PRAMANA — journal of physics

Vol. 78, No. 5
May 2012
pp. 719–728

Nucleation rate of the quark-gluon plasma droplet at finite quark chemical potential

D S GOSAIN[1,*], S SOMORENDRO SINGH[1] and AGAM K JHA[2]
[1]Department of Physics and Astrophysics, University of Delhi, Delhi 110 007, India
[2]Department of Physics, Kirori Mal College, University of Delhi, Delhi 110 007, India
*Corresponding author. E-mail: dharmender.gosain@gmail.com

MS received 14 January 2011; revised 14 September 2011; accepted 23 December 2011

Abstract. The nucleation rate of quark-gluon plasma (QGP) droplet is computed at finite quark chemical potential. In the course of computing the nucleation rate, the finite size effects of the QGP droplet are taken into account. We consider the phenomenological flow parameter of quarks and gluons, which is dependent on quark chemical potential and we calculate the nucleation rate of the QGP droplet with this parameter. While calculating the nucleation rate, we find that for low values of quark phenomenological parameter γ_q, nucleation rate is negligible and when γ_q increases, nucleation rate increases significantly.

Keywords. Quark-gluon plasma; quark-hadron phase transition; nucleation rate.

PACS Nos 12.38.Mh; 25.75.Ld; 25.75.Nq

1. Introduction

Quantum chromodynamics (QCD), a true theory of strong interactions, is a key [1] to understand the phase transition from a confined hadronic matter to a deconfined matter of free quarks and gluons. Extracting some meaningful results using the core QCD is a very complicated task, and so it becomes imperative to devise some phenomenological models to fetch information regarding this new matter called quark-gluon plasma (QGP). One of the model extensively used in the literature is the statistical model. This model assumes that the QGP–hadron system is at quasistatic equilibrium enabling applicability of equilibrium statistical mechanics to this complicated system. This assumption has been successfully employed by many pioneers in this field [2,3].

Although this phenomenon is complicated, it is widely accepted that experiments at Relativistic Heavy Ion Collider (RHIC) at BNL and LHC at CERN indicate the formation of this highly dense matter called quark-gluon plasma. The energy density of the deconfined matter may reach a very high value due to large fluctuations in the system. If the energy density in a particular finite region of space is more than some critical value

DOI: 10.1007/s12043-012-0265-x; ***e*Publication: 20 March 2012**

(1–2 GeV/fm^3), then in that small space the matter is more readily described as a quark-gluon plasma rather than as a hot gas of hadrons. It means that the formation of QGP takes place in a small finite region and the effect of finite size is necessary to explain the evolution of the system [4]. Another important issue is that while calculating the nucleation rate of the QGP droplet, it is necessary to consider the finite size of hadrons [5]. These corrections are properly taken care of in the workout of this paper.

Now, in the framework of the homogeneous nucleation theory, the nucleation rate of the plasma droplet can be estimated as [6]

$$I = I_0 \exp\left(-\frac{\Delta F_c}{T}\right), \tag{1}$$

where I_0 is the prefactor, T is the temperature and ΔF_c is the change in free energy due to the formation of a critical size plasma droplet (i.e. critical free energy). The prefactor was first computed by Kapusta and Cernai in a course-grained effective field theory approximation to QCD and this type of work is further extensively studied to the finite baryonic density by Venugopalan and Vischer [6]. The prefactor influences the growth rate and statistical fluctuations. It also accounts in the calculation of the available phase space.

In the present study, the nucleation rate of the QGP droplet from hadronic medium is calculated at finite quark chemical potential taking into consideration the surface tension and the shape contribution of the QGP droplet. We used Ramanathan *et al* [7,8] statistical model in which the effective QCD potential is considered, to construct the density of states for quarks and gluons. These types of potential models give a very successful explanation in the case of particle multiplicities.

This paper is organized as follows: in §2, we briefly explain the density of states at finite quark chemical potential. We calculate the nucleation rate in §3. Finally, we present the result and conclusion in §4.

2. Density of states at finite quark chemical potential

We construct the density of state at finite quark chemical potential following the procedures of the Thomas–Fermi electronic model of atom and Bethe's model of nucleons [9]. The density of states for relativistic particles like quarks and gluons at these chemical potential is defined as

$$\rho_{q,g}(k,\mu) = \left[\frac{v(2m)^{3/2}}{2\pi^2}\right][-V_{\text{eff}}(k,\mu)]^{1/2} \cdot \left[-\frac{dV_{\text{eff}}(k,\mu)}{dk}\right], \tag{2}$$

where k is the relativistic four momentum of the quarks and gluons, v is the volume of the droplet and V_{eff} is a suitable effective confining potential, depending on finite chemical potential, set up between the current quarks–antiquarks and gluons in the system. It is defined as

$$V_{\text{eff}}(k,\mu) = \left(\frac{8\pi}{27k}\right)\gamma'_{g,q}\left(\frac{1}{\ln(1+k^2/\Lambda^2)}\right)T^2 - \frac{m_0^2}{2k}. \tag{3}$$

$\gamma'_{g,q}$ is the phenomenological flow parameter of quarks and gluons and Λ is the QCD parameter taken as 150 MeV.

In this effective potential we use the chemical potential through the phenomenological flow parameters of quarks. The parameter γ_{q}' is obtained by modifying our earlier value of γ_{q}. It is done by replacing γ_{q} by $\gamma_{\mathrm{q}}[1+\frac{\mu^2}{\pi^2T^2}]$. We are representing this modified value by γ_{q}'. The modification is inspired by the work of ref. [10] and this modified value enhances the nucleation rate of the QGP droplet. Moreover, the gluon parameter remains the same as the earlier one, i.e. γ_{g}' as γ_{g}. The value of γ_{g} is fixed as $1/3$, which is also chosen by Peshier. The effective potential is obtained through the thermal Hamiltonian [11] as

$$H(k,T)=[k^2+m(T)^2]^{1/2}, \tag{4}$$

which in the large k limit can be expressed as

$$H(k,T)=k+\frac{m_0^2}{2k}-\frac{(m_0^2-m^2(T))}{2k}, \tag{5}$$

where $m(T)$ is the thermal-dependent quark or gluon mass.

3. Nucleation rate at finite quark chemical potential

The nucleation process is naturally driven by statistical fluctuations. It is determined by the critical free energy difference between the two phases. The free energy difference between the quark and the hadronic phases, by considering the curvature and shape of the droplet, can be approximated as [4,7,8]

$$\begin{aligned}\Delta F \;=\; &-\frac{4\pi}{3}R^3[P_{\mathrm{q,g}}(T,\mu_{\mathrm{B}})-P_{\mathrm{had}}(T,\mu_{\mathrm{B}})]\\ &+4\pi R^2\sigma+\;8\pi CR+\;\tau_{\mathrm{critical}}T\ln\left[1+\frac{4\pi R^3}{3}s_{\mathrm{q,g}}\right].\end{aligned} \tag{6}$$

In the equation, there are the usual volume or pressure contribution, surface contribution proportional to the surface tension σ, curvature and shape contribution. As the system undergoes a transition near the temperature $T=170$ MeV, the suitable surface tension obtained near the strong interacting phase is around $50\mathrm{MeV/fm}^2$, which is very much closer to the lattice calculation of surface tension [12]. The shape contribution is an entropy term on account of the fluctuations in droplet shape. τ_{critical} is called the Fisher critical exponent, for which we choose to have a value of 2.2 and $s_{\mathrm{q,g}}$ is the entropy of the QGP droplet.

Since the prefactor is valid for the case where curvature is not important and there exists no derivation of the prefactor with curvature term in the literature, for the present calculation we have not considered the curvature term.

The pressure difference is attributed to $P_{\mathrm{q,g}}(T,\mu_{\mathrm{B}})$, which is the sum of pressure contributed by all the quarks and gluons and $P_{\mathrm{had}}(T,\mu_{\mathrm{B}})$, the sum of pressure contributed by all the hadrons in the hadronic medium with the suitable finite volume correction. In the present calculation, we consider three flavours of quarks, their antiparticles and gluons for the QGP. For the hadronic sector, the pions, nucleons, Δ, λ, Σ, Ξ and the corresponding antiparticles are considered. We have used the current (dynamic) quark masses $m_u=m_d=0$ MeV and $m_s=150$ MeV.

The pressure contribution of quarks and gluons can be calculated by using the equation

$$P_i = \pm\frac{T g_i}{v}\int \mathrm{d}k\rho_{\mathrm{q,g}}(k,\mu)[\ln(1\pm\mathrm{e}^{-(\sqrt{m_i^2+k^2}-\mu_i)/T})+\ln(1\pm\mathrm{e}^{-(\sqrt{m_i^2+k^2}+\mu_i)/T}]\,. \tag{7}$$

In the expression, the fermionic and bosonic particles are indicated by the upper and lower signs. $\rho_{\mathrm{q,g}}(k,\mu)$ is the density of states for the particular particle i (quarks, gluons etc.). g_i and μ_i are the appropriate degeneracy factor and chemical potential. v is the volume of QGP system. The values of the quark chemical potential μ_{q} are taken as 100, 200, 300 and 400 MeV and the electric charge, chemical potential and strange quark chemical potential are considered to be zero.

The pion pressure is given as

$$P_\pi^0 = -\left(\frac{3T}{2\pi^2}\right)\int_0^\infty k^2\mathrm{d}k\,\ln(1-\mathrm{e}^{-\sqrt{m_\pi^2+k^2}/T})\,. \tag{8}$$

Pressure and baryon density contributed by the baryons and antibaryons are given as

$$P_{\mathrm{B}}^0 = -\left(\frac{T}{2\pi^2}\right)g_i\int_0^\infty \mathrm{d}k k^2[\ln(1+\mathrm{e}^{-(\sqrt{m_i^2+k^2}\mp\mu_i)/T})]\,, \tag{9}$$

$$n_{\mathrm{B}}^0 = \frac{g_i}{2\pi^2}\int_0^\infty \mathrm{d}k\;k^2\left[\frac{1}{\exp\left[\left(\sqrt{m_i^2+k^2}\mp\mu_i\right)/T\right]}\right], \tag{10}$$

where g_i, m_i and μ_i are appropriate degeneracy, mass and chemical potential for baryons and antibaryons. Another important point is that hadrons are not considered as point particles. They have finite volume and the above relations are only for point particles. So we need appropriate corrections for these point particle hadrons. The correction factor for these hadrons are taken from the Kouno and Takagi approach [5]. The corrected pressure and number density for baryons are given as

$$P_{\mathrm{h}} = \frac{P_{\mathrm{B}}^0}{1+n_{\mathrm{B}}^0 V_{\mathrm{h}}} + \frac{P_{\bar{\mathrm{B}}}^0}{1+n_{\bar{\mathrm{B}}}^0 V_{\mathrm{h}}} + P_\pi^0\,, \tag{11}$$

$$n_{\mathrm{B}} = \frac{n_{\mathrm{B}}^0}{1+n_{\mathrm{B}}^0 V_{\mathrm{h}}} - \frac{n_{\bar{\mathrm{B}}}^0}{1+n_{\bar{\mathrm{B}}}^0 V_{\mathrm{h}}}\,, \tag{12}$$

where P_{B}^0, $P_{\bar{\mathrm{B}}}^0$, n_{B}^0 and $n_{\bar{\mathrm{B}}}^0$ are the pressure and number density for all the point-like baryons (i.e. nucleons, Δ, λ, Σ, Ξ) and corresponding antibaryons. V_{h} is the hadronic hard core volume and it is defined as $V_{\mathrm{h}} = \frac{4}{3}\pi R^3$, where R is the hadronic hard core radius, which is suitably chosen as $R = 0.9$ fm [5].

Now, we use eq. (1) for calculating the nucleation rate the QGP droplet. The first term in the right-hand side is the prefactor I_0 which is given as

$$I_0 = \frac{\kappa}{2\pi}\Omega_0\,. \tag{13}$$

The dynamical prefactor κ is given by

$$\kappa = \frac{2\sigma}{(\Delta\omega)^2 (R_c)^3} \left[\lambda T + 2 \left(\frac{4}{3}\eta + \zeta \right) \right] , \tag{14}$$

where λ is the thermal conductivity and η , ζ are viscosities of the hadronic phase. The bulk viscosity ζ is very small compared to shear viscosity η and can be neglected. $\Delta\omega$ is the difference in enthalpy densities of the two phases. For these dissipative coefficients we have used the parametrization of Danielwicz [3,13].

$$\eta = \left(\frac{1700}{T} \right)^2 \left(\frac{n}{n_0} \right)^2 + \frac{22}{1 + T^2/1000} \left(\frac{n}{n_0} \right)^{0.7} + \frac{5.8 T^{1/2}}{1 + 160/T^2} , \tag{15}$$

$$\lambda = \left(\frac{0.15}{T} \right) \left(\frac{n}{n_0} \right)^{1.4} + \frac{0.02}{1 + T^4/7 \times 10^6} \left(\frac{n}{n_0} \right)^{0.4} + \frac{0.0225 T^{1/2}}{1 + 160/T^2} . \tag{16}$$

To a first approximation the statistical prefactor Ω_0 is:

$$\Omega_0 = \frac{2}{3\sqrt{3}} \left(\frac{\sigma}{T} \right)^{3/2} \left(\frac{R_c}{\xi_h} \right)^4 . \tag{17}$$

ξ_h is the correlation length in the hadronic phase which is taken as 0.7 fm and n_0 is the normal nuclear matter density.

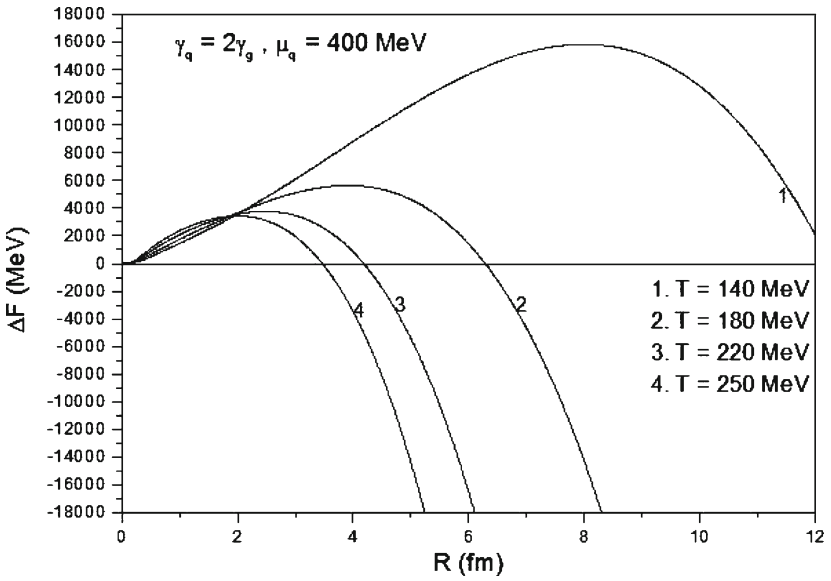

Figure 1. The free energy ΔF as a function of droplet radius R, at $\gamma_g = 1/3$ and $\gamma'_q = 2(1 + \frac{\mu^2}{\pi^2 T^2})\gamma_g$, i.e. $\gamma_q = 2\gamma_g$ for different temperatures, at quark chemical potential $\mu_q = 400$ MeV.

4. Result and conclusion

In this paper, we have calculated the nucleation rate of QGP droplet from the hadronic medium at finite baryon density. For evaluating the nucleation rate, it is required to find the critical free energy change when QGP droplet is formed in the hadronic medium. The nucleation rate depends on the change in critical free energy.

The parameters γ_g and γ_q' are phenomenologically used, with certain modifications due to finite quark chemical potential. The whole set of parameters for quarks are considered after fixing the gluon parameter as the Peshier value, $\gamma_g = 1/3$.

In figures 1–3, we show that with the increase in the value of γ_q there is a decrease in critical free energy and a decrease in critical radius. We find that for very high value of γ_q, critical free energy and critical radius are much lower indicating that droplets are highly unstable. For a vanishingly small value of ΔF_c, hadron–QGP transition takes place spontaneously without the formation of QGP droplets, which is observed in the figures as we increase the parametrization value from $\gamma_q = 2\gamma_g$ to $\gamma_q = 6\gamma_g$. The formation of droplet size is also effected by the quark chemical potential. We fix the quark chemical potential as $\mu_q = 400$ MeV in the figures because, as we increase the quark chemical potential μ_q, the critical free energy and critical radius decrease. For μ_q larger than 400 MeV, the critical free energy and critical radius corresponding to $\gamma_q = 6\gamma_g$ and subsequent higher values of γ_q, are very small and for μ_q smaller than 400 MeV, free energy corresponding to $\gamma_q = 2\gamma_g$ and subsequent lower values, increases without showing any critical value.

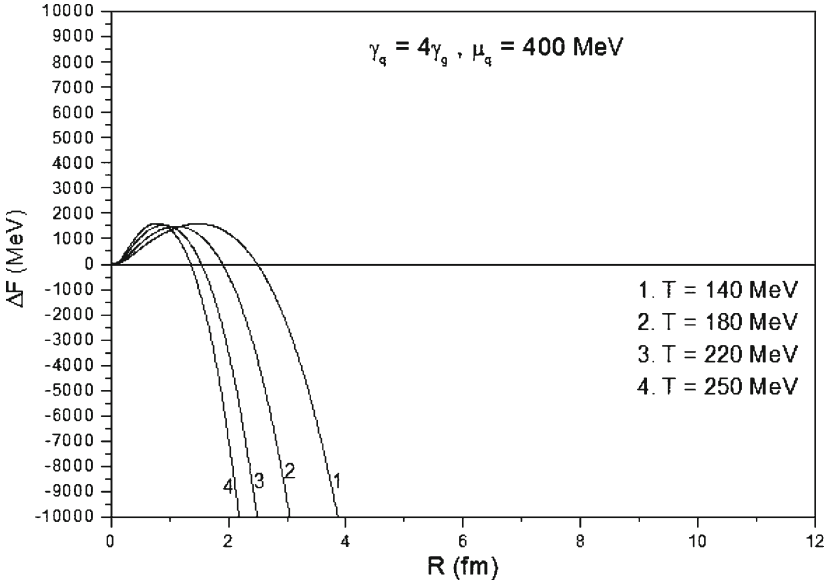

Figure 2. The free energy ΔF as a function of droplet radius R, at $\gamma_g = 1/3$ and $\gamma_q' = 4(1 + \frac{\mu^2}{\pi^2 T^2})\gamma_g$ for different temperatures, at quark chemical potential $\mu_q = 400$ MeV.

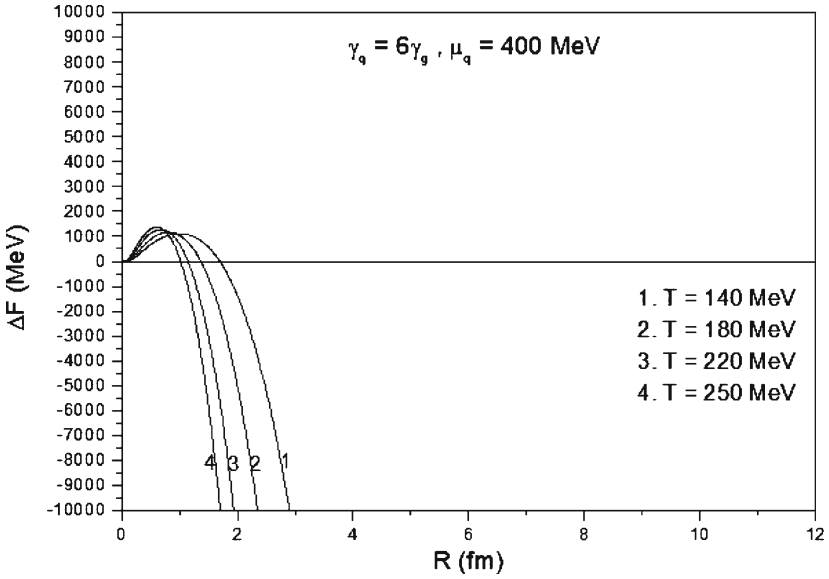

Figure 3. The free energy ΔF as a function of droplet radius R, at $\gamma_{\rm g} = 1/3$ and $\gamma_{\rm q}' = 6(1 + \frac{\mu^2}{\pi^2 T^2})\gamma_{\rm g}$ for different temperatures, at quark chemical potential $\mu_{\rm q} =$ 400 MeV.

Then in figure 4, the variation of nucleation rate with temperature is shown at $\gamma_{\rm q} = 4\gamma_{\rm g}, \gamma_{\rm g} = \frac{1}{3}$ for various values of quark chemical potential. Although the nucleation picture is discussed for $\gamma_{\rm q} = 4\gamma_{\rm g}, \gamma_{\rm g} = \frac{1}{3}$ only, the physical outcome will remain the same for any other value, for example, $\gamma_{\rm q} = 6\gamma_{\rm g}, \gamma_{\rm g} = \frac{1}{3}$. It is not reliable to take any higher values for quark phenomenological parameters since for them the droplet radius is small and hence prefactor estimation will be unreliable. At the same time, it is not right to take the quark phenomenological parameter too low, i.e. $\gamma_{\rm q} \leq 2\gamma_{\rm g}$ since for these low values the nucleation rate vanishes.

From the figure we can recognize the well-known fact that the transition temperature decreases with increase in chemical potential. We would like to mention here that in the context of the present model, the modification in the phenomenological parameter of $\gamma_{\rm q}$ and the dependence of inter-quark potential on quark chemical potential, show this trend of decreasing the transition temperature with chemical potential and this decreasing phenomena cannot be observed in the case of simple quark and gluon flow parameters.

Moreover, in figures 5 and 6 we investigate the variation of the exponential factor and nucleation rate of the QGP droplet with the quark phenomenological parameters. These figures are plotted for 180 MeV temperature and 100 MeV quark chemical potential but the physical outcome will remain the same for any other value of temperature and quark chemical potential. In figure 5, for low values of $\gamma_{\rm q}'$ the exponential factor in nucleation rate equation is vanishingly small and hence nucleation rate vanishes. For higher values of $\gamma_{\rm q}'$, the exponential factor increases. This is also supported by figure 6. Figure 6 clearly shows that with the increase of $\gamma_{\rm q}'$, nucleation rate increases but for very large values of $\gamma_{\rm q}'$,

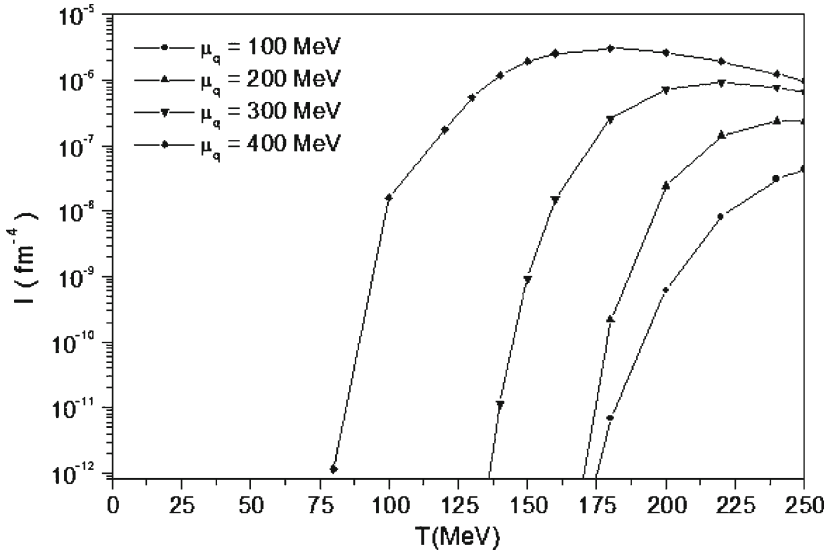

Figure 4. The nucleation rate of plasma droplets as a function of temperature. Curves are for different quark chemical potentials.

it is ambiguous to predict anything about the change in the nucleation rate as the critical radius obtained will be very low when the value of γ_q' is very large. In such a situation the calculation of the prefactor is found to be invalid.

We conclude that even such a simplified model can grasp lots of physics behind hadron–QGP phase transition. The crucial role played by the phenomenological flow parameters

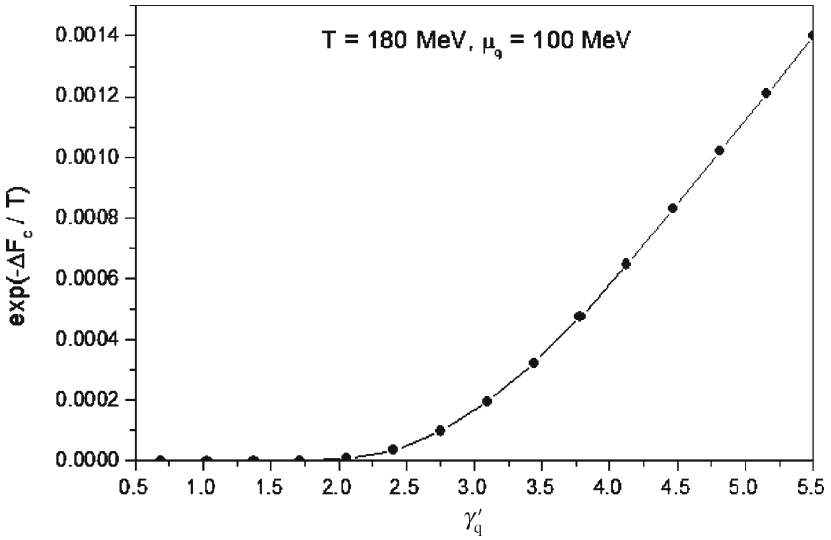

Figure 5. Variation of $\exp(-\Delta F_c/T)$ with respect to γ_q' at a fixed temperature $T = 180$ MeV and quark chemical potential $\mu_q = 100$ MeV.

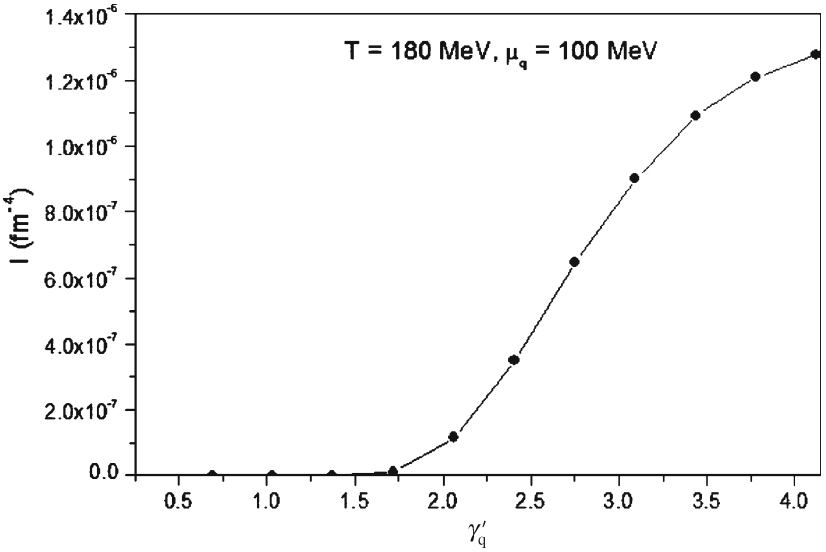

Figure 6. Variation of I (nucleation rate) with respect to γ_q' at a fixed temperature $T = 180$ MeV and quark chemical potential $\mu_q = 100$ MeV.

in developing a statistical model for hot plasma system of QGP indicates their need and importance. It is demonstrated that these parameters should vary with quark chemical potential and temperature.

Acknowledgements

The authors are very thankful to Dr R Ramanathan and Dr K K Gupta for their support in preparing this manuscript. One of the authors D S Gosain is highly grateful to the Department of Physics and Astrophysics for providing an opportunity to do research and to CSIR for providing financial support.

References

[1] C Y Wong, *Introduction to high-energy heavy ion collisions* (World Scientific, Singapore, 1994)
L P Csernai, *Introduction to relativistic heavy-ion collisions* (Wiley, New York, 1994)
R C Hwa, *Quark-gluon plasma* (World Scientific, Singapore, 1990)
[2] H Satz, CERN-TH-2590, 18pp (1978)
F Karsch, E Laermann, A Peikert, Ch Schmidt and S Stickan, *Nucl. Phys.* **B94**, 411 (2001)
F Karsch and H Satz, *Nucl. Phys.* **A702**, 373 (2002)
[3] L P Csernai, J I Kapusta and E Osnes, *Phys. Rev.* **D67**, 045003 (2003)
J I Kapusta, R Venugopal and A P Vischer, *Phys. Rev.* **C51**, 901 (1995)
L P Csernai and J I Kapusta, *Phys. Rev. Lett.* **69**, 737 (1992)
L P Csernai, J I Kapusta, Gy Kluge and E E Zabrodin, *Z. Phys.* **C58**, 453 (1993)

[4] I Mardor and B Svetitsky, *Phys. Rev*. **D44**, 878 (1991)
Gregers Neergard and Jes Madsen, *Phys. Rev*. **D60**, 054011 (1999)
M G Mustafa and A Ansari, *Z. Phys.* **C57**, 51 (1993)
[5] H Kouno and F Takagi, *Z. Phys.* **C42**, 209 (1989)
S Uddin and C P Singh, *Z. Phys.* **C53**, 319 (1992)
[6] L P Cernai and J I Kapusta, *Phys. Rev*. **D46**, 1379 (1992)
R Venugopalan and A P Vischer, *Phys. Rev*. **E49**, 5849 (1994)
[7] S S Singh, D S Gosain, Y Kumar and A K Jha, *Pramana – J. Phys.* **74**, 27 (2010)
[8] R Ramanathan, Y K Mathur, K K Gupta and Agam K Jha, *Phys. Rev*. **C70**, 027903 (2004)
R Ramanathan, K K Gupta, Agam K Jha and S Singh, *Pramana – J. Phys.* **68**, 757 (2007)
[9] A D Linde, *Nucl. Phys.* **B216**, 421 (1983)
E Fermi, *Zeit F. Physik* **48**, 73 (1928)
L H Thomas, *Proc. Camb. Phil. Soc.* **23**, 542 (1927)
H A Bethe, *Rev. Mod. Phys.* **9**, 69 (1937)
[10] A Meng Zhao *et al*, *Mod. Phys. Lett.* **A25**, 47 (2010)
V M Bannur, *Eur. Phys. J.* **C50**, 629 (2007)
[11] G D Yen and M I Gorenstien, *Phys. Rev*. **C59**, 2788 (1999)
A Peshier, B Kampfer, O P Pavlenko and G Soff, *Phys. Lett.* **B337**, 235 (1994)
[12] K Kajantie, L Karkkainen and K Rummukainen, *Nucl. Phys.* **B333**, 100 (1990)
T Bhattacharya, A Gocksck, C Korthals Altes and R D Pisarski, *Phys. Rev. Lett.* **66**, 998 (1991)
[13] P Danielewicz, *Phys. Lett.* **B146**, 168 (1984)
M G Mustafa, D K Srivastava and B Sinha, *Euro. Phys. J.* **C5**, 711 (1998)