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Relativistic effects on the modulational instability of electron plasma waves in quantum plasma

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Abstract. Relativistic effects on the linear and nonlinear properties of electron plasma waves are investigated using the one-dimensional quantum hydrodynamic (QHD) model for a two-component electron–ion dense quantum plasma. Using standard perturbation technique, a nonlinear Schrödinger equation (NLSE) containing both relativistic and quantum effects has been derived. This equation has been used to discuss the modulational instability of the wave. Through numerical calculations it is shown that relativistic effects significantly change the linear dispersion character of the wave. Unlike quantum effects, relativistic effects are shown to reduce the instability growth rate of electron plasma waves.

Keywords. Relativistic effects; modulational instability; electron plasma wave; quantum plasma.

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1. Introduction

Many authors have studied the linear and nonlinear properties of plasma waves. However, most of these investigations are confined to classical nonrelativistic plasma. Particle velocities in some plasmas may become comparatively high; in some cases it may even approach the speed of light. For such plasmas, it becomes important to consider the relativistic effects. In fact, relativistic effect may significantly modify the linear and nonlinear behaviour of plasma waves. Relativistic plasma can be formed in many practical situations e.g. in space–plasma phenomena [1], the plasma sheet boundary of Earth's magnetosphere [2], Van Allen radiation belts [3] and laser–plasma interaction experiments [4]. The relativistic motion in plasmas is assumed to exist during the early evolution of the Universe [5]. Studies on relativistic effects on ion-acoustic solitary waves are many. Das and Paul [6] first investigated the ion-acoustic solitary waves in relativistic plasma and showed that relativistic effect is important for the formation of ion-acoustic

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solitary waves in the presence of streaming motion of plasma particles. Subsequently, many authors [7–12] considered various parameters together with the relativistic effect for studying ion-acoustic solitary waves and obtained some fascinating results which are important in laboratory and space plasmas. Recently, Saeed *et al* [13] have shown that in electron–positron–ion plasma, increase in the relativistic streaming factor causes the soliton amplitude to thrive and its width shrinks. El-Labany *et al* [14] have shown that relativistic effect can modify the condition of modulational instability of ion-acoustic waves in warm plasma with nonthermal electrons. Han *et al* [15] have studied the existence of ion-acoustic solitary waves and their interaction in weakly relativistic two-dimensional thermal plasma. Sahu and Roy Chowdhury [16] have studied electron acoustic solitons in relativistic plasma with nonthermal electrons. Gill *et al* [17] have studied the amplitude and width variations of ion-acoustic solitons in relativistic electron–positron–ion plasma. Only a few studies related to the relativistic effects on electron plasma waves can be found in the literature. Recently, Bharuthram and Yu [18] have shown that relativistic electron plasma waves can propagate as quasistationary nonlinear waves as well as solitary waves.

All the above works on relativistic effects on plasma waves have been reported for classical plasma. But in plasmas, where the density is quite high and temperature is very low, the thermal de Broglie wavelength may become comparable to the interparticle distances. In such situations, quantum effects come in the picture due to the overlapping of wave functions of the neighbouring particles. These quantum effects can modify the linear and nonlinear characteristics from that found in the corresponding classical plasma. Such quantum effects in plasma become important in a variety of environments such as dense cosmological and astrophysical plasma [19,20], Earth's magnetosphere, Van Allen radiation belts, laser–solid density plasma interactions [21] etc. As a newly emerging field in plasma physics, quantum plasmas have received much attention. During the past few years, there has been a great deal of interest in the investigation of linear and nonlinear properties of various wave modes in quantum plasmas [22–24]. Many authors have studied quantum effects on the linear and nonlinear properties of ion-acoustic and dust-acoustic waves [25–27]. Recently, we have studied the effect of quantum diffraction on the electron plasma waves and it has been found that quantum effects can significantly modify the modulational instability conditions and the instability growth rates of finite-amplitude electron plasma waves [28]. In nonrelativistic classical plasma, electron plasma waves are modulationally stable for all wavelengths [29] whereas in nonrelativistic quantum plasma, electron plasma waves become modulationally unstable in two distinct regions of wavenumbers [28]. Quantum effects can significantly change the instability growth rate and also alter the stability and instability domains of the wavenumber. It has been found that the relativistic variation of mass has significant contribution to the nonlinear behaviour of electron plasma waves even in the classical limit [30]. There are practical situations such as intense laser–solid interaction experiments, some astrophysical situations and presumably in the early period of the evolution of the Universe where both the quantum and relativistic effects may become important for consideration. There has been a great deal of renewed interest on this area of physics, particularly due to the intense laser–plasma interaction experiments. So it is interesting to study the combined effects of quantum corrections and relativistic motion on plasma waves. Recently, Sahu [31] has studied quantum ion-acoustic solitary waves in weakly relativistic plasma. Gill *et al* [32] have studied ion-acoustic shock waves in a relativistic electron–positron–ion quantum plasma. But so far as we know, no work has been done on the modulational instability of electron plasma waves in quantum plasma including relativistic effects. The motivation of the present paper is to study the relativistic effect on the amplitude modulation of electron plasma waves in quantum plasma. Quantum relativistic plasma is a growing field of research. Researchers are trying to develop suitable models to study such plasmas. Quantum systems can be described under certain conditions by classical equations in which potential energy is replaced by an effective potential incorporating quantum effects. Bohm potential is an example of this strategy which plays an important role in formulating quantum hydrodynamics. Bohm defined the wave function $\Psi(x, t)$ as

$$\Psi(x, t) = R(x, t) \exp\left[iS(x, t)/\hbar\right],$$

where R is the real amplitude and S is the real phase. Then, using Schrödinger equation he obtained equation for S which apart from the usual classical potential contains an additional potential term which is called Bohm potential [33].

Because of its simplicity and numerical efficiency, the quantum hydrodynamic (QHD) model developed by Hass [34] has been widely used in the non-relativistic regime. This model includes forces associated with quantum statistical electron pressure and the quantum force involving tunnelling of degenerate electrons through Bohm potential [33]. The quantum correction in this model is obtained from the moments of non-relativistic Wigner function. For quantum corrections in relativistic plasma, we may rely on the moments of a relativistic Wigner function such as the one described by Bialynncki-Birula *et al* [35] and Shin [36]. Thus the QHD model for non-relativistic plasma can be generalized to at least weakly relativistic regime. The QHD model has previously been used by some authors [31,32] to study nonlinear ion-acoustic waves in quantum relativistic regime. Using the one-dimensional quantum hydrodynamic (QHD) model for two-component electron–ion dense quantum plasma we have studied the linear and nonlinear properties of electron plasma waves including weakly relativistic effect and ion motion. It is shown that the relativistic effects can significantly change the linear and nonlinear properties of electron plasma waves in quantum plasma. For a given wavenumber k the frequency ω of the electron plasma wave decreases significantly with the increase in the streaming velocity of the plasma particles. The relativistic effect is shown to reduce the nonlinear steepening of the electron plasma waves in quantum plasma. The modulational instability growth rate of electron plasma waves is found to decrease with the increase in the streaming velocity of electrons. A similar observation was made by Spatschek [30] for relativistic electron plasma waves in classical plasma. The group dispersion effect is found to increase sharply with wavenumber k in the low-k region; it quickly attains a maximum and then drops slightly to attain an almost constant value in the high k-region. However, in this high k-region, the group dispersion decreases significantly with the increase in the streaming velocity of electrons. Ion motion, though found to be unimportant in the high k-region, has significant effect in the low k-region in changing the stability/instability domains of the wavenumber.

The paper is organized in the following way: In §2, the basic set of quantum hydrodynamic equations are presented, including relativistic effects. In §3 we derive the NLS equation using the standard multiple scale perturbation technique by taking into account weakly relativistic effects. In §4 we consider the possibility of modulational instability of the wave. In the last section we discuss the results and give some concluding remarks.

2. Basic equations

We consider weakly relativistic plasma consisting of electrons and ions with a streaming motion along the x-axis. We assume that the plasma particles behave as a one-dimensional Fermi gas at zero temperature and therefore the pressure law [37] is

$$p_j = \frac{m_j V_{\mathrm{F}j}^2}{3n_{j0}^2} n_j^3, \tag{1}$$

where $j = \mathrm{e}$ for electron and $j = \mathrm{i}$ for ions; m_j is the mass; $V_{\mathrm{F}j} = \sqrt{2k_{\mathrm{B}} T_{\mathrm{F}j}/m_j}$ is the Fermi speed, $T_{\mathrm{F}j}$ is the Fermi temperature and k_{B} is the Boltzmann constant; n_j is the number density with the equilibrium value n_{j0}. The set of QHD equations describing the dynamics of the electron plasma waves in the model plasma under consideration are given by

$$\frac{\partial n_j}{\partial t} + \frac{\partial (n_j u_j)}{\partial x} = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x}\right)(u_j \gamma_j) = -\frac{q_j}{m_j}\frac{\partial \phi}{\partial x} - \frac{1}{m_j n_j}\frac{\partial p_j}{\partial x} + \frac{\hbar^2}{2m_j^2 \gamma_j}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_j}}\frac{\partial^2 \sqrt{n_j}}{\partial x^2}\right], \tag{3}$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_{\mathrm{e}} - n_{\mathrm{i}}), \tag{4}$$

where u_j, q_j and p_j are respectively the fluid velocity, charge and pressure of the jth species, $q_{\mathrm{e}} = -e$, $q_{\mathrm{i}} = e$, $\gamma_{\mathrm{e}} = (1 - u_{\mathrm{e}}^2/c^2)^{1/2}$ is the relativistic factor for electrons, $\gamma_i = 0$ for ions, c is the velocity of light in free space, $\hbar$ is the Planck's constant divided by 2π and ϕ is the electrostatic wave potential. Electrons, because of their lighter mass, attain relativistic speed more easily than the heavier ions. For this we consider the ion motion to be non-relativistic. It may also be noted that following the widely adopted practice and for simplicity, we have included relativistic effect only in the equation of motion (3). This is justified as we are considering ultra-cold plasma with weakly relativistic effect. Moreover, it must also be kept in mind that the quantum relativistic plasma is a growing field of research in which models are also being developed.

We now use the following normalization:

$$x \to x\omega_{\mathrm{pe}}/V_{\mathrm{Fe}}, \quad t \to t\omega_{\mathrm{pe}}, \quad \phi \to e\phi/2k_{\mathrm{B}}T_{\mathrm{Fe}}, \quad n_j \to n_j/n_0$$

and

$$u_j \to u_j/V_{\mathrm{Fe}},$$

where $\omega_{\mathrm{pe}} = \sqrt{4\pi n_0 e^2/m_{\mathrm{e}}}$ is the electron plasma oscillation frequency and V_{Fe} is the Fermi speed of electrons. The normalization gives us the following simplified set of equations for ions and electrons as:

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \frac{\partial (n_{\mathrm{i}} u_{\mathrm{i}})}{\partial x} = 0, \tag{5}$$

$$\left(\frac{\partial}{\partial t}+u_{\mathrm{i}}\frac{\partial}{\partial x}\right)u_{\mathrm{i}}=-\mu\frac{\partial\phi}{\partial x}-\sigma n_{\mathrm{i}}\frac{\partial n_{i}}{\partial x}+\frac{\mu^{2}H^{2}}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_{\mathrm{i}}}}\frac{\partial^{2}\sqrt{n_{\mathrm{i}}}}{\partial x^{2}}\right], \tag{6}$$

$$\frac{\partial(n_{\mathrm{e}})}{\partial t}+\frac{\partial(n_{\mathrm{e}}u_{\mathrm{e}})}{\partial x}=0, \tag{7}$$

$$\left(\frac{\partial}{\partial t}+u_{\mathrm{e}}\frac{\partial}{\partial x}\right)(u_{\mathrm{e}}\gamma_{\mathrm{e}})=\frac{\partial\phi}{\partial x}-n_{\mathrm{e}}\frac{\partial n_{\mathrm{e}}}{\partial x}+\frac{H^{2}}{2\gamma_{\mathrm{e}}}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_{\mathrm{e}}}}\frac{\partial^{2}\sqrt{n_{\mathrm{e}}}}{\partial x^{2}}\right], \tag{8}$$

$$\frac{\partial^{2}\phi}{\partial x^{2}}=(n_{\mathrm{e}}-n_{\mathrm{i}}), \tag{9}$$

where $H=\hbar\omega_{\mathrm{pe}}/2k_{\mathrm{B}}T_{\mathrm{Fe}}$ is a nondimensional quantum parameter proportional to the quantum diffraction, $\mu=m_{\mathrm{e}}/m_{\mathrm{i}}$ is the ratio of electron to ion mass and $\sigma=T_{\mathrm{Fi}}/T_{\mathrm{Fe}}$ is the ratio of ion to electron Fermi temperatures. The quantum diffraction parameter H is proportional to the ratio of the plasma energy $\hbar\omega_{\mathrm{pe}}$ (energy of an elementary excitation associated with an electron plasma wave) to the Fermi energy $k_{\mathrm{B}}T_{\mathrm{Fe}}$.

3. Derivation of the NLSE

We assume that the ions, because of their heavier mass, cannot respond to the high-frequency components of the field quantities. They respond only to the slowly-varying part of the field quantities generated through nonlinear interactions of high-frequency waves. With these ideas in mind we make the following Fourier expansion of the field quantities:

$$\begin{bmatrix}n_{\mathrm{e}}\\u_{\mathrm{e}}\\\phi\end{bmatrix}=\begin{bmatrix}1\\u_{0}\\0\end{bmatrix}+\varepsilon^{2}\begin{bmatrix}n_{\mathrm{e}0}\\u_{\mathrm{e}0}\\\phi_{0}\end{bmatrix}+\sum_{s=1}^{\infty}\varepsilon^{s}\left\{\begin{bmatrix}n_{\mathrm{es}}\\u_{\mathrm{es}}\\\phi_{\mathrm{s}}\end{bmatrix}.\exp(is\psi)+\begin{bmatrix}n_{\mathrm{es}}^{*}\\u_{\mathrm{es}}^{*}\\\phi_{\mathrm{s}}^{*}\end{bmatrix}.\exp(-is\psi)\right\} \tag{10a}$$

$$\begin{bmatrix}n_{\mathrm{i}}\\u_{\mathrm{i}}\end{bmatrix}=\begin{bmatrix}1\\u_{0}\end{bmatrix}+\varepsilon^{2}\begin{bmatrix}n_{\mathrm{i}0}\\u_{\mathrm{i}0}\end{bmatrix} \tag{10b}$$

in which $\psi=kx-\omega t$ (ω, k being the normalized wave frequency and wavenumber respectively), the field quantities $n_{\mathrm{e}0}$, $u_{\mathrm{e}0}$, ϕ_0, n_{es}, u_{es}, ϕ_{s}, $n_{\mathrm{i}0}$ and $u_{\mathrm{i}0}$ are assumed to vary slowly with x and t, i.e., they are supposed to be functions of

$$\xi=\varepsilon(x-c_{\mathrm{g}}t)\quad\text{and}\quad\tau=\varepsilon^{2}t, \tag{11}$$

where ε is a smallness parameter and c_{g} is the normalized group velocity.

Now substituting the expansion (10a, b) in eqs (5)–(9) and then equating from both sides the coefficients of $\exp(\mathrm{i}\psi)$, $\exp(\mathrm{i}2\psi)$ and terms independent of ψ, we obtain three sets of equations which we call I, II and III. To solve these three sets of equations we make the following perturbation expansion for the field quantities $n_{\mathrm{e}0}$, $u_{\mathrm{e}0}$, ϕ_0, n_{es}, u_{es}, ϕ_{s}, $n_{\mathrm{i}0}$ and $u_{\mathrm{i}0}$ which we denote by A:

$$A=A^{(1)}+\varepsilon A^{(2)}+\varepsilon^{2}A^{(3)}+\cdots. \tag{12}$$

Solving the lowest order equations obtained from the set of equations I after substituting the expansion (12) we get

$$n_{e1}^{(1)} = -k^2\phi_1^{(1)}, \quad u_{e1}^{(1)} = -k\,(\omega - ku_0)\,\phi_1^{(1)} \tag{13}$$

and the normalized linear dispersion relation as

$$\gamma_3(\omega - ku_0)^2 = 1 + k^2 + \gamma_2\frac{H^2k^4}{4}, \tag{14}$$

where

$$\gamma_3 = 1 + \frac{3u_0^2}{2c^2} \quad \text{and} \quad \gamma_2 = 1 - \frac{u_0^2}{2c^2}.$$

The dispersion relation (14) in dimensional form becomes

$$\gamma_3(\omega - ku_0)^2 = \omega_{pe}^2 + k^2V_{Fe}^2 + \gamma_2\frac{H^2k^4V_{Fe}^4}{4\omega_{pe}^2}. \tag{15}$$

Equation (15) describes the relativistic quantum counterpart of the classical electron plasma wave dispersion relation. In the absence of relativistic effect ($\gamma_2 = \gamma_3 = 1$), the dispersion relation (15) reduces to that of quantum electron plasma waves in a non-drifting ($u_0 = 0$) plasma as described in ref. [29]. Also note that for a non-drifting non-relativistic classical ($H = 0$) plasma, the dispersion relation (15) reduces to the well-known dispersion relation of electron plasma waves, provided electron Fermi velocity V_{Fe}

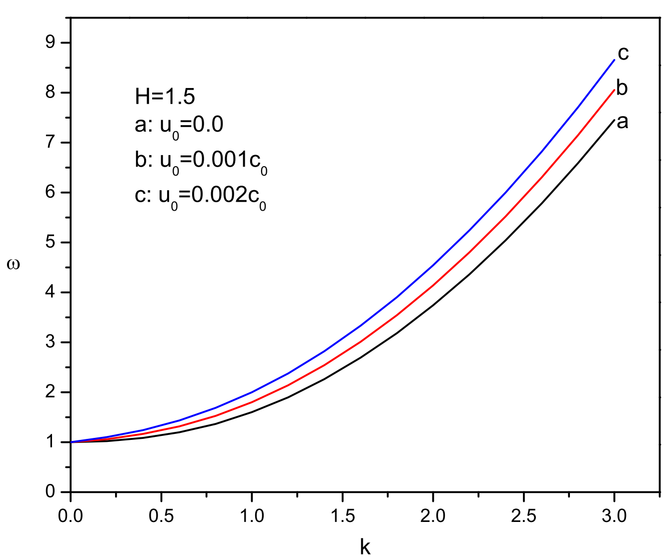

Figure 1. Linear dispersion curve for different values of the streaming factor for a given quantum diffraction parameter $H = 1.5$ with $\sigma = 0.001$.

is replaced by the electron thermal velocity V_e. For a relativistic classical cold streaming plasma the dispersion relation (15) reduces to

$$\omega = ku_0 \pm \omega_{pe}/\sqrt{1 + 3u_0^2/c^2}$$

which indicates two modes of propagation under the condition $ku_0 > \omega_{pe}/\sqrt{1 + 3u_0^2/c^2}$.

The dispersion relation (14) is plotted in figure 1 both in the non-relativistic and relativistic limits with the quantum diffraction parameter $H = 1.5$ and for different values of the streaming velocity of plasma particles. Obviously the wave frequency is enhanced by the relativistic effects.

The group velocity $c_g = d\omega/dk$ is obtained from the dispersion relation (14) as

$$c_g = \frac{k + (\gamma_2 H^2 k^3/2)}{\sqrt{\gamma_3 \left(1 + k^2 \left(1 + (\gamma_2 H^2 k^2/4)\right)\right)}} + u_0. \tag{16}$$

The second harmonic quantities in the lowest order can be obtained from the solutions of the lowest-order equations obtained from the set of equations II after substituting the perturbation expansion (12). Thus we get

$$\begin{aligned}
\phi_2^{(1)} &= -b_2 \phi_1^{(1)2}, \\
n_{e2}^{(1)} &= 4k^2 b_2 \phi_1^{(1)2}, \\
u_{e2}^{(1)} &= (\omega - ku_0)\left(4b_2 k - k^3\right)\phi_1^{(1)2},
\end{aligned} \tag{17}$$

where

$$b_2 = \frac{\left[2(\omega - ku_0)^2 k^3 \gamma_3 + k^5\left(1 - \dfrac{\gamma_2 H^2 k^2}{4}\right) + k^2(\omega - ku_0)^2\{k\gamma_3 - 3\gamma_1(\omega - ku_0)\} - \dfrac{k^6 H^2 \gamma_1}{4}(\omega - ku_0)\right]}{[8(\omega - ku_0)^2\gamma_3 - 8k^3(1 + H^2 k^2 \gamma_2) - 2k]} \tag{18}$$

in which $\gamma_1 = u_0/c^2$.

The zeroth harmonic quantities are obtained from the solutions of the lowest-order equations obtained from the set of equations III after substituting the perturbation expansion (12).

$$\begin{aligned}
\phi_0^{(1)} &= b_0 \phi_1^{(1)2}, \\
n_{e0}^{(1)} &= n_{i0}^{(1)} = b_1 \phi_1^{(1)2}, \\
u_{e0}^{(1)} &= [b_1(c_g - u_0) - 2(\omega - ku_0)k^3]\phi_1^{(1)2}, \\
u_{i0}^{(1)} &= b_1(c_g - u_0)\phi_1^{(1)2},
\end{aligned} \tag{19}$$

where

$$b_0 = \frac{k^2\left[2\gamma_3\left(c_g-u_0\right)k(\omega-ku_0)+\left\{\gamma_3+3\gamma_1\left(u_0-c_g\right)\right\}(\omega-ku_0)^2 + \left(1+\frac{\gamma_2 H^2 k^2}{4}\right)k^2+\frac{3\gamma_1 H^2 k^3}{4}(\omega-ku_0)\right]}{\left[(c_g-u_0)^2\gamma_3-1\right]}$$

$$b_1 = \frac{\mu b_0}{[(c_g-u_0)^2\gamma_3-\sigma]}. \tag{20}$$

The first harmonic quantities in the second order are obtained from the solutions (13) by replacing $-i\omega$ by $(-i\omega-\varepsilon c_g(\partial/\partial\xi)+\varepsilon^2(\partial/\partial\tau))$ and ik by $(ik+\varepsilon\,\partial/\partial\xi)$ and then picking out order ε terms. Thus we obtain

$$\phi_1^{(2)} = 0,$$
$$n_{e1}^{(2)} = 2ik\frac{\partial\phi_1^{(1)}}{\partial\xi},$$
$$u_{e1}^{(2)} = i(\omega+kc_g-2ku_0)\frac{\partial\phi_1^{(1)}}{\partial\xi}. \tag{21}$$

Now collecting coefficients of ε^3 from both sides of the sets of equations I after substituting the perturbation expansion (12) we get a set of equations for the first harmonic quantities in the third order. Using the above solutions and after proper elimination,

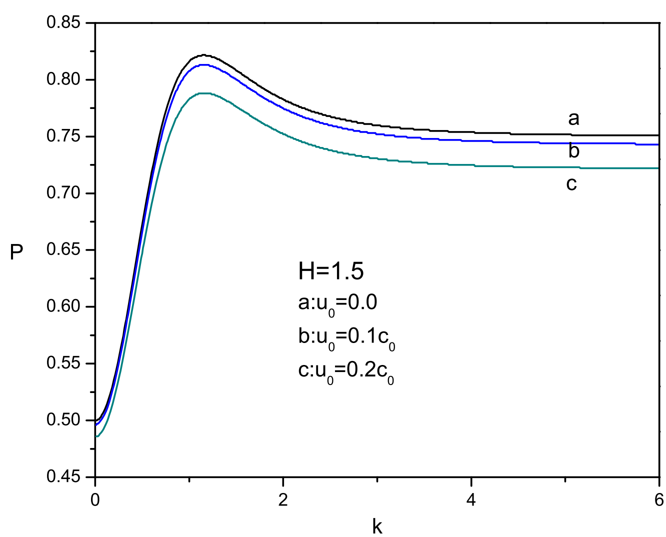

Figure 2. Variation of the group dispersion coefficient (P) with wavenumber (k) for different values of the streaming factor for a given quantum diffraction parameter $H=1.5$ with $\sigma=0.001$.

we obtain the desired nonlinear Schrödinger equation (NLSE) describing the nonlinear evolution of the wave amplitude as

$$i\frac{\partial \alpha}{\partial \tau} + P\frac{\partial^2 \alpha}{\partial \xi^2} = Q\alpha^2\alpha^*, \tag{22}$$

where $\alpha = \phi_1^{(1)}$, the group dispersion coefficient

$$P = \frac{1}{2}\frac{\mathrm{d}c_\mathrm{g}}{\mathrm{d}k} = \frac{[1-\gamma_3(c_\mathrm{g}-u_0)^2 + (3/2)\,\gamma_2 H^2 k^2]}{2\gamma_3\,(\omega - ku_0)} \tag{23}$$

and the nonlinear coefficient

$$Q = \frac{[\gamma_3(\omega - ku_0) f_1 + k(f_2 + f_3)]}{2k^2\gamma_3(\omega - ku_0)}, \tag{24}$$

where

$$f_1 = -k\left[(\omega - ku_0)\left\{b_1 k - 3k^5 + 8b_2 k^3\right\} + b_1 k^2\left(c_\mathrm{g} - u_0\right)\right],$$

$$f_2 = \begin{bmatrix} \left(-k^2\,(\omega - ku_0)\left\{b_1\left(c_\mathrm{g} - u_0\right) - 2\,(\omega - ku_0)\,k^3\right\}\right. \\ \left. -\gamma_3 k^2\,(\omega - ku_0)^2\left(4b_2 k - k^3\right)\right)\left(\gamma_3 + \dfrac{3u_0^2}{c^2}\right) \\ -\dfrac{9u_0 k^4}{2c^2}\,(\omega - ku_0)^3 - b_1 k^3 - 4b_2 k^5 \end{bmatrix}$$

and

$$f_3 = \frac{k^4 H^2}{8}\begin{bmatrix} \gamma_2\left\{b_1 k^2 + 20 b_2 k^4\right\} - \dfrac{1}{c^2}\left(2k^2 u_0\left\{b_1\left(c_\mathrm{g} - u_0\right) - 2k^3(\omega - ku_0)\right\}\right) \\ +\dfrac{1}{c^2}\left\{72 u_0 k^3 b_2(\omega - ku_0) + k^4(\omega - ku_0)^2\left(1 - \dfrac{2}{c^2}\right)\right\} \end{bmatrix}.$$

Note that both the coefficients P and Q depend on the relativistic effects through the terms γ_2, γ_3 and u_0. The group dispersion coefficient P has been plotted in figure 2 for different values of streaming velocity u_0. Obviously, the group dispersion effect increases sharply with wavenumber k in the low k-region; it quickly attains a maximum value and then drops slightly to attain an almost constant value in the high k-region. The group dispersion effect is almost independent of u_0 in the low k-region but it decreases significantly with the increase in the streaming velocity u_0 in the high k-region. Thus relativistic effects lead to less dispersive electron plasma wave.

4. Modulational instability

The amplitude modulation of electron plasma waves is described by the NLS equation with quantum corrections and relativistic effects. In connection with the nonlinear propagation of waves of various types, it has been found that in certain cases conversion of an initially uniform wave train into a spatially modulated wave becomes energetically favourable. In such cases the wave becomes modulationally unstable. Now it is well known that when $PQ > 0$ the wave train is modulationally stable, whereas when $PQ < 0$

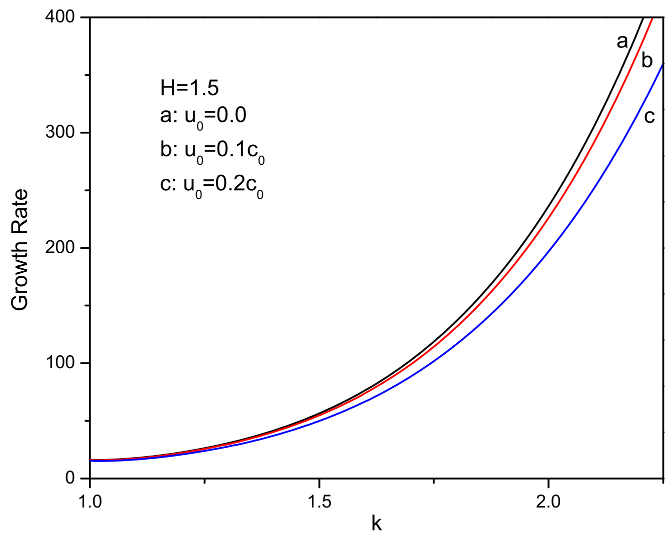

Figure 3. Variation of the growth rate of instability with wavenumber (k) for different values of the streaming factor for a given quantum diffraction parameter $H = 1.5$ with $\sigma = 0.001$.

the wave train becomes modulationally unstable. The growth rate of this instability depends on the wavenumber of modulation and it attains a maximum value of $g_\mathrm{m} = |Q|\,\alpha_0^2$ corresponding to the wavenumber $l_\mathrm{m} = |Q/P|^{1/2}\,|\alpha_0|$ of the modulation. Numerical computation of P and Q for different values of k using expressions (23) and (24), for a fixed value of the quantum diffraction parameter ($H = 1.5$), and other system parameters shows that $PQ < 0$ for all values of k except a small range of Δk of k in the low k-region. This means that the electron plasma waves in relativistic quantum plasma are modulationally unstable for all values of k except a small range of Δk of k in the low k-region. This type of result was also obtained in non-relativistic plasma in our previous work [28]. The stability region Δk in the wavenumber domain is found to be not much sensitive to the variation in stream velocity of plasma particles. In the high k-region the wave is modulationally unstable. Here we find through numerical calculations that the maximum growth rate of instability (g_m) decreases with the increase in the relativistic stream velocity u_0. Thus, the relativistic effect tends to reduce the nonlinear steepening of the wave giving lower growth rate of instability. This is opposite to the effect produced by quantum diffraction because in our previous work we have shown that the quantum effects make electron plasma waves more unstable giving higher growth rate of instability (figure 3).

5. Discussions and conclusions

In this paper we have used the one-dimensional quantum hydrodynamic (QHD) model to investigate relativistic effects on the linear and nonlinear properties of quantum electron plasma waves in a two-component electron–ion dense quantum plasma including the effects of ion motion. To the best of our knowledge, so far no work, including both the relativistic and quantum effects has been reported on the modulational instability of electron plasma waves. We have considered a weakly relativistic situation. Electrons, because of their lighter mass, attain relativistic speed more easily than the heavier ions. For this, we consider ion motion to be non-relativistic and introduce relativistic effects on electron motion through the parameter γ_e. The model includes quantum statistical effect through the equation of state (1) and quantum diffraction effect through the parameter H. Quantum effects in plasma become important when thermal de Broglie wavelength becomes much larger than the average interparticle distance. For ions, because of their heavier mass, quantum effects can be neglected in most practical situations. Because of this we have not considered quantum effects on ion motion.

Relativistic effects are shown to significantly influence the linear and nonlinear properties of electron plasma waves. For a given k, the frequency of the wave decreases significantly with the increase in the drift velocity of plasma particles. Unlike quantum effects which tend to increase the nonlinear effects, relativistic effects tend to reduce the nonlinear steepening of electron plasma waves in the high k-region. Thus, the relativistic effects weaken the nonlinear quantum effects. In the low k-region, relativistic effects introduce no significant additional changes in the nonlinear properties of electron plasma waves in quantum plasma. The group dispersion effect is shown to be almost independent of relativistic drift velocity u_0 in the low k-region, but in the high k-region the group dispersion effect decreases significantly with the increase in u_0. Numerical calculations including ion motion show that ion motion has very little effect on the linear and nonlinear properties of electron plasma waves in the high k-region but in the low k-region it has significant effect in changing the stability and instability regions in the wavenumber domain [29].

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References

[1] C Grabbe, *J. Geophys. Res.* **94**, 17299 (1989)
[2] J I Vette, *Summary of particle population in the magnetosphere* (Reidel, Dordrecht, 1970) p. 305
[3] H Ikezi, *Phys. Fluids* **16**, 1668 (1973)
[4] M Marklund and P K Shukla, *Rev. Mod. Phys.* **78**, 591 (2006)
[5] K A Holcomb and T Tajima, *Phys. Rev.* **D40**, 3809 (1989)
[6] G C Das and S N Paul, *Phys. Fluids* **28**, 823 (1985)
[7] Y Nejoh, *J. Plasma Phys.* **38**, 439 (1987)
[8] H H Kuehl and G Y Zhang, *Phys. Fluids* **B3**, 26 (1991)
[9] A Roychowdhury, G Pakira and S N Paul, *IEEE Trans. Plasma Sci.* **26**, 987 (1998)
[10] T S Gill, A S Bains and N S Saini, *Cand. J. Phys.* **87**, 861 (2009)
[11] B C Kalita, R Das and H K Sarmah, *Phys. Plasmas* **18**, 012304 (2011)
[12] H Pakzad, *Astrophys. Space Sci.* **332**, 269 (2011)

[13] R Saeed, A Shah and M N Haq, *Phys. Plasmas* **17**, 102301 (2010)
[14] S K El-Labany, M S Abdel Krim, S A El-Warraki and W F El-Taibany, *Chin. Phys.* **12**, 759 (2003)
[15] J N Han, S L Du and W S Duan, *Phys. Plasmas* **15**, 112104 (2008)
[16] B Sahu and R Roy Chowdhury, *Phys. Plasmas* **13**, 072302 (2006)
[17] T S Gill, A Singh, H Kaur, N S Saini and P Bala, *Phys. Lett.* **A361**, 364 (2007)
[18] R Bharuthram and M Y Yu, *Astrophys. Space Sci.* **207**, 197 (1993)
[19] M Opher, L O Silva, D E Dauger, V K Decyk and J M Dawson, *Phys. Plasmas* **8**, 2544 (2001)
[20] Y D Jung, *Phys. Plasmas* **8**, 3842 (2001)
[21] D Kremp, T Bornath, M Bonitz and M Schalanges, *Phys. Rev.* **E60**, 4725 (1999)
[22] P K Shukla and B Eliasson, *Phys. Rev. Lett.* **96**, 245001 (2006)
[23] F Hass, L G Garcia, J Goedert and G Manfredi, *Phys. Plasmas* **10**, 3858 (2003)
[24] G Manfredi, *Fields Inst. Commun.* **46**, 263 (2005)
[25] A P Misra and C Bhowmik, *Phys. Plasmas* **14**, 012309 (2007)
[26] S A Khan and A Mushtaq, *Phys. Plasmas* **14**, 083703 (2007)
[27] A P Misra and A Roy Chowdhury, *Phys. Plasmas* **13**, 072305 (2006)
[28] B Ghosh, S Chandra and S N Paul, *Phys. Plasmas* **18**, 012106 (2011)
[29] T Kakutani and W Sugimoto, *Phys. Fluids* **17**, 1617 (1974)
[30] K H Spatschek, *Phys. Rev.* **A16**, 2470 (1977)
[31] B Sahu, *Pramana – J. Phys.* **76**, 933 (2011)
[32] T S Gill, A S Bains and C Bedi, *J. Phys. Conf. Ser.* **208**, 012040 (2010)
[33] C L Gardner and C Ringhofer, *Phys. Rev.* **E53**, 157 (1996)
[34] F Hass, *Phys. Plasmas* **12**, 062117 (2005)
[35] I Bialynicki-Birula, P Gornicki and J Rafelski, *Phys. Rev.* **D44**, 1825 (1991)
[36] G Shin, *J. Korean Phys. Soc.* **29**, 571 (1996)
[37] G Manfredi and F Haas, *Phys. Rev.* **B64**, 075316 (2001)