PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 79, No. 2
August 2012
pp. 199–209

Parallel decoherence in composite quantum systems

M DUGIĆ[1,*] and J JEKNIĆ-DUGIĆ[2]
[1]Department of Physics, Faculty of Science, Kragujevac, Serbia
[2]Department of Physics, Faculty of Science and Mathematics, Niš, Serbia
*Corresponding author. E-mail: dugic@open.telekom.rs

MS received 3 October 2011; revised 27 January 2012; accepted 8 February 2012

Abstract. For the standard quantum Brownian motion (QBM) model, we point out the occurrence of simultaneous (parallel), mutually irreducible and autonomous decoherence processes. Besides the standard Brownian particle, we show that there is at least another system undergoing the dynamics described by the QBM model. We do this by selecting the two mutually irreducible, global structures (decompositions into subsystems) of the composite system of the QBM model. The generalization of this observation is a new, challenging task in the foundations of the decoherence theory. We do not place our findings in any interpretational context.

Keywords. Quantum decoherence; quantum Brownian motion; quantum structure; entanglement relativity.

PACS Nos 03.65.Yz; 03.65.Ud; 03.65.Ta

1. Introduction

"In particular, one issue which has been often taken for granted is looming big, as a foundation of the whole decoherence program. It is the question of what are the 'systems' which play such a crucial role in all the discussions of the emergent classicality. (...) [A] compelling explanation of what are the systems – how to define them given, say, the overall Hamiltonian in some suitably large Hilbert space – would be undoubtedly most useful." (p. 1820 in ref. [1]).

In this paper, we consider the two specific structures (decompositions, partitions into subsystems) of the standard quantum Brownian motion (QBM) set-up [2–5] and we obtain QBM effect for both the structures considered. The structures are mutually irreducible (i.e. can *not* be obtained from each other by decomposing, grouping or permutations of the constituent subsystems) and global (do *not* have even a single degree of freedom in common). The structures are mutually linked by the linear canonical transformations (LCTs) and thus are dynamically independent, autonomous structures of the one and the same composite system of the QBM model that cannot be obtained from each other via the 'coarse graining' [6] operation.

The LCT is a universal physical method. Already the unitary evolution regroups or separates the constituent subsystems, thus locally changing the structure of a composite system. In quantum decoherence, the LCTs are sometimes used to ease the calculation [5], while (kinematically) regrouping or decomposing the subsystems may shed new light on the mechanism of decoherence [7,8]. A change in the environmental degrees of freedom reveals some subtleties such as the 'system-size' dependence of decoherence [5,9] and can help in distinguishing the robust (the preferred 'pointer basis' [4,5,10,11]) states for the open system [12–14]. However, these models and considerations refer to the local structures that share the degrees of freedom or can be reduced to each other. This makes the structures considered mutually dynamically coupled or dependent, which is not our objective.

Paradigmatic to our considerations is the hydrogen atom (HA) model. The quantum theory of the hydrogen atom relies on the transformations of the electron's (e) and the proton's (p) degrees of freedom to introduce the atom centre-of-mass (CM) and the 'relative position (R)' degrees of freedom. Due to the absence of coupling between CM and R, one obtains the separation of the variables and the exact solution of the atomic internal energy and eigenstates. The two structures of HA, e + p and CM + R, are mutually irreducible and global like those of the QBM set-up we discuss below.

In §2, we derive our main result on the parallel decoherence: at variance with the standard wisdom, we point out that a composite quantum system can be described by (may host) the different, simultaneously existing and mutually independent quasiclassical (global) structures. In §3 we generalize our considerations and we emphasize that the 'parallel decoherence' launches a new task in the foundations of the decoherence theory. In general, this task can be formidable yet possibly of a wider scientific interest. Section 4 is Discussion, where we emphasize: as long as one can rely on our model-dependent finding, if the standard decoherence program provides the 'appearance of a Classical World', our results suggest the 'appearance of the Classical Worlds'. Nevertheless, we do not enter into any interpretational details. Section 5 is Conclusion.

2. Parallel decoherence in the QBM set-up

"Note that decoherence derives from the presupposition of the existence and the possibility of a division of the world into 'system(s)' and 'environment'." (p. 83 in ref. [11]).

By 'decoherence' we mean the 'environment-induced selection' of the preferred ('pointer basis') states of an open quantum system [4,5,10,11,15]. The effect of decoherence refers to certain (typically 'collective') observables (formally subsystems) of a larger (open) system S in unavoidable interaction with its environment E. The interaction in S + E system and its strength determine a preferred set of (not necessarily orthogonal) pointer basis states of the open system S that bear robustness – these states exhibit the 'least' response to the environmental influence.

For a sufficiently large environment, the robustness of the pointer basis states gives rise to the quasidiagonal form of the system's state $\hat{\rho}_S$ (in the pointer basis representation) and the effect of decoherence appears to be irreversible in practice [4,16]. This apparent irreversibility is the physical basis of both the approximate quasiclassical dynamics of the decohered system (a subsystem of S) as well as the classical-information contents of the open system S.

This typical attitude of the decoherence theory in introducing a composite system $\mathcal{C}$ by grouping the actual systems, S and E ($\mathcal{C} = \mathrm{S} + \mathrm{E}$) is at variance with our concern here. Actually, we admit that the existence of a (practically closed) composite system $\mathcal{C}$ whose structure appearing through the alternative decompositions into subsystems is yet to be defined. The different structures are mutually linked by some LCTs and we investigate the occurrence of decoherence for some alternative structures.

In some abstract terms, our task can be readily formulated: whether or not, the LCTs can provide the occurrence of decoherence for an alternative structure of a composite system? While this is the subject of the next section, here we just emphasize: as the occurrence of decoherence is directly related to the interaction term in the Hamiltonian of the composite system [15], the LCTs should preserve the desired characteristics [15] of the interaction even for the alternative structure. In this section, we are concerned with the concrete, QBM model of the composite system.

In order to emphasize our basic observation, we first deal with the simplified model of a pair of one-dimensional systems S, E linearly interacting through their respective position operators $\hat{x}_{\mathrm{S}}$ and $\hat{x}_{\mathrm{E}}$.

The Hamiltonian is given by

$$\hat{H} = \frac{\hat{p}_{\mathrm{S}}^2}{2m_{\mathrm{S}}} + \frac{\hat{p}_{\mathrm{E}}^2}{2m_{\mathrm{E}}} + \frac{m_{\mathrm{E}}\omega^2}{2}\hat{x}_{\mathrm{E}}^2 - C\hat{x}_{\mathrm{S}}\hat{x}_{\mathrm{E}} \equiv \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{E}} + \hat{H}_{\mathrm{S+E}}. \tag{1}$$

The standard LCT that introduces the centre-of-mass (CM) and the 'relative positions' (R) observables ($\hat{X}_{\mathrm{CM}} = (m_{\mathrm{S}}\hat{x}_{\mathrm{S}} + m_{\mathrm{E}}\hat{x}_{\mathrm{E}})/(m_{\mathrm{S}} + m_{\mathrm{E}})$ and $\hat{\rho}_{\mathrm{S}} = \hat{x}_{\mathrm{S}} - \hat{x}_{\mathrm{E}}$, respectively) give rise to

$$\begin{aligned}\hat{H} =& \frac{\hat{p}_{\mathrm{CM}}^2}{2(m_{\mathrm{S}} + m_{\mathrm{E}})} + c_1\hat{X}_{\mathrm{CM}}^2 + \frac{\hat{p}_{\mathrm{R}}^2}{2\mu} + c_2\hat{\rho}_{\mathrm{R}}^2 \\ &- c_3\hat{X}_{\mathrm{CM}}\hat{\rho}_{\mathrm{R}} \equiv \hat{H}_{\mathrm{CM}} + \hat{H}_{\mathrm{R}} + \hat{H}_{\mathrm{CM+R}},\end{aligned} \tag{2}$$

$c_1 = m_{\mathrm{E}}\omega^2/2 - C$, $c_2 = m_{\mathrm{S}}\mu\omega^2/2(m_{\mathrm{S}}+m_{\mathrm{E}}) + C\mu/(m_{\mathrm{S}}+m_{\mathrm{E}})$, $c_3 = C(m_{\mathrm{E}}-m_{\mathrm{S}})/(m_{\mathrm{S}}+m_{\mathrm{E}}) + \mu\omega^2$ and $\mu = m_{\mathrm{S}}m_{\mathrm{E}}/(m_{\mathrm{S}} + m_{\mathrm{E}})$ is the reduced mass, with the constraint $C < m_{\mathrm{E}}\omega^2/2$. This way, the composite system $\mathcal{C} = \mathrm{S} + \mathrm{E}$ is formally redefined to introduce the alternative structure defined by the 'new' subsystems CM and R. Certainly, $\mathrm{S} + \mathrm{E} = \mathcal{C} = \mathrm{CM} + \mathrm{R}$ and the Hamiltonian eqs (1) and (2) is the composite system's Hamiltonian, $\hat{H} \equiv \hat{H}_{\mathrm{C}}$.

The two structures, S + E and CM + R, are mutually irreducible (cannot be obtained from each other by decomposing or grouping the constituent subsystems) and 'global' (not having common degrees of freedom). The models are formally similar: the interaction terms are exactly of the same form that distinguishes the position-eigenstates as the candidate pointer basis states [15] for both S and CM. For the many-particles environment (see below), both the open systems can be described by the following master equation [4,5]:

$$\imath\hbar\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = [\hat{H}, \hat{\rho}] - \imath\Lambda[\hat{x}, [\hat{x}, \hat{\rho}]], \tag{3}$$

which models the position-observable ($\hat{x}$) measurement. Therefore, the models, albeit simplified, suggest the possible simultaneous and mutually independent existence of the preferred states for the open systems of both decompositions.

Let us extend the model eq. (1) by identifying $\hat{x}_{\mathrm{E}}$ with the collective variable $\sum_i \kappa_i \hat{x}_{Ei}$ for the harmonic bath (E). Then the model eq. (1) resembles the Caldeira–Leggett model [2] for the quantum Brownian motion (QBM). For this model, the occurrence of decoherence is a well-established result [2–5]. Physically, the bath E acts as a 'quantum apparatus' measuring the S's position-observable $\hat{x}_{\mathrm{S}}$. Interestingly enough, for the initial Gaussian states for S, the occurrence of decoherence for the linear position-observables coupling (also for eqs (1) and (2)) is largely independent of the details, such as the appearance/non-appearance and the kind of external field $V(\hat{x}_{\mathrm{S}})$ in the model, the strength of interaction, the spectral density or the bath's temperature as well as the presence of (classical or quantum) correlations in the initial state of the composite system [3,5]. The Gaussian states (that include the standard 'coherent states') appear as the approximate pointer basis.

Let us formally consider the Caldeira–Leggett model:

$$\begin{aligned}\hat{H} &= \frac{\hat{p}_{\mathrm{S}}^2}{2m_{\mathrm{S}}} + V(\hat{x}_{\mathrm{S}}) + \sum_i \left(\frac{\hat{p}_{\mathrm{E}i}^2}{2m_i} + \frac{m_i\omega_i^2\hat{x}_{\mathrm{E}i}^2}{2}\right) \pm \hat{x}_{\mathrm{S}} \sum_i \kappa_i \hat{x}_{\mathrm{E}i} \\ &\equiv \hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{E}} + \hat{H}_{\mathrm{S+E}},\end{aligned} \tag{4}$$

where the index i enumerates the environmental 'particles', and the $\pm$ sign is in accord with the variations of the model in the literature. The Hamiltonian eq. (4) generates unitary dynamics for the initially separable state of the composite system, $\hat{\rho}_{\mathrm{C}} = \hat{\rho}_{\mathrm{S}} \otimes \hat{\rho}_{\mathrm{Eth}}$, where $\hat{\rho}_{\mathrm{Eth}}$ denotes the thermal-equilibrium state of the environment E. So, we consider the standard, linear QBM set-up with the separable initial state of the composite system $\mathcal{C}$.

Now, we apply the standard LCTs introducing the centre-of-mass (CM) and the relative-position variables for the whole composite system $\mathcal{C}$, where the set of relative positions is collectively denoted as the subsystem R, $\{\hat{\rho}_{\mathrm{R}\alpha}\}$. Then, the inverse transformations give $\hat{x}_i = \hat{X}_{\mathrm{CM}} + \sum_\alpha \omega_{\alpha i}\hat{\rho}_{\mathrm{R}\alpha}$ and $\hat{x}_1 \equiv \hat{x}_{\mathrm{S}}$, $\omega_{1i} \equiv \omega_{\mathrm{S}i}$; ωs can be positive/negative real constants.

For the 'new' structure CM + R one obtains

$$\begin{aligned}\hat{H} &= \frac{\hat{P}_{\mathrm{CM}}^2}{2M} + \frac{1}{2}M\Omega_{\mathrm{CM}}^2\hat{X}_{\mathrm{CM}}^2 + \sum_\alpha \left(\frac{\hat{p}_{\mathrm{R}\alpha}^2}{2\mu_\alpha} + \frac{1}{2}\mu_\alpha\nu_\alpha^2\hat{\rho}_{\mathrm{R}\alpha}^2\right) \\ &\quad + \hat{V}_{\mathrm{R}} \pm \hat{X}_{\mathrm{CM}} \sum_\alpha \sigma_\alpha \hat{\rho}_{\mathrm{R}\alpha}\end{aligned} \tag{5}$$

for the two relevant models, of the free particle and of the harmonic oscillator as the open system S. Introducing the total (CM) mass M, the standard reduced masses μ_α and the 'mass polarization' constants $C_{\alpha\alpha'} = m_{\alpha+1}m_{\alpha'+1}/M$, the constants appearing in eq. (5) are as follows:

(i) For the *free particle* ($V(\hat{x}_S) = 0$): $M\Omega_{CM}^2/2 = \sum_i (\pm\kappa_i + m_i\omega_i^2/2)$, $\mu_\alpha \nu_\alpha^2/2 = \pm\omega_{\alpha S}\sum_i \kappa_i\omega_{\alpha i} + \sum_i m_i\omega_i^2\omega_{\alpha i}^2/2$, and $\sigma_\alpha = \sum_i(\kappa_i\omega_{\alpha i} + \kappa_i\omega_{\alpha S} + m_i\omega_i^2\omega_{\alpha i})$. The internal interaction term $\hat{V}_R = \sum_{\alpha\neq\alpha'}[C_{\alpha\alpha'}\hat{p}_{R\alpha}\hat{p}_{R\alpha'}/\mu_\alpha\mu_{\alpha'} + (\Omega_{\alpha\alpha'} + \omega_{\alpha S}\Omega_{\alpha'})\hat{\rho}_{R\alpha}\hat{\rho}_{R\alpha'}]$; $\Omega_\alpha = \sum_i \kappa_i\omega_{\alpha i}$ and $\Omega_{\alpha\alpha'} = \sum_i m_i\omega_i^2\omega_{\alpha i}\omega_{\alpha' i}/2$. The conditions of positivity, $M\Omega_{CM}^2/2 > 0$ and $\mu_\alpha\nu_\alpha^2/2 > 0$, exhibit the subtleties concerning the choice of the physically interesting LCT.

(ii) For the *harmonic oscillator* ($V(\hat{x}_S) = m_S\omega_S^2\hat{x}_S^2/2$): the harmonic part for S adds the terms appearing by virtue of the inverse transformation (see above): $\hat{x}_S = \hat{X}_{CM} + \sum_\alpha \omega_{\alpha S}\hat{\rho}_{R\alpha}$. Particularly, the harmonic term for S obtains the form: $m_S\omega_S^2\hat{X}_{CM}^2/2 + \sum_\alpha m_S\omega_S^2\omega_{\alpha S}^2\hat{\rho}_{R\alpha}^2/2 + \sum_{\alpha\neq\alpha'} m_S\omega_S^2\omega_{\alpha S}\omega_{\alpha' S}\hat{\rho}_{R\alpha}\hat{\rho}_{R\alpha'}/2 + \hat{X}_{CM}\sum_\alpha m_S\omega_S^2\omega_{\alpha S}\hat{\rho}_{R\alpha}$. By adding this sum to the Hamiltonian for the free particle (Case (i)) one obtains the Hamiltonian of the general form eq. (5).

To this end, it is essential to note that the two structures of the composite system, S + E and CM + R, do not follow from each other via decomposing, grouping or the permutations of subsystems (degrees of freedom) operations. As the two structures do not have even a single degree of freedom in common, they are 'global', as distinct from the 'local' structures emphasized in Introduction [5–8,11–13]. The two open systems, S and CM, are one-dimensional systems and cannot be decomposed – they do not posses any structure of their own. So, the two structures are mutually irreducible and their unitary (Schrödinger) dynamics are mutually independent and autonomous.

Compare the two models, eqs (4) and (5), that equally apply to both the Cases (i) and (ii). The simple exchange of CM and R in eq. (5) by S and E gives a formal variation of eq. (4). Both open systems (S and CM) are one-dimensional. Then, by virtue of the LCTs, the respective environments (E and R) bear the same number of degrees of freedom – the same complexity and ability to provide 'genuine decoherence'. Both environments are harmonic-oscillator systems. The interaction terms are of exactly the same form. Therefore the related spectral densities are also of the same form.

The differences come about as follows. First for the Hamiltonian forms that differ in the values of the parameters (the masses and the characteristic frequencies of the oscillators), there is a new harmonic term for CM system relative to the system S, and there is the (small-norm) term $\hat{V}_R$ involving the couplings for the new-environment's (R's) oscillators. Second, the variable transformations typically induce a change in quantum state of the composite system: if a state is separable for one structure, it is typically entangled for the alternative structure – the entanglement relativity [17–24].

Nevertheless, as we show in Appendix, all these distinctions do not change the conclusion presented for the simple model. Actually, in Appendix we show that the composite system's Hamiltonian $\hat{H}$ generates for the fixed initial state $\hat{\rho}_C$ the two, simultaneously (in parallel) occurring and mutually irreducible and independent decoherence processes for the two open systems S and CM. While the details regarding the occurrence of decoherence (such as the decoherence time, the recurrence time or the state fluctuations) may be different for different structures, one can say: just like the open system S, the open system CM is a 'Brownian particle' for its respective structure. As the two decoherence processes unfold simultaneously (in parallel) and are mutually irreducible and independent (autonomous), we emphasize our main result as follows: the isolated composite system $\mathcal{C}$ hosts at least two simultaneously and mutually independently occurring (the parallel) decoherence processes that amount to the approximate quasiclassical behaviour of the subsystems, i.e. of the two, mutually irreducible and dynamically autonomous 'Brownian particles', S and CM.

3. Some general considerations

Let us present our considerations in more abstract terms.

The LCTs can be formally presented in a compact form as

$$\begin{aligned}\hat{X}_{\mathrm{S}'\alpha} &= X(\hat{x}_{\mathrm{S}i}, \hat{p}_{\mathrm{S}i}; \hat{\xi}_{\mathrm{E}j}, \hat{\pi}_{\mathrm{E}j}), \quad \hat{P}_{\mathrm{S}'\alpha} = P(\hat{x}_{\mathrm{S}i}, \hat{p}_{\mathrm{S}i}; \hat{\xi}_{\mathrm{E}j}, \hat{\pi}_{\mathrm{E}j}) \\ \hat{\Xi}_{\mathrm{E}'\beta} &= \Xi(\hat{x}_{\mathrm{S}i}, \hat{p}_{\mathrm{S}i}; \hat{\xi}_{\mathrm{E}j}, \hat{\pi}_{\mathrm{E}j}), \quad \hat{\Pi}_{\mathrm{E}'\beta} = \Pi(\hat{x}_{\mathrm{S}i}, \hat{p}_{\mathrm{S}i}; \hat{\xi}_{\mathrm{E}j}, \hat{\pi}_{\mathrm{E}j}).\end{aligned} \tag{6}$$

In eq. (6) appear the (continuous) position and the momentum observables of the subsystems, which are indicated by the indices to the observables appearing in eq. (6). The LCTs do not assume any constraints on the degrees of freedom and the number of the degrees of freedom is conserved; the tensor-product structures of the $\mathcal{C}$'s Hilbert state fulfill the equality $\otimes_{i=1}^{\nu_{\mathrm{S}}} H_{\mathrm{S}i} \otimes_{j=1}^{\nu_{\mathrm{E}}} H_{\mathrm{E}j} = \otimes_{p=1}^{\nu_{\mathrm{S}'}} H_{\mathrm{S}'p} \otimes_{q=1}^{\nu_{\mathrm{E}'}} H_{\mathrm{E}'q}$, while $\nu_{\mathrm{S}} + \nu_{\mathrm{E}} = \nu_{\mathrm{S}'} + \nu_{\mathrm{E}'}$. The LCTs are global if the related structures, $\mathcal{S} = \{\hat{x}_{\mathrm{S}i}, \hat{\xi}_{\mathrm{E}j}\}$ and $\mathcal{S}' = \{\hat{X}_{\mathrm{S}'\alpha}, \hat{\Xi}_{\mathrm{E}'\beta}\}$, do not have a single degree of freedom in common, $\mathcal{S} \bigcap \mathcal{S}' = 0$, and both the two structures obey the Schrödinger law and are dynamically mutually independent. Further on, as in §2, we consider the mutually irreducible structures.

In general terms, our task refers to a pair of (global) decompositions, $\mathrm{S}+\mathrm{E}$ and $\mathrm{S}'+\mathrm{E}'$, of a composite system $\mathcal{C}$, whose Hamiltonian, $\hat{H}_{\mathrm{C}}$, can be written as

$$\hat{H}_{\mathrm{S}} + \hat{H}_{\mathrm{E}} + \hat{H}_{\mathrm{SE}} = \hat{H}_{\mathrm{C}} = \hat{H}_{\mathrm{S}'} + \hat{H}_{\mathrm{E}'} + \hat{H}_{\mathrm{S}'\mathrm{E}'}. \tag{7}$$

Due to the interaction $\hat{H}_{\mathrm{SE}}$, the entanglement in $\mathrm{S}+\mathrm{E}$ is expected to be of the general form $\sum_i \alpha_i |\psi_i\rangle_{\mathrm{S}} |\chi_i\rangle_{\mathrm{E}}$, e.g. an instantaneous Schmidt form of state $|\Phi\rangle_{\mathrm{C}}$ of the composite system $\mathcal{C}$. On the other hand, for the separable interaction [15] $\hat{H}_{\mathrm{S}'\mathrm{E}'}$, one obtains for the same state $|\Phi\rangle_{\mathrm{C}}$ (in the same instant of time) another form $\sum_j \beta_j |\phi_j\rangle_{\mathrm{S}'} |\varphi_j\rangle_{\mathrm{E}'}$, still with the equality:

$$\sum_i \alpha_i |\psi_i\rangle_{\mathrm{S}} |\chi_i\rangle_{\mathrm{E}} = |\Phi\rangle_{\mathrm{C}} = \sum_j \beta_j |\phi_j\rangle_{\mathrm{S}'} |\varphi_j\rangle_{\mathrm{E}'}. \tag{8}$$

Independently of their physical contents, the equalities like eq. (8) emphasize a challenging mathematical task. In the formal mathematical context, deriving the LHS (RHS) of eq. (8) from the RHS (LHS) of eq. (8) is an open issue weakly investigated so far. For certain simple models, one can show [21,22] that a state given in a separable form for the decomposition $\mathrm{S}+\mathrm{E}$ bears quantum entanglement regarding another decomposition $\mathrm{S}'+\mathrm{E}'$; the decompositions being related by certain LCTs. As our dynamical arguments can hardly cover these methodological gaps for obtaining exact (kinematical) forms of the $\mathcal{C}$'s state for different decompositions, we do not report any progress in this regard.

In the respective position-representations of eq. (6), eq. (8) reads (up to a constant) as

$$\sum_i \alpha_i \psi_i(x_{\mathrm{S}m})\chi(X_{\mathrm{E}n}) = \sum_j \beta_j \phi_j(\xi_{\mathrm{S}'p})\varphi_j(\Xi_{\mathrm{E}'q}). \tag{9}$$

Of course, the presence of entanglement is not sufficient for the occurrence of decoherence. But decoherence requires entanglement. Interestingly enough, the occurrence of decoherence for the QBM model of §2 is quite independent of the initial correlations in the composite system ([3,5], and references therein).

Now, one may pose the following question: Does the parallel decoherence apply to a general system? In answering this question, we emphasize: The above analysis bears some subtlety as the global LCTs can completely change the character of the model of the composite system. Then, in general, the task of theoretically predicting decoherence may be challenging.

To see this, we refer to the simple yet paradigmatic models of the hydrogen atom, and of the QBM model of §2. Let us first emphasize that the kinetic terms are of the same form for every subsystem. However, the external fields for the constituent subsystems as well as their mutual interactions nontrivially change. For the hydrogen atom, as it is well-known, the (Coulomb) interaction present for the $e + p$ decomposition disappears in the CM + R decomposition – it becomes the external (Coulomb) field for the 'relative particle' (R). Regarding the QBM model (§2), both the external fields as well as the interaction for the subsystems can change, relative to the original decomposition. Both CM and R are 'placed' in the quadratic external potentials. If such potentials are present in the original model, then the characteristic frequencies are changed. The interaction in CM + R decomposition is formally the same as for S + E decomposition, yet with different strength. These examples illustrate the following general rule: all (but the kinetic) terms of the Hamiltonian for a decomposition can contribute to all (but the kinetic) the terms of the Hamiltonian for an alternative decomposition.

Of course, the changes in the form of interaction and in its strength [15] provide different backgrounds for the possible occurrence of decoherence for different global structures. The change in character and strength of interaction can give rise to a change in the approximations/physical assumptions for the alternative decomposition(s), e.g., if the 'weak coupling' and/or the 'rotating wave' (the 'secular') approximations [5] are valid for the original decomposition, this need not be the case for an alternative global decomposition of the composite system; similarly, the 'spectral density' [5] can change for different decompositions. On the other hand, even if the 'original environment' is in thermal equilibrium, the 'new environment' need not be even stationary. Finally, the global LCT can change the character of the quantum state of the composite system: a separable state for one decomposition typically obtains entangled form for some alternative decomposition [17–24]. Then, a completely positive dynamics for one decomposition (S + E) can become non-completely positive dynamics for another structure (S′ + E′).

Therefore, we answer the above-posed question as follows: investigating the parallel occurrence of decoherence is a new challenging task in the foundations of the decoherence theory. Bearing in mind the details that may determine the dynamics of open systems, the occurrence of decoherence for the alternative structures, in general, cannot be guaranteed. Rather, it should be separately considered for a class of similar models of open systems and their environments.

4. Discussion

Global and mutually irreducible structures (partitions into subsystems) of an isolated composite system are crucial for our results and observations. While decoherence regarding the local and/or mutually reducible structures may bear some subtlety yet to be discovered, the parallel decoherence as introduced in this article is the characteristic of the mutually global and irreducible structures. The 'parallel decoherence' means simultaneous, mutually non-intersecting (dynamically independent) unfolding of decoherence for different structures. So, at variance with the standard view, a composite system may host different, mutually independent global quasiclassical structures. The concept of the (global) quasiclassical structure [4,5,10,11] is relative. We believe that this relativity of 'structure' may enrich the classical concept of complexity [25] and may be of interest for a number of disciplines, e.g., bearing in mind eq. (6), a comparison between the two open systems, S and CM, is vague, not only on the intuitive ground. Actually, the two sets of states appearing in eq. (9), $\{\psi_i(x_{\mathrm{S}m})\}$ and $\{\phi_j(\xi_{\mathrm{S}'p})\}$, do not belong to the same probability space, neither e.g. $\int |\phi_j(\xi_{\mathrm{S}'p})|^2 \Pi_n \mathrm{d}x_{En}$ can be interpreted as the probability density for S. Consequently, the complexity of the two semiclassical structures may be different both in the classical [25] as well as in the quantum-mechanical context ([26], and references therein). To this end, the work is in progress and the results will be presented elsewhere.

Regarding our QBM model of §2, one may pose the following question: Are there only two possible decompositions which give rise to pointer states? Section 2 and Appendix implicitly answer this question. Actually, formally every linear canonical transformation, not involving the momentums, preserves the linear position–position coupling and the physical kind of the environment that are essential for our finding. Unless the self-Hamiltonians for the new subsystems appear non-realistic, one obtains the same conclusion. So, for such types of LCTs, while the details may be different, we can answer the question: there is more than two decompositions supporting the QBM effect.

The decoherence-based structure analysis is not restricted to the 'massive' quantum particles. The LCT can be defined for both the 'massive particles' (e.g. atoms) interacting with a quantum field (e.g. the electromagnetic field [27]) and the interacting quantum fields. While the details can be different, as long as the reduced dynamics is Markovian and the coupling is linear in the transformations-related observables, our finding of the parallel occurrence of decoherence may be expected to be valid. The details in this regard will be presented elsewhere.

Bearing in mind the global structures, our main result can be described as given in Introduction: if decoherence establishes 'the appearance of a classical world' [4] (e.g. $\mathrm{S}+\mathrm{E}$), our findings suggest 'the appearance of the classical world*s*' (e.g. $\mathrm{S}+\mathrm{E}$ and $\mathrm{CM}+\mathrm{R}$). As a corollary of the standard decoherence theory [4,5,10,11], the parallel occurrence of decoherence opens the following question: This parallel decoherence implies that the emergent classical world is not unique, which does not seem to be supported by our general observations. Does then the parallel decoherence suggest the requirement of a further selection process?

The answer to this question is essentially interpretational. Detailed analysis and arguments in this regard require some space. Here we just emphasize: If for some interpretational reasons only one structure is expected to be physically realistic, a selection rule is needed as emphasized by Zanardi's [18] "Without further physical assumption, no partition has an ontologically superior status with respect to any other.", and by Halliwell (chapter 3 in ref. [28]), "However, for many macroscopic systems, and in particular for the universe as a whole, there may be no natural split into distinguished subsystems and the rest, and another way of identifying the naturally decoherent variables is required." Further details can be found in ref. [29].

5. Conclusion

We launch a search for the occurrence of decoherence regarding different global and mutually irreducible decompositions into subsystems of an isolated composite quantum system. For the QBM-similar models, we obtain the occurrence of decoherence for an alternative decomposition of the composite system '(open) system plus environment'. Physically, this finding provides us with the observation of the parallel (simultaneous) occurrence of decoherence thus exhibiting relativity of the basic physical concept of '(global) semiclassical structure'. Being at variance with the standard view to 'physical structure', our findings open some conceptual issues yet to be explored as well as a new route in describing the composite physical systems.

Appendix

The two models, eqs (4) and (5), differ in (a) the values of the model parameters such as the masses and the characteristic frequencies, (b) (non)appearance of the external fields for the respective open systems as well as (c) appearance of the internal interaction for the new environment, $\hat{V}_{\mathrm{R}}$, which makes the model eq. (5) non-linear. Finally, as the two models are mutually related by the variables (the LCTs) transformations, the state for the composite system $\mathcal{C}$, which is assumed to be separable regarding the original structure $\mathrm{S} + \mathrm{E}$ is now (d) expected to be of the non-separable form for $\mathrm{CM} + \mathrm{R}$ structure.

While the points (a) and (b) are particularly trivial [2–5], the points (c) and (d) should be carefully examined in the context of the occurrence of decoherence. We should first emphasize that the point (c) can be straightforwardly managed by the proper linear transformations, $\hat{\rho}_{\mathrm{R}\alpha} = \sum_l \lambda_{l\alpha} \hat{Q}_{\mathrm{R}l}$, introducing the normal coordinates $\hat{Q}_{\mathrm{R}l}$. Then the environmental Hamiltonian $\hat{H}_{\mathrm{R}}$ is linearized, i.e. it obtains the form $\hat{H}_{\mathrm{R}} = \sum_l (\hat{P}_{\mathrm{R}l}^2/2 + \omega_l^2 \hat{Q}_{\mathrm{R}l}^2/2)$ that removes the 'nonlinear term' of the form of $\hat{V}_{\mathrm{R}}$ in eq. (5). As this is the linear transformation referring only to the environment R, the coupling term $\hat{X}_{\mathrm{CM}} \sum_\alpha \sigma_\alpha \hat{\rho}_{\mathrm{R}\alpha}$ in eq. (5) acquires another linear form $\hat{X}_{\mathrm{CM}} \sum_l \lambda_l \hat{Q}_{\mathrm{R}l}$ that keeps the form of the 'spectral density' [5].

Regarding the point (d): the introduction of normal coordinates for R introduces further change in the R's reduced state; e.g. a separable state becomes non-separable (entanglement relativity [17–24]). But this does not constitute any problem here as the environment is traced out and the tracing-out operation is basis-independent.

Therefore, the linearization of the Hamiltonian $\hat{H}_{\mathrm{R}}$ in eq. (5) gives the form of the Hamiltonian $\hat{H}$ of the fully isomorphic form as the 'original' form (eq. (4)). Then the standard results of the QBM theory [2–5] directly provide the following conclusion: like the open system S (eq. (4)), the open system CM (eq. (5)) is subject to the QBM effect, i.e. to the occurrence of decoherence, which distinguishes the related Gaussian states as the preferred states for the system CM. The 'decoherence function', $\Gamma(t)$, is of the same form for both structures (see e.g. eq. (4.226) in ref. [5]):

$$\Gamma(t) \approx -|\alpha - \beta|^2 \Lambda(t)/2, \tag{10}$$

for a pair of two 'coherent states', $|\alpha\rangle$ and $|\beta\rangle$. In the long time limit, $\Gamma(t)$ is dominated by the overlap, $|\alpha - \beta|^2$ [5]. Nevertheless, the 'long time limit' may refer to even mutually incomparable time intervals for the two structures. So, one may wonder if there is significantly different 'decoherence times' for the two structures, i.e. for the two decoherence processes. To see this is not the case, some care is needed.

Actually, even if for an instant of time t for which the ratio of $\Lambda(t)$ and $\Lambda'(t)$ for the two structures is far from unity, there is always the possibility also to change the first factor in eq. (10) in order to obtain $\Gamma(t)$ to be of the same order for both structures: $|\alpha - \beta|^2 \Lambda(t) \sim |\alpha' - \beta'|^2 \Lambda'(t)$. Physically, it means that, in such cases, the same 'decoherence time' refers to different Gaussian states for the two structures. So, the decoherence times are of the same order of magnitude for the two structures, yet in general for the different pairs of the respective Gaussian states.

In effect, the unique unitary dynamics for the composite system $\mathcal{C}$ – generated by the unique system-Hamiltonian for the unique initial state – hides the two, mutually independent, irreducible and simultaneously occurring decoherence processes for the two, mutually irreducible open systems, S and CM, that are the subsystems of the mutually irreducible global structures of the composite system $\mathcal{C}$.

Acknowledgements

The work in this paper is financially supported by the Ministry of Science, Serbia, Grant No. 171028.

References

[1] W H Zurek, *Philos. Trans. R. Soc. London, Ser.* **A356**, 1793 (1998)
[2] A O Caldeira and A J Leggett, *Phys. Rev.* **A31**, 1059 (1985)
[3] L D Romero and J P Paz, *Phys. Rev.* **A55**, 4070 (1997)
[4] D Giulini, E Joos, C Kiefer, J Kupsch, I-O Stamatescu and H D Zeh, *Decoherence and the appearance of a classical world in quantum theory* (Springer, Berlin, 1996)
[5] H P Breuer and F Petruccione, *The theory of open quantum systems* (Clarendon Press, Oxford, 2002)
[6] M Gell-Mann and J B Hartle, *Phys. Rev.* **D47**, 3345 (1993)
[7] F C Lombardo and P I Villar, *Int. J. Mod. Phys.* **B20**, 2952 (2006)
[8] J C Flores, *J. Phys. A: Math. Gen.* **31**, 8629 (1998)
[9] G M Palma, K-A Suominen and A Ekert, *Proc. R. Soc. London* **A452**, 567 (1996)
[10] W H Zurek, *Rev. Mod. Phys.* **75**, 715 (2003)
[11] M Schlosshauer, *Rev. Mod. Phys.* **76**, 1267 (2004)
[12] S Banerjee and J Kupsch, *J. Phys. A: Math. Gen.* **38**, 5237 (2005)
[13] L S Schulman, *Phys. Rev.* **A57**, 840 (1998)
[14] L S Schulman, *Phys. Rev. Lett.* **92**, 210404 (2004)

[15] M Dugić, *Phys. Scr.* **56**, 560 (1997)
[16] H D Zeh, *Seminaire Poincaré* **2**, 1 (2005)
[17] M Dugić, arXiv:quant-ph/9903037v1 (1999)
[18] P Zanardi, *Phys. Rev. Lett.* **87**, 077901 (2001)
[19] M Dugić and J Jeknić, *Int. J. Theor. Phys.* **45**, 2215 (2006)
[20] M Dugić and J Jeknić-Dugić, *Int. J. Theor. Phys.* **47**, 805 (2008)
[21] A C De la Torre *et al*, *Eur. J. Phys.* **31**, 325 (2010)
[22] N L Harshman and S Wickramasekara, *Phys. Rev. Lett.* **98**, 080406 (2007)
[23] M O Terra Cunha, J A Dunningham and V Vedral, *Proc. R. Soc.* **A463**, 2277 (2007)
[24] E Ciancio, P Giorda and P Zanardi, *Phys. Lett.* **A354**, 274 (2006)
[25] S Y Auyang, *Foundations of complex-system theories in economics, evolutionary biology, and statistical physics* (Cambridge University Press, Cambridge, 1998)
[26] C Mora, H Briegel and J Kupsch, *Int. J. Quant. Inf.* **5**, 729 (2007)
[27] B Bellomo, G Campogno and F Petruccione, *J. Phys. A: Math. Gen.* **38**, 10203 (2005)
[28] S Saunders, J Barrett, A Kent and D Wallace (eds.), *Many worlds? Everett, quantum theory, and reality* (Oxford University Press, Oxford, 2010)
[29] J Jeknić-Dugić, M Dugić and A Francom, arXiv:1109.6424v1[quant-ph] (2011)