oscillations of the whole bulk material. In composite materials, both the BAW characteristics and the ME effect are therefore realized by using the idea of average product properties through various connectivities described already. However, almost all experimental and theoretical research works are focussed on investigations of the ME effect in various composites. Therefore, this paper deals with the new wave characteristic which contains the SH-BAW velocity. It is possible to state that this work proposes a new parameter (wave characteristic) which can be used to classify the magnetoelectric (ME) effect in piezoelectromagnetic (composite) materials.

2. Wave characteristics for the ME effect

The wave characteristic discussed here is given by [13]

$$\Delta = V_{\mathrm{tem}} - V_{\mathrm{new}}, \tag{2}$$

where V_{tem} is the speed of shear-horizontal bulk acoustic wave (SH-BAW) coupled with both the electrical and magnetic potentials

$$V_{\text{tem}} = V_{t4}(1 + K_{\text{em}}^2)^{1/2}, \tag{3}$$

$$K_{\text{em}}^2 = \frac{\mu e^2 + \varepsilon h^2 - 2\alpha e h}{C(\varepsilon\mu - \alpha^2)}, \tag{4}$$

where K_{em}^2 is the coefficient of magnetoelectromechanical coupling (CMEMC), V_{t4} is the speed of SH-BAW when the CMEMC vanishes, i.e. $V_{t4} = \sqrt{C/\rho}$, ρ being the composite mass density. In expression (2), V_{new} is the new SH-SAW velocity discovered by the author in ref. [13]:

$$V_{\text{new}} = V_{\text{tem}}\sqrt{1 - b^2} \tag{5}$$

$$b = \frac{\alpha^2}{\varepsilon\mu}\frac{K_{\text{em}}^2 - (eh/\alpha C)}{1 + K_{\text{em}}^2}. \tag{6}$$

Here, C, e, and h denote the elastic stiffness constant, piezoelectric constant, and piezomagnetic coefficient, respectively.

Evaluation of inequality (1) for the CMEMC in expression (4) can also be useful for composite systems because it is obvious that $V_{\text{tem}} \to \infty$ in expression (3) if $\alpha^2 \to \varepsilon\mu$, which means that V_{tem} can be significantly larger than the speed of light in vacuum. As well known, SH-SAW velocities are significantly smaller than the speed of light. The case of $\alpha^2 > \varepsilon\mu$ for which $V_{\text{tem}} < V_{t4}$ due to $K_{\text{em}}^2 < 0$ is also interesting. This can hold true for $\mu < 0$. The usual situation is $V_{\text{tem}} > V_{t4}$ because $V_{\text{tem}}(h = \alpha = 0) = V_{te} > V_{t4}$ for pure piezoelectrics and $V_{\text{tem}}(e = \alpha = 0) = V_{tm} > V_{t4}$ for pure piezomagnetics. Therefore, it is expected that $V_{\text{tem}} > V_{t4}$ can be true for some piezoelectric/piezomagnetic multilayered structures (composites). However, some experimental works, see refs [14,15], reported the studies of left-handed artificial materials (metamaterials) in the frequency region 1–100 THz, and even above [14]. They have an interest in the region when $\mu < 0$ and $\varepsilon < 0$ ($\varepsilon\mu > 0$). Hence, it is also possible to compare α^2 with $\varepsilon\mu$ in eq. (1) for the CMEMC in eq. (4). Note that $\mu < 0$ and $\varepsilon < 0$ can result in $V_{\text{tem}} < V_{t4}$ for real values of e and h in eq. (3), but can result in $V_{\text{tem}} > V_{t4}$ for imaginary values of e and h (or $\alpha^2 > \varepsilon\mu$) in eq. (3). Note that theoretical approaches exist which use complex material constants to describe wave propagation in multilayered structures [16]. However, many experimental reports like refs [14,15] do not provide the complete set of material constants for the investigated unique composites. The following section analytically investigates the first and second derivatives of the parameter Δ with respect to α.

3. The derivatives of the parameter Δ

The first derivative of the parameter Δ with respect to the magnetoelectric constant α for a composite can be evaluated by

$$\frac{\partial\Delta}{\partial\alpha} = \frac{V_{t4}}{2(1 + K_{\text{em}}^2)^{1/2}}\frac{\partial K_{\text{em}}^2}{\partial\alpha} + \frac{bV_{\text{tem}}}{\sqrt{1 - b^2}}\frac{\partial b}{\partial\alpha} - \sqrt{1 - b^2}\frac{\partial V_{\text{tem}}}{\partial\alpha}, \tag{7}$$

where

$$\frac{\partial K_{\mathrm{em}}^2}{\partial\alpha} = \frac{2(\alpha K_{\mathrm{em}}^2 - eh/C)}{\varepsilon\mu - \alpha^2} \tag{8}$$

and

$$\frac{\partial b}{\partial\alpha} = \frac{2b}{\alpha} - b\frac{(\partial K_{\mathrm{em}}^2/\partial\alpha)}{1+K_{\mathrm{em}}^2} + \frac{\alpha^2}{\varepsilon\mu}\frac{((\partial K_{\mathrm{em}}^2/\partial\alpha) + (eh/\alpha^2 C))}{1+K_{\mathrm{em}}^2}. \tag{9}$$

After obtaining the first derivatives of K_{em}^2 and b with respect to α, one can find that Δ has an extreme point when

$$\frac{\partial V_{\mathrm{tem}}}{\partial\alpha} = \frac{\partial V_{\mathrm{new}}}{\partial\alpha}. \tag{10}$$

Furthermore, one can obtain the following second partial derivative:

$$\frac{\partial^2\Delta}{\partial\alpha^2} = \frac{\partial^2 V_{\mathrm{tem}}}{\partial\alpha^2} - \frac{\partial^2 V_{\mathrm{new}}}{\partial\alpha^2}, \tag{11}$$

where

$$\frac{\partial^2 V_{\mathrm{tem}}}{\partial\alpha^2} = -\frac{V_{t4}}{4(1+K_{\mathrm{em}}^2)^{3/2}}\left(\frac{\partial K_{\mathrm{em}}^2}{\partial\alpha}\right)^2 + \frac{V_{t4}}{2(1+K_{\mathrm{em}}^2)^{1/2}}\frac{\partial^2 K_{\mathrm{em}}^2}{\partial\alpha^2} \tag{12}$$

$$\begin{aligned}\frac{\partial^2 V_{\mathrm{new}}}{\partial\alpha^2} &= \sqrt{1-b^2}\frac{\partial^2 V_{\mathrm{tem}}}{\partial\alpha^2} - \frac{2b}{\sqrt{1-b^2}}\frac{\partial V_{\mathrm{tem}}}{\partial\alpha}\frac{\partial b}{\partial\alpha} - \frac{bV_{\mathrm{tem}}}{\sqrt{1-b^2}}\frac{\partial^2 b}{\partial\alpha^2}\\ &\quad -\frac{V_{\mathrm{tem}}}{(1-b^2)^{3/2}}\left(\frac{\partial b}{\partial\alpha}\right)^2.\end{aligned} \tag{13}$$

In eq. (7), the first partial derivative of the CMEMC [12] is defined by eq. (8) and the second partial derivative of the CMEMC [12] is defined as follows:

$$\frac{\partial^2 K_{\mathrm{em}}^2}{\partial\alpha^2} = \frac{2K_{\mathrm{em}}^2 + 4\alpha(\partial K_{\mathrm{em}}^2/\partial\alpha)}{\varepsilon\mu - \alpha^2}. \tag{14}$$

In eq. (13), the first partial derivative of the function $b(\alpha)$ with respect to the electromagnetic constant α is defined by eq. (9) and the second partial derivative of $b(\alpha)$ with respect to α can be expressed as follows:

$$\begin{aligned}\frac{\partial^2 b}{\partial\alpha^2} &= \frac{2}{\alpha}\frac{\partial b}{\partial\alpha} - \frac{2b}{\alpha^2} - \frac{1}{1+K_{\mathrm{em}}^2}\left(\frac{\partial b}{\partial\alpha}\frac{\partial K_{\mathrm{em}}^2}{\partial\alpha} + b\frac{\partial^2 K_{\mathrm{em}}^2}{\partial\alpha^2}\right)\\ &\quad + \frac{b}{\left(1+K_{\mathrm{em}}^2\right)^2}\left(\frac{\partial K_{\mathrm{em}}^2}{\partial\alpha}\right)^2 + \frac{\alpha}{\varepsilon\mu(1+K_{\mathrm{em}}^2)}\left(2\frac{\partial K_{\mathrm{em}}^2}{\partial\alpha} + \alpha\frac{\partial^2 K_{\mathrm{em}}^2}{\partial\alpha^2}\right)\\ &\quad - \frac{\alpha^2}{\varepsilon\mu(1+K_{\mathrm{em}}^2)^2}\frac{\partial K_{\mathrm{em}}^2}{\partial\alpha}\left(\frac{\partial K_{\mathrm{em}}^2}{\partial\alpha} + \frac{eh}{\alpha^2 C}\right).\end{aligned} \tag{15}$$

An inflexion point of the function $\Delta(\alpha)$ can be determined using the following equality:

$$\frac{\partial^2 V_{\text{tem}}}{\partial\alpha^2} = \frac{\partial^2 V_{\text{new}}}{\partial\alpha^2}. \tag{16}$$

The parameter Δ in eq. (2) for the ME effect can be derived from the following boundary conditions: (i) mechanically free, electrically and magnetically open surface and (ii) mechanically free, electrically and magnetically closed surface. The realization of these boundary conditions is described in ref. [17]. However, the SH-SAW velocity V_{new} in eq. (5) is not the single characteristic for the above-mentioned two cases. Therefore, the following section describes two alternative SH-SAW characteristics for the boundary conditions.

4. The other SH-SAWs for the boundary conditions

For Case (i), the following velocity can be calculated:

$$V_{\text{PMESM}} = V_{\text{tem}} \left[1 - \left(\frac{K_{\text{em}}^2 - K_{\text{e}}^2}{1 + K_{\text{em}}^2} \right)^2 \right]^{1/2} \tag{17}$$

which was first obtained by Melkumyan [18]. So, V_{PMESM} in eq. (17) is called the piezomagnetic exchange surface Melkumyan wave or PMESM wave. In eq. (17), the well-known coefficient of the electromechanical coupling (CEMC) for purely piezoelectric materials is defined by $K_{\text{e}}^2 = e^2/\varepsilon C$.

For Case (ii), the following SH-SAW velocity is derived:

$$V_{\text{PEESM}} = V_{\text{tem}} \left[1 - \left(\frac{K_{\text{em}}^2 - K_{\text{m}}^2}{1 + K_{\text{em}}^2} \right)^2 \right]^{1/2} \tag{18}$$

which was also obtained by Melkumyan [18]. Also, V_{PEESM} in eq. (18) stands for the piezoelectric exchange surface Melkumyan wave or PEESM wave. The term $K_{\text{em}}^2 - K_{\text{m}}^2$ in eq. (18) represents a subtraction of K_{m}^2 for the purely piezomagnetic phase from K_{em}^2 of a coupled piezoelectromagnetic phase. In eq. (18), the coefficient of the magnetomechanical coupling (CMMC) for pure piezomagnetics is defined as $K_{\text{m}}^2 = h^2/\mu C$.

The following section provides some results of theoretical investigations and discussion of the characterization of piezoelectromagnetic composites.

5. Results and discussion

Figure 1 shows Δ vs. α for the composites listed in table 1. Note that refs [19,20] did not provide values of α for the composites. According to ref. [20], $BaTiO_3$–$CoFe_2O_4$ composite composed of piezoelectric $BaTiO_3$ inclusions and piezomagnetic (magnetostrictive) $CoFe_2O_4$ matrix represents a typical particulate composite. The material constants for the two-phase composite material such as (2–2) PZT-5H–Terfenol-D composite significantly differ from those for the single-phase bulk materials, namely PZT-5H and Terfenol-D. It is apparent that a composite possesses its own unique set of material constants $\{C, e, h, \varepsilon, \mu, \alpha\}$. Also, the material constants can strongly depend on the

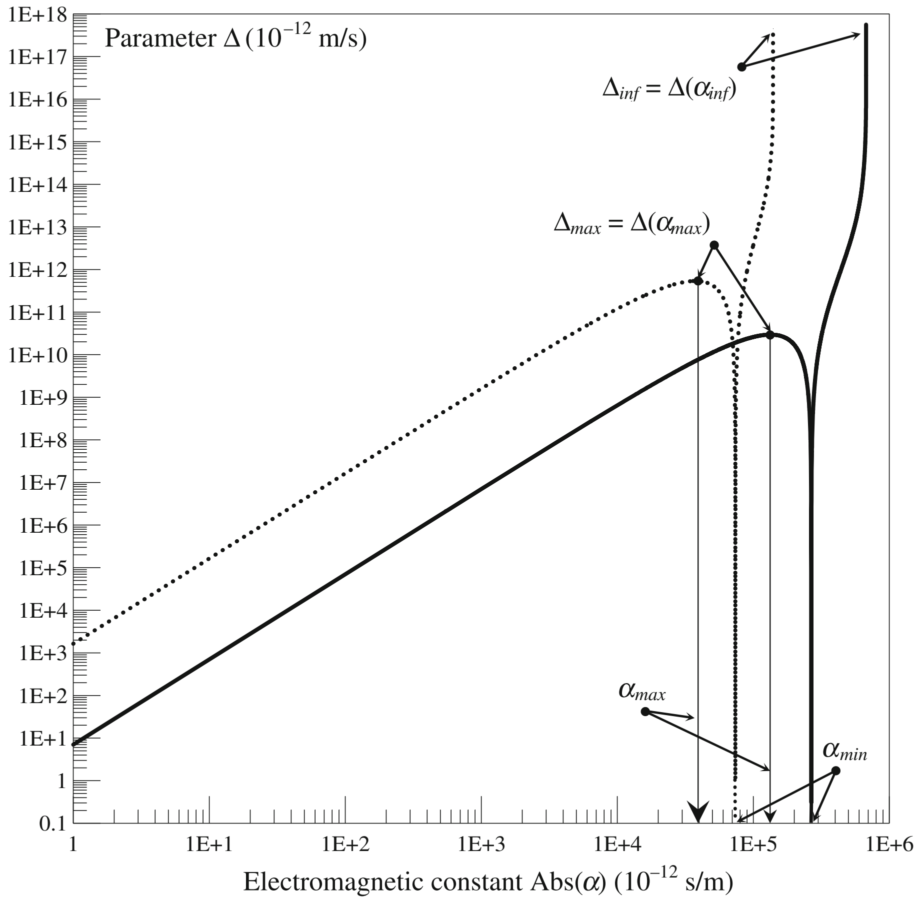

Figure 1. Δ (pm/s) vs. $|\alpha|$ (ps/m) for the composites listed in table 1. The solid line is for $BaTiO_3$–$CoFe_2O_4$. $\alpha_{\rm max}$, $\alpha_{\rm min}$, and $\alpha_{\rm inf}$ indicate the situations when Δ is maximum, minimum, and infinite, respectively.

structure of piezoelectromagnetics, for instance, 0–3, 1–3, 2–2 composites. It is expected that working modes, for instance, the direction of applied field or strain can also cause significant changes in the constants.

It is also possible to expand figure 1 by adding two curves of Δ vs. α, namely two suitable sample composites: the first with $\varepsilon\mu \sim 0.1\varepsilon\mu$ (PZT-5H–Terfenol-D) and the

Table 1. The material constants for the composites used.

Composite material	ρ (kg/m^3)	C, 10^{10} (N/m^2)	e (C/m^2)	h (T)	ε, 10^{-10} (F/m)	μ, 10^{-6} (N/A^2)	$\varepsilon\mu$, 10^{-16}
$BaTiO_3$–$CoFe_2O_4$	5730	4.40	5.80	275	56.4	81.0	4568.4
PZT-5H–Terfenol-D	8500	1.45	8.50	83.8	75.0	2.61	195.75

second with $\varepsilon\mu \sim 10\varepsilon\mu$ ($BaTiO_3$–$CoFe_2O_4$) because $\varepsilon\mu$ (PZT-5H–Terfenol-D) $\sim 0.1\varepsilon\mu$ ($BaTiO_3$–$CoFe_2O_4$). The resulting graph can serve as a classification tool for composites with $\alpha > 0$ which possess the unique behaviour with clear extreme points shown in figure 1 and can be naturally divided into two groups: (1) those with $0 < \alpha < \alpha_{\min}$ and (2) those with sqr($\alpha_{\min}$) $< \alpha^2 \sim \varepsilon\mu$ ($\alpha_{\min} < \alpha < \alpha_{\inf}$). For the first group $\alpha_{\max} < \alpha_{\min}$ giving $\Delta_{\max}$ which is probably the most interesting case in the group. $\Delta = \Delta_{\max}$ demonstrates the largest possible instability of the SH-BAW propagation in such composites. The second group can be restricted by the condition $\alpha < \alpha_{\inf}$, but not $\alpha < \infty$. It is possible to restrict this group because the sophisticated case of $\alpha > \alpha_{\inf}$ is not treated in this report. In this second group there is a very dramatic dependence of Δ on α, namely Δ is very quickly changed from zero to infinity. This cannot be good for some applications but can be good for others, for instance, for sensors. The problem is to reach such large values of α. It is expected that this natural boundary $\Delta(\alpha = \alpha_{\min}) = 0$ is not strict because the value of Δ is also very quickly changed just below the value of $\alpha_{\min}$. Therefore, it is possible to write $\alpha \sim \alpha_{\min}$ for the boundary between the two groups.

This classification is coupled with the wave properties contrary to the existing methods reviewed in refs [7,8] which evaluate only the single constant α for comparison of different composite materials: the higher is the value of α, the stronger is the magnetoelectric coupling. Moreover, the review paper [7] ignored the well-known bulk wave characteristic (3) of piezoelectromagnetic composites. According to figure 1, it is thought that the evaluation of α can be enough for the same composite, but not so when different composites are compared which have their own boundary values such as $\alpha = 0\,(\Delta = 0)$, $\alpha = \alpha_{\min}\,(\Delta = 0)$, and $\alpha^2 = \varepsilon\mu\,(\Delta = \infty)$. Indeed, it is not necessary to create unique composites with the constant α as high as possible to distinguish the SH-BAW and SH-SAW from each other (to get maximum Δ denoted by $\Delta_{\max}$ in figure 1) in the phase velocity measurements. For this purpose, it is necessary to collect composites with smallest values of $\varepsilon\mu$ and corresponding values of α^2 just below the values of $\varepsilon\mu$. Also, the maximum $\Delta = \Delta_{\max}$ for $\alpha = \alpha_{\max} < \alpha_{\min}$ can be significantly higher for composites with smaller $\varepsilon\mu$ (see figure 1 and table 1). Note that in this report only eq. (1) is treated for simplicity, but not the sophisticated cases of $\alpha^2 = \varepsilon\mu$ and $\alpha^2 > \varepsilon\mu$. It is clearly seen in expression (3) that the case of $\alpha^2 = \varepsilon\mu$ when $\alpha = \alpha_{\inf}$ creates infinite value for the SH-BAW V_{tem}. Indeed, this phenomenon can be seen only in piezoelectromagnetics, but not in pure piezoelectrics and pure piezomagnetics.

Table 1 lists the material properties of $BaTiO_3$–$CoFe_2O_4$ and PZT-5H–Terfenol-D composites. From table 1, the value of $\varepsilon\mu$ for $BaTiO_3$–$CoFe_2O_4$ is approximately one order larger than that for PZT-5H–Terfenol-D. Different positions of $\Delta_{\max}$ of the function $\Delta(\alpha)$ are shown in figure 1. As expected, the maximum of $\Delta(\alpha)$ for PZT-5H–Terfenol-D composite is approximately one order larger than that for $BaTiO_3$–$CoFe_2O_4$ composite. Note that $\Delta \rightarrow \infty$ occurs when $\alpha^2 \rightarrow \varepsilon\mu$. It is worth noticing that $V_{\mathrm{new}}(\alpha = \alpha_{\min}) = V_{\mathrm{tem}}$ indicates that no SH-SAWs can exist at a very large value of $\alpha = \alpha_{\min}$ where $\alpha_{\max}^2 < \alpha_{\min}^2 < \varepsilon\mu$. Indeed, $V_{\mathrm{new}} = V_{\mathrm{tem}}$ occurs when $b = 0$ for $K_{\mathrm{em}}^2 - eh/(\alpha C) = 0$ in eq. (6), and a large value of α around $\alpha_{\min}$ decreases Δ to very small values less than pm/s. However, large values of $\alpha^2 \sim \varepsilon\mu$ were not reported. On the other hand, $\alpha > \alpha_{\max}$ is still possible. The composites with $\alpha > \alpha_{\max}$ form a unique class and will be found in the future when suitable piezoelectromagnetic composites with $\varepsilon\mu < 10^{-16}$ and $\alpha \sim 10^{-9}$ are experimentally found. The materials with $\alpha \sim \alpha_{\min}$ can be classified as those in which the SH-SAW propagation is unstable, but the SH-BAW propagation is preferable due to the strong magnetoelectric effect. This cannot mean that the magnetoelectric effect is missing for $\alpha \sim \alpha_{\min}$ because $\alpha \neq 0$. This can be unlike the case of $\alpha = 0$. It is thought that both the cases of $\alpha \sim \alpha_{\min}$ and $\alpha \to 0$ can be experimentally compared. In general, for a small value of $\alpha < \alpha_{\max}$, Δ has an approximately linear dependence on α (see figure 1). In refs [21,22], the ME constant $\alpha = -3.6 \times 10^{-8}$ s/m for laminated $BaTiO_3$–$CoFe_2O_4$ composite was used. For $\alpha < 0$, Δ in eq. (2) has no extreme points and V_{new} ($\alpha < 0$) is never equal to V_{tem}. This means that for $\alpha < 0$, such SH-SAWs with V_{new} in eq. (5) can always exist.

Indeed, Δ is very small and can be even significantly smaller than mm/s. It is obvious that Δ represents an indicator of the instability of the SH-BAW V_{tem}. The simplest case of the instability [23] is the classical surface Bleustein–Gulyaev waves [24,25] in a purely piezoelectric (or piezomagnetic) monocrystal of class 6 *mm*. Also, Δ strongly depends on the magnetoelectric constant α. It is well-known that the SH-BAW V_{tem} can be unstable and reduce to the new SH-SAW V_{new} due to the coupling of the piezoelectromagnetic waves with the electrical and magnetic potentials. Probably, the SH–BAW and SH–SAW can independently propagate, and some experimental problems to measure the phase velocities (V_{ph}) with high accuracy occur. An improved optical method for measuring both the phase and group velocities described in [26] allows one to measure V_{ph} with an accuracy of ∼2 m/s. Also, SH-SAWs can easily be produced by electromagnetic acoustic transducers (EMATs) [27]. The EMATs have many advantages over traditional piezoelectric transducers [28,29]. These experimental tools of the SH-SAW (SH-BAW) propagation investigations in the piezoelectromagnetics can be used now itself. It is well-known that SH-SAWs can be used in sensors and for the non-destructive testing and evaluation of the piezoelectromagnetics. The recent book [30] discusses many possible applications of the materials possessing the magnetoelectric effect.

Figure 2 shows $\partial\Delta/\partial\alpha$ vs. α for the piezoelectromagnetic composite materials listed in table 1. $\partial\Delta/\partial\alpha$ is equal to zero at the extreme points of $\Delta(\alpha)$ in figure 1. The local minimum of the function $\Delta(\alpha = \alpha_{\min})$ corresponds to the least value on the whole domain. In figure 2, the extreme points of $\partial\Delta/\partial\alpha$ are observed at $\alpha = 15346.47 \times 10^{-12}$ s/m and 59717.73×10^{-12} s/m for PZT-5H–Terfenol-D composite, and at $\alpha = 54857.05 \times 10^{-12}$ s/m and $213187.84 \times 10^{-12}$ s/m for $BaTiO_3$–$CoFe_2O_4$ composite. These points are also seen in figure 3 which graphically shows $\partial^2\Delta/\partial\alpha^2$ vs. α. In figure 3, the extreme points of $\partial^2\Delta/\partial\alpha^2$ are observed at $\alpha = 36000 \times 10^{-12}$ s/m for PZT-5H–Terfenol-D and at $\alpha = 133980 \times 10^{-12}$ s/m for $BaTiO_3$–$CoFe_2O_4$. Indeed, the curves for the two composites are quite similar in nature. The difference occurs in the locations of the maximum and minimum values. In figure 2, both the solid and dashed lines have the extreme points. However, the solid line is very smooth for the scale used. Therefore, the arrows demonstrate the extreme points in figures 2 and 3. Also, figure 3 shows that $\alpha = 0$ in figure 2 cannot represent an extreme point because equality (10) cannot be fulfilled.

Following ref. [12], it is also possible to briefly discuss $\partial\Delta/\partial\alpha$ and $\partial^2\Delta/\partial\alpha^2$. The first derivative $\partial\Delta/\partial\alpha$ has dimension of $(\text{m/s})^2$ and represents some squares in the corresponding two-dimensional (2D) space of velocities. Therefore, extreme point values of $\partial\Delta/\partial\alpha$ naturally represent possible extreme values for the 2D-space. Analogically, $\partial^2\Delta/\partial\alpha^2$ has dimensions of $(\text{m/s})^3$ and the extreme point values represent extreme volumes for the 3D-space. Note that $\partial\Delta/\partial(\alpha^2)$ and $\partial\Delta/\partial(\varepsilon\mu)$ have the same dimension, but

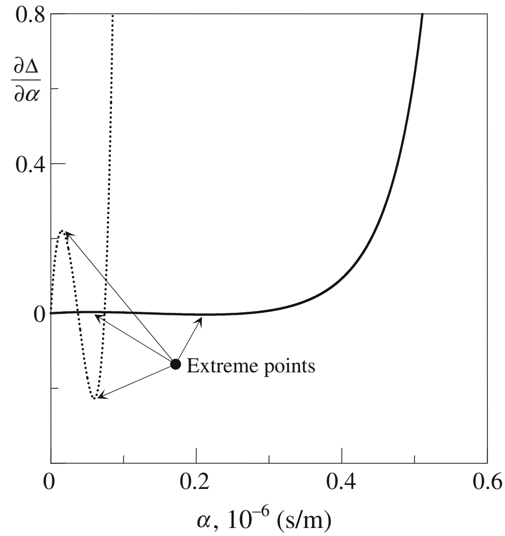

Figure 2. $(\partial\Delta/\partial\alpha) \times 10^{12}$ $(m/s)^2$ vs. α for the composites listed in table 1. The solid line is for $BaTiO_3$–$CoFe_2O_4$. The arrows show the positions of the extreme points for which the exact values are given in §5.

different 3D-spaces of velocities. Indeed, $\varepsilon\mu \gg a^2$ for many cases, and one can investigate the dependence of Δ on $\varepsilon\mu$. In this case one deals with the extended set of material constants such as $\{\rho, C, e, h, \varepsilon, \mu, \alpha, \varepsilon\mu\}$ because the additional parameter $\varepsilon\mu$ defined in eq. (1) represents $f(\varepsilon, \mu) = \varepsilon\mu$.

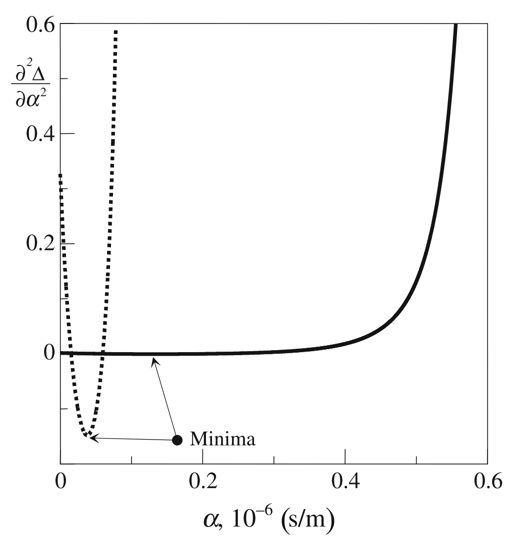

Figure 3. $(\partial^2\Delta/\partial\alpha^2) \times 10^{12}$ $(m/s)^3$ vs. α for the composites listed in table 1. The solid line is for $BaTiO_3$–$CoFe_2O_4$. The arrows show the positions of the minima, for which the exact values are given in §5.

6. Conclusion

This report acquainted researchers with the wave characteristic of the difference between the velocity V_{tem} of the SH-BAW coupled with both the electrical and the magnetic potentials and the velocity V_{new} of seven new SH-SAWs recently discovered by the author in ref. [13]. It can be used for characterizing piezoelectromagnetic (composite) materials. The evaluation of the wave characteristic can be useful together with the evaluation of measured value of α, because Δ depends on all the material constants of piezoelectromagnetics. It also strongly depends on the structure of piezoelectromagnetics (0–3, 1–3, 2–2 composites) and working mode, for instance, the direction of applied field or strain. It was also discussed that the single characteristic such as V_{new} can be used instead of the two alternative characteristics V_{PEESM} and V_{PMESM} for the two sets of boundary conditions. The relatively complex dependence $\Delta(\alpha)$ as an indicator of the instability of the SH-BAW V_{tem} was also discussed. It was shown that the sign of α can dramatically change the dependence of Δ on α because the extreme points can exist only for $\alpha > 0$. Also, the first and second derivatives of Δ with respect to α were graphically investigated.

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References

[1] J Ryu, Sh Priya, K Uchino and H-E Kim, *J. Electroceramics* **8**, 107 (2002)
[2] C L Zhang, W Q Chen, J Y Li and J S Yang, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **56**, 1046 (2009)
[3] M Zeng, S W Or and H L W Chan, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **57**, 2147 (2010)
[4] Sh-X Dong, J-F Li and D Viehland, *Trans. Ultrason. Ferroelectr. Freq. Control* **51**, 793 (2004)
[5] Ü Özgür, Ya Alivov and H Morkoç, *J. Mater. Sci. Mater. Electron.* **20**, 911 (2009)
[6] M Avellaneda and G Harshe, *J. Intelligent Mater. Syst. Struct.* **5**, 501 (1994)
[7] G Srinivasan, *Ann. Rev. Mater. Res.* **40**, 153 (2010)
[8] M Fiebig, *J. Phys. D: Appl. Phys.* **38**, R123 (2005)
[9] A R Annigeri, N Ganesan and S Swarnamani, *Smart Materials and Structures* **15**, 459 (2006)
[10] J Aboudi, *Smart Materials and Structures* **10**, 867 (2001)
[11] P S Neelakanta, *Handbook of electromagnetic materials* (CRC Press, New York, 1995)
[12] A A Zakharenko, Analytical investigation of surface wave characteristics of piezoelectromagnetics of class 6 mm, *International Scholarly Research Network (ISRN) Applied Mathematics* (India) Vol. 2011, DOI: 10.5402/2011/408529 (free on-line access)
[13] A A Zakharenko, *Propagation of seven new SH-SAWs in piezoelectromagnetics of class 6 mm*, Saarbruecken–Krasnoyarsk, LAP LAMBERT Academic Publishing GmbH & Co. KG, 76 pages, 2010, ISBN: 978-3-8433-6403-4
[14] A Ishikawa, T Tanaka and S Kawata, *Phys. Rev. Lett.* **95**, 237401 (2005)

[15] T F Gundogdu, I Tsiapa, A Kostopoulos, G Konstantinidis, N Katsarakis, R S Penciu, M Kafesaki, E N Economou, Th Koschny and C M Soukoulis, *Appl. Phys. Lett.* **89**, 084103 (2006)

[16] C Potel, S Devolder, A Ur-Rehman, J-F Belleval, J-M Gherbezza, O Leroy and M Wevers, *J. Appl. Phys.* **86**, 1128 (1999)

[17] V I Al'shits, A N Darinskii and J Lothe, *Wave Motion* **16**, 265 (1992)

[18] A Melkumyan, *Int. J. Solids and Structures* **44**, 3594 (2007)

[19] B-L Wang and Y-W Mai, *Int. J. Solids and Structures* **44**, 387 (2007)

[20] T J-Ch Liu and Ch-H Chue, *Composite Structures* **72**, 254 (2006)

[21] J Y Li, *Int. J. Eng. Sci.* **38**, 1993 (2000)

[22] X-F Li, *Int. J. Solids and Structures* **42**, 3185 (2005)

[23] Yu V Gulyaev, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **45**, 935 (1998)

[24] J L Bleustein, *Appl. Phys. Lett.* **13**, 412 (1968)

[25] Yu V Gulyaev, *Sov. Phys. J. Exp. Theoret. Phys. Lett.* **9**, 37 (1969)

[26] E A Kolosovskii, A V Tsarev and I B Yakovkin, *Acoustical Phys. Rep.* (Moscow) **44**, 793 (1998)

[27] R Ribichini, F Cegla, P B Nagy and P Cawley, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **57**, 2808 (2010)

[28] R B Thompson, Physical principles of measurements with EMAT transducers, in: *Physical acoustics* edited by W P Mason and R N Thurston (Academic Press, New York, NY, 1990) Vol. 19, pp. 157–200

[29] M Hirao and H Ogi, *EMATs for science and industry: Non-contacting ultrasonic measurements* (Kluwer Academic, Boston, MA, 2003)

[30] A A Zakharenko, *Seven new SH-SAWs in cubic piezoelectromagnetics*, Saarbruecken–Krasnoyarsk, LAP LAMBERT Academic Publishing GmbH & Co. KG, 172 pages, 2011, ISBN: 978-3-8473-3485-9