

String cosmology in LRS Bianchi type-II dusty Universe with time-decaying vacuum energy density Λ

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Abstract. A model of a cloud formed by massive strings is used as a source of LRS Bianchi type-II with time-decaying vacuum energy density Λ . To construct string cosmological models, we have used the energy–momentum tensor for such strings as formulated by Letelier (1983). The high nonlinear field equations have been solved for two types of strings: (i) massive string and (ii) Nambu string. The expansion θ in the model is assumed to be proportional to the shear σ . This condition leads to $A = \beta B^m$, where A and B are the metric coefficients, m is a constant and β is an integrating constant. Our models are in accelerating phase which is consistent with the recent observations of supernovae type-Ia. The physical and geometrical behaviour of these models are also discussed.

Keywords. LRS Bianchi type-II models; Nambu string; massive string.

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1. Introduction

The problem of the structure formation in the Universe is an important challenge in cosmology. There are two theories for the structure formation of the Universe: (i) which is based on the amplification of quantum fluctuations in a scalar field during inflation and (ii) which is based on symmetry breaking phase transition in the early Universe which leads to the formation of topological defects such as domain walls, cosmic strings, monopoles, textures and other ‘hybrid’ creatures. However, domain walls and monopoles are disastrous for the cosmological models but strings, on the other hand, causes no harm, but can lead to very interesting astrophysical consequences [1]. The cosmic strings play a vital role in the formation of galaxies. They have been created when the symmetry between the strong and electroweak forces is broken due to the phase transition in the early Universe ($t \sim 10^{-36}$ s) [1] as the temperature falls down below some critical temperature ($T_{\text{GUT}} = 10^{28}$ K) as predicted by grand unified theories (GUT) [1–6]. The existence of a large-scale network of strings in the early Universe does not contradict the present-day

observations. The vacuum strings may generate sufficient density fluctuations to explain the galaxy formation [7]. The cosmic strings have coupled stress-energy to the gravitational field. Therefore, the study of gravitational effects of such strings will be interesting. The general relativistic treatment of strings was initiated by Letelier [8,9]. Here we have considered gravitational effects, arisen from strings by coupling stress-energy of strings to the gravitational field. Letelier [9] defined the massive strings as geometric strings (massless) with particles attached along its expansion.

The strings that form the cloud are massive strings instead of geometrical strings. Each massive string is formed by a geometrical string with particles attached along its extension. Hence, the strings that form the cloud are the generalization of Takabayasis relativistic model of strings (called p -strings). This is the simplest model wherein we have particles and strings together. In principle, we can eliminate the strings and end up with a cloud of particles. This desirable property of the model of a string cloud can be used in cosmology since strings are not observed at the present time of evolution of the Universe (see [10–13]).

There are significant observational evidences that show that the expansion of the Universe is undergoing a late time acceleration (Perlmutter *et al* [14–16]; Riess *et al* [17,18]; Garnavich *et al* [19,20]; Schmidt *et al* [21]; Efstathiou *et al* [22]; Spergel *et al* [23]; Allen *et al* [24]; Sahni and Starobinsky [25]; Peebles and Ratra [26]; Padmanabhan [27]; Lima and Maia [28]). This, in other words, amounts to saying that in the context of Einstein's general theory of relativity some sort of dark energy, constant or which varies only slowly with time and space, dominates the current composition of cosmos. The origin and nature of such an accelerating field poses a completely open question. Recently, Riess *et al* [29] have presented an analysis of 156 SNe including a few at $z > 1.3$ from the Hubble Space Telescope (HST) 'GOOD ACS' Treasury survey. They conclude that for present acceleration $q_0 < 0$ ($q_0 \approx -0.7$). Observations (Riess *et al* [29]; Knop *et al* [30]) of type-Ia supernovae (SNe) allow us to probe the expansion history of the Universe leading to the conclusion that the expansion of the Universe is accelerating. Observations strongly favour a small and positive value of the effective cosmological constant with magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$ at the present epoch.

Some of the recent discussions on the cosmological constant 'problem' and on cosmology with a time-varying cosmological constant point out that in the absence of any interaction with matter or radiation, the cosmological constant remains a constant. However, in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying Λ can be found. This entails that energy has to be conserved by a decrease in the energy density of the vacuum component followed by a corresponding increase in the energy density of matter or radiation.

In recent times, the study of anisotropic string cosmological models have generated a lot of research interest. Reddy [31,32], Reddy and Naidu [33], Reddy *et al* [34,35], Rao *et al* [36–39], Pradhan [40,41], Pradhan and Mathur [42], Pradhan *et al* [43–46], Amirhashchi and Hishamuddin [47–49], Tripathi *et al* [50,51] have studied string cosmological models in different contexts. Recently, Amirhashchi and Zainuddin [47] have obtained LRS Bianchi type-II string dust cosmological models for perfect fluid distribution in general relativity. Motivated by the situations discussed above, in this paper, we shall focus upon the problem of establishing a formalism for studying the massive string

in LRS Bianchi II space-time. In this paper, we have investigated a new and general solution for Bianchi type-III cosmological model for a cloud of strings and time-varying cosmological constant which is different from the other solutions. The paper is organized as follows. The metric and the field equations are presented in §2. In §3, we deal with solution of the field equations with cloud of strings and cosmological constant. In §3.1 the field equations have been solved for massive string. We describe some physical and geometric properties of the model in §3.1.1. In §3.2 the field equations have been solved for Nambu or geometric string. We describe some physical and geometric properties of the model in §3.2.1. Finally, in §4, concluding remarks are given.

2. The metric and field equations

We consider the LRS Bianchi type-II metric in the form

$$ds^2 = -dt^2 + B^2(dx + z dy)^2 + A^2(dy^2 + dz^2), \quad (1)$$

where A, B are functions of t only. The energy–momentum tensor for a cloud of strings is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j, \quad (2)$$

where u_i and x_i satisfy the condition

$$u_i u^i = -x^i x_i = -1, \quad u^i x_i = 0, \quad (3)$$

where ρ is the proper energy density for a cloud of strings with particles attached to them, λ is the string tension density, u^i is the four-velocity of the particles and x^i is a unit space-like vector representing the direction of strings. In a co-moving coordinate system, we have

$$u^i = (0, 0, 0, 1), \quad x^i = \left(\frac{1}{B}, 0, 0, 0 \right). \quad (4)$$

The particle density of the configuration is given by

$$\rho = \rho_p + \lambda. \quad (5)$$

The Einstein's field equations (with $8\pi G = 1$ and $C = 1$)

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -T_{ij}, \quad (6)$$

for the metric (1) lead to the following system of equations:

$$2 \frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4} \frac{B^2}{A^4} = \lambda - \Lambda, \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4} \frac{B^2}{A^4} = -\Lambda, \quad (8)$$

$$2 \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{1}{4} \frac{B^2}{A^4} = \rho - \Lambda, \quad (9)$$

where the overdot stands for the first and double overdot for the second derivative with respect to t .

The average scalar factor a for LRS Bianchi type-II is given by

$$a = (A^2 B)^{1/3}. \tag{10}$$

A volume scale factor V is defined as

$$V = a^3 = A^2 B. \tag{11}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{12}$$

where $H_1 = \dot{A}/A$ and $H_2 = \dot{B}/B$ are the directional Hubble's parameters in the directions of x , y and z respectively.

From (11) and (12), we obtain

$$H = \frac{1}{3} \frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3} \left(\frac{\dot{B}}{B} + 2 \frac{\dot{A}}{A} \right). \tag{13}$$

An important observational quantity is the deceleration parameter q , which is defined as

$$q = - \frac{\ddot{a}a}{\dot{a}^2}. \tag{14}$$

The scalar expansion θ , the shear scalar σ^2 and the average anisotropy parameter A_m are defined as

$$\theta = u^i_{;i} = \frac{\dot{B}}{B} + 2 \frac{\dot{A}}{A}, \tag{15}$$

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{2} \left[2 \frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} \right] - \frac{\theta^2}{6}, \tag{16}$$

$$A_m = \frac{1}{3} \sum_i^3 \left(\frac{\Delta H_i}{H} \right)^2, \tag{17}$$

where $\Delta H_i = H_i - H$ ($i = 1, 2, 3$).

3. Solution of the field equations

The field equations (7)–(9) are a system of three equations with five unknown parameters A , B , ρ , λ , Λ . Two additional constraints relating these parameters are required to obtain explicit solutions of the system. First we assume that the expansion θ in the model is proportional to the shear σ . This condition leads to

$$A = \beta B^m, \tag{18}$$

where m is a constant and β is a constant of integration.

From eqs (7) and (9) we get

$$\lambda = 2 \frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4} \frac{B^2}{A^4} + \Lambda, \quad (19)$$

and

$$\rho = 2 \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{1}{4} \frac{B^2}{A^4} + \Lambda. \quad (20)$$

The highly non-linear field equations can be solved for the following two types of geometric strings.

3.1 Case I: Massive string

In this case we assume that the sum of the rest energy density and the tension density for a cloud of strings vanishes [52–54], i.e.

$$\rho + \lambda = 0. \quad (21)$$

From eqs (19)–(21) we obtain

$$2 \frac{\dot{A}\ddot{B}}{AB} + 2 \frac{\dot{A}^2}{A^2} + 2 \frac{\ddot{A}}{A} = \frac{B^2}{A^4} - 2\Lambda. \quad (22)$$

Using (18), eq. (22) can be written as

$$2\ddot{B} + 4m \frac{\dot{B}^2}{B} = \frac{B^{3-4m}}{m\beta} - \frac{2\Lambda}{m} B. \quad (23)$$

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = ff'$, where $f' = df/dB$. Hence eq. (23) takes the form

$$\frac{d}{dB}(f^2) + \frac{4m}{B} f^2 = \frac{B^{3-4m}}{m\beta} - \frac{2\Lambda}{m} B. \quad (24)$$

Equation (24), after integrating, reduces to

$$f^2 = \left(\frac{dB}{dt} \right)^2 = \frac{1}{4\beta m} B^{4(1-m)} - \frac{\Lambda}{m(1+2m)} B^2 + NB^{-4m}, \quad (25)$$

where N is an integrating constant. Equation (25) can be written in the following form:

$$f^2 = aB^{4(1-m)} - bB^2 + NB^{-4m}. \quad (26)$$

To get a deterministic solution, we assume $m = \frac{1}{2}$. In this case eq. (26) takes the form

$$f^2 = MB^2 + NB^{-2}, \quad (27)$$

where

$$M = a - b. \quad (28)$$

Therefore, we find

$$\frac{dB}{\sqrt{MB^2 + NB^{-2}}} = dt. \quad (29)$$

Integrating eq. (29), we obtain

$$B^2 = \left(\frac{N}{M}\right)^{1/2} \sinh[2\sqrt{M}(t + \alpha)], \quad (30)$$

and

$$A^2 = \beta^2 \left(\frac{N}{M}\right)^{1/4} \sinh^{1/2}[2\sqrt{M}(t + \alpha)], \quad (31)$$

where $M > 0$ without any loss of generality.

Thus the metric (1) reduces to

$$\begin{aligned} ds^2 = & -dt^2 + \left(\frac{N}{M}\right)^{1/2} \sinh[2\sqrt{M}(t + \alpha)](dx + z dy)^2 \\ & + \beta^2 \left(\frac{N}{M}\right)^{1/4} \sinh^{1/2}[2\sqrt{M}(t + \alpha)](dy^2 + dz^2). \end{aligned} \quad (32)$$

After using a suitable transformation of coordinates, the model (32) reduces to

$$\begin{aligned} ds^2 = & -dt^2 + \left(\frac{N}{M}\right)^{1/2} \sinh(2\sqrt{MT})(dx + z dy)^2 \\ & + \beta^2 \left(\frac{N}{M}\right)^{1/4} \sinh^{1/2}(2\sqrt{MT})(dy^2 + dz^2). \end{aligned} \quad (33)$$

3.1.1 *The geometric and physical significance of the model.* The energy density (ρ), the string tension (λ), the particle density (ρ_p) and the vacuum energy density (Λ) for the model (33) are given by

$$\rho = \frac{5}{4}M \coth^2(2\sqrt{MT}) - \frac{1}{4\beta^2} + \Lambda \quad (34)$$

$$\lambda = -\frac{5}{4}M \coth^2(2\sqrt{MT}) + \frac{1}{4\beta^2} - \Lambda \quad (35)$$

$$\rho_p = \frac{5}{2}M \coth^2(2\sqrt{MT}) - \frac{1}{2\beta^2} + 2\Lambda \quad (36)$$

$$\Lambda = \frac{(8(-1/4\beta^{-2} - 3/2\beta^{-1})\beta + 5(\coth(2\sqrt{MT}))^2)}{(-16\beta + 10(\coth(2\sqrt{MT}))^2\beta)}. \quad (37)$$

From eqs (34) and (36), we observe that the energy conditions, $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied under conditions

$$\coth^2(2\sqrt{MT}) \geq \frac{1 - 4\beta^4\Lambda}{5M\beta^4}. \quad (38)$$

We also observe that string tension density $\lambda \geq 0$ and cosmological constant $\Lambda \geq 0$ under conditions

$$\coth^2(2\sqrt{MT}) \leq \frac{1 - 4\beta^4\Lambda}{5M\beta^4} \quad (39)$$

and

$$\frac{\coth^2(2\sqrt{MT})}{2} \geq \left(\frac{1 + 3\beta}{\beta} \right) \geq \frac{8}{10}$$

or

$$\frac{\coth^2(2\sqrt{MT})}{2} \leq \left(\frac{1 + 3\beta}{\beta} \right) \leq \frac{8}{10}, \tag{40}$$

respectively. From eq. (34), it is noted that the proper energy density $\rho(t)$ is a decreasing function of time and it approaches a small positive value at the present epoch. This behaviour is clearly depicted in figure 1 as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well-known situations.

From eq. (35) and also by comparing eqs (38) and (39), it is found that the tension density λ is always negative. This behaviour of λ is clearly shown in figure 3. It is pointed out by Letelier [9] that λ may be positive or negative. When $\lambda < 0$, the string phase of the Universe disappears, i.e. we have an anisotropic fluid of particles.

From eq. (37), we see that the cosmological term Λ is a decreasing function of time and it approaches a small positive value at late time. From figure 2, we note this behaviour of cosmological term Λ in the model. Recent cosmological observations suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type-Ia supernova suggest that our Universe may be an accelerating one with induced cosmological density through the cosmological Λ -term. Thus, our model is consistent with the results of recent observations.

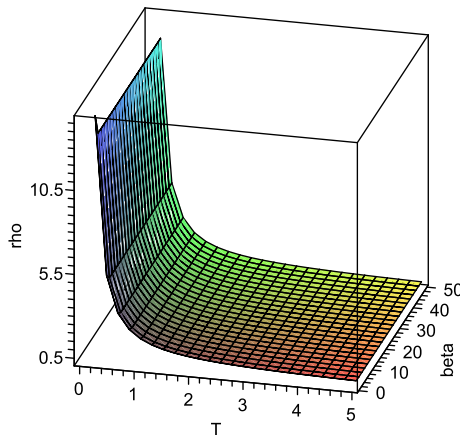


Figure 1. The plot of energy density ρ vs. T and β .

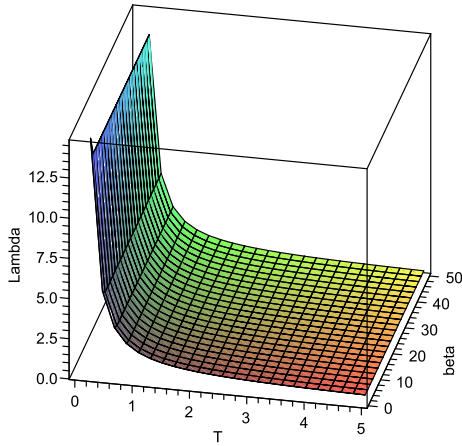


Figure 2. The plot of vacuum energy density Λ vs. T and β .

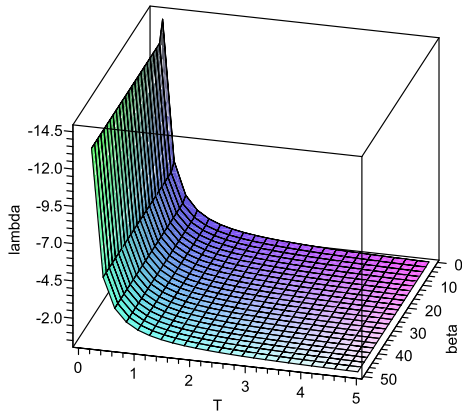


Figure 3. The plot of tension density λ vs. T and β .

The expressions for the scalar of expansion θ , magnitude of shear σ^2 , proper volume V , deceleration parameter q and the average anisotropy parameter A_m for the model (33) are given by

$$\theta^2 = 4M \coth^2(2\sqrt{MT}), \tag{41}$$

$$\sigma^2 = \frac{1}{12}M \coth^2(2\sqrt{MT}), \tag{42}$$

$$V = \beta^2 \left(\frac{N}{M}\right)^{1/2} \sinh(2\sqrt{MT}), \tag{43}$$

$$q = - \left[\frac{(4M/3) - (8M/9) \coth^2(2\sqrt{MT})}{\frac{4}{9}M \coth^2(2\sqrt{MT})} \right], \tag{44}$$

$$A_m = 2 \left(\frac{m-1}{2m+1} \right)^2 = \frac{1}{8}. \quad (45)$$

The rates of expansion H_i in the x , y and z directions are given by

$$H_1 = H_2 = \frac{\sqrt{M}}{2} \coth(2\sqrt{MT}), \quad H_3 = \sqrt{M} \coth(2\sqrt{MT}). \quad (46)$$

Hence the average generalized Hubble's parameter is given by

$$H = \left(\frac{2m+1}{3} \right) \sqrt{M} \coth(2\sqrt{MT}) = \frac{2}{3} \sqrt{M} \coth(2\sqrt{MT}). \quad (47)$$

To indicate whether a model inflates or not one can find sign of deceleration parameter q . A negative sign $-1 \leq q < 0$, indicates inflations whereas positive sign of q corresponds to standard decelerating model. From (44), we observe that

$$q < 0, \quad \text{if } \coth^2(2\sqrt{MT}) < \frac{3}{2}, \quad (48)$$

and

$$q > 0, \quad \text{if } \coth^2(2\sqrt{MT}) > \frac{3}{2}. \quad (49)$$

The model (33) starts with a big bang at $T = 0$. The expansion in the model decreases as time increases. The proper volume of the model increases as time increases. Since σ/θ is constant, the model does not approach isotropy. There is a point-type singularity in the model at $T = 0$ [55]. When $\coth^2(2\sqrt{MT}) < \frac{3}{2}$, the solution gives accelerating model of the Universe whereas when $\coth^2(2\sqrt{MT}) > \frac{3}{2}$, our solution represents decelerating model of the Universe. It is also observed that at $T = (1/2\sqrt{M}) \coth^{-1}(\sqrt{3})$, q approaches -1 as in the case of de-Sitter Universe.

In this case, from eqs (35) and (36), we obtain

$$\frac{\rho_p}{|\lambda|} = 2 > 1. \quad (50)$$

Thus, in our model, the Universe is dominated by massive strings throughout the whole process of evolution [1,56].

3.2 Case II: Nambu string

In this case we assume

$$\rho - \lambda = 0. \quad (51)$$

This corresponds to the state equation for a cloud of massless geometric (Nambu) strings, i.e. $\rho_p = 0$. Therefore, in this case, from eqs (19) and (20), we obtain

$$2 \frac{\dot{A}\dot{B}}{AB} - 2 \frac{\ddot{A}}{A} + \frac{1}{2} \frac{B^2}{A^4} = 0. \quad (52)$$

Putting (18) in (52) we get

$$2\ddot{B} + 2(m-2)\frac{\dot{B}^2}{B} = \frac{B^{3-4m}}{2m\beta^4}. \quad (53)$$

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = ff'$, where $f' = df/dB$. This leads to

$$\frac{d}{dB}(f^2) + \frac{2(m-2)}{B}f^2 = \frac{B^{3-4m}}{2m\beta^4}. \quad (54)$$

Equation (54), after integrating, reduces to

$$\frac{dB}{dt} = \sqrt{-\frac{B^{4(1-m)}}{4m^2\beta^4} + LB^{-2(m-2)}}, \quad (55)$$

where L is an integrating constant.

Hence the metric (1) reduces to

$$ds^2 = \frac{dB^2}{(-\frac{B^{4(1-m)}}{4m^2\beta^4} + LB^{-2(m-2)})} + B^2(dx + z dy)^2 + \beta^2 B^{2m}(dy^2 + dz^2). \quad (56)$$

After making suitable transformation of coordinates, the metric (56) reduces to

$$ds^2 = \frac{dT^2}{(-\frac{T^{4(1-m)}}{4m^2\beta^4} + LT^{-2(m-2)})} + T^2(dx + z dy)^2 + \beta^2 T^{2m}(dy^2 + dz^2). \quad (57)$$

3.2.1 *The geometric and physical significance of the model.* The energy density (ρ), the string tension (λ), the particle density (ρ_p) and the vacuum energy density Λ for the model (57) are given by

$$\rho = \lambda = m(m+2) \left(\frac{1}{4m^2\beta^4} - LT^{2m} \right)^2 T^{-2(4m-3)} + \frac{1}{4\beta^4} T^{-2(2m-1)} + \Lambda, \quad (58)$$

$$\rho_p = 0, \quad (59)$$

$$\Lambda = \left[\left(\frac{3m^2 - 4}{16m^4\beta^8} \right) - \left(\frac{2m^2 + m + 4}{2m^2\beta^4} \right) LT^{2m} - (m^2 - 2m + 4)L^2 T^{4m} \right] \times T^{-2(4m-3)} - \frac{1}{4\beta^4} T^{-2(2m-1)}. \quad (60)$$

From (58), we observe that the energy condition $\rho > 0$ is satisfied under the condition

$$m(m+2) > 0. \quad (61)$$

From eq. (60), it is found that the vacuum energy density $\Lambda > 0$ when $T \geq T_c$, where T_c is the critical time given by

$$T_c = \left[\frac{1}{2\beta^2 \sqrt{\left[\left(\frac{3m^2-4}{16m^4\beta^8} \right) - \left(\frac{2m^2+m+4}{2m^2\beta^4} \right) LT^{2m} - (m^2 - 2m + 4)L^2T^{4m} \right]}} \right]^{1/2(m-1)}. \quad (62)$$

From eq. (58) we note that the proper energy density $\rho(t)$ is a decreasing function of time and it approaches a small positive value at the present epoch. This behaviour is clearly depicted in figure 4 as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well-known situations.

From eq. (60), it is clear that the cosmological constant Λ is a decreasing function of time and it approaches a small positive value at late time which is supported by results from supernova observations recently obtained by high-z supernova team and supernova cosmological project [14–21]. We note this behaviour of cosmological term Λ in figure 5.

The expressions for the scalar of expansion θ , magnitude of shear σ^2 , proper volume V , deceleration parameter q and the average anisotropy parameter A_m for the model (57) are given by

$$\theta = (2m + 1) \left(-\frac{1}{4m^2\beta^4} + LT^{2m} \right) T^{3-4m}, \quad (63)$$

$$\sigma^2 = \frac{(2m - 1)^2}{\sqrt{3}} \left(-\frac{1}{4m^2\beta^4} + LT^{2m} \right)^2 T^{-2(4m-3)}, \quad (64)$$

$$V = \beta^2 T^{2m+1}, \quad (65)$$

$$q = -\frac{3}{2m + 1} \left[\frac{\left(\frac{1-m}{48m^4\beta^8} \right) + \left(\frac{7m-10}{6m^2\beta^4} \right) LT^{2m} + \left(\frac{10-4m}{3} \right) L^2 T^{4m}}{\left(-\frac{1}{4m^2\beta^4} + LT^{2m} \right)^2} \right], \quad (66)$$

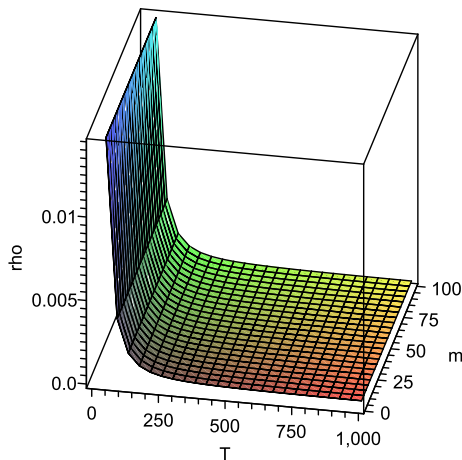


Figure 4. The plot of energy density ρ vs. T and m .

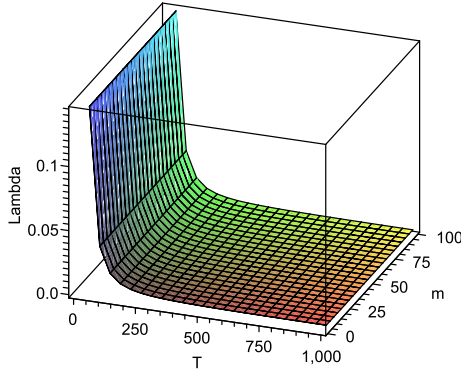


Figure 5. The plot of cosmological constant Λ vs. T and m .

$$A_m = 2 \left(\frac{m - 1}{2m + 1} \right)^2. \tag{67}$$

The rates of expansion H_i in the x , y and z directions are given by

$$H_1 = H_2 = m \left(-\frac{1}{4m^2\beta^4} + LT^{2m} \right) T^{3-4m},$$

$$H_3 = \left(-\frac{1}{4m^2\beta^4} + LT^{2m} \right) T^{3-4m}. \tag{68}$$

Hence the average generalized Hubble's parameter is given by

$$H = \left(\frac{2m + 1}{3} \right) \left(-\frac{1}{4m^2\beta^4} + LT^{2m} \right) T^{3-4m}. \tag{69}$$

From eq. (66), we observe that

$$q < 0, \quad \text{if } T^{4m} \left[2(2m - 5)L - \left(\frac{7m - 10}{2m^2\beta^4} \right) T^{-2m} \right] < \left(\frac{1 - m}{16m^4L\beta^8} \right), \tag{70}$$

and

$$q > 0, \quad \text{if } T^{4m} \left[2(2m - 5)L - \left(\frac{7m - 10}{2m^2\beta^4} \right) T^{-2m} \right] > \left(\frac{1 - m}{16m^4L\beta^8} \right). \tag{71}$$

Also we note that for

$$T^{4m} \left[(2m - 3)L - 3 \left(\frac{m - 1}{2m^2\beta^4} \right) T^{-2m} \right] = \left(\frac{m + 2}{48m^4L\beta^8} \right), \tag{72}$$

$q = -1$ as in the case of de-Sitter Universe.

The model (57) starts with a big bang at $T = 0$. For $m > 3/2$, the expansion in the model decreases as time increases. The proper volume of the model increases as time increases. There is a point-type singularity in the model at $T = 0$ [55]. Since σ/θ is constant, this model does not approach isotropy. From (66) it is clear that our model represents an accelerating Universe for the condition given by eq. (70) and a decelerating model of the Universe under condition given by eq. (71).

In this case, from eqs (58) and (59), we obtain

$$\frac{\rho_p}{|\lambda|} = 0. \tag{73}$$

Hence, in this case the strings dominate over the particles.

4. Conclusion

In this paper we have presented a new solution of Einstein’s field equations for LRS Bianchi type-II space-time with a cloud of strings in the presence of a time-varying cosmological constant. We have considered two cases: (i) massive string and (ii) Nambu string. In both cases our models start with a big bang at $T = 0$ and in both our models are in accelerating phase under appropriate conditions. In Case (i) the Universe is dominated by massive strings throughout the whole process of evolution and λ is always negative whereas in Case (ii) the string dominates over the particle and λ is always positive. In both cases the models do not approach isotropy except for $m = 1$. Our models are realistic and new to the others.

It is also possible to describe cosmological constant on the basis of thermodynamics. By thermodynamics we know that

$$\tau dS = d(\rho V) + \rho dV, \tag{74}$$

where V is the proper volume and τ is the temperature. Therefore

$$\tau \dot{S} = \dot{\rho} + \rho \left(\frac{\dot{B}}{B} + 2\frac{\dot{A}}{A} \right) - \lambda \frac{\dot{A}}{A}. \tag{75}$$

As, in Riemannian geometry without cosmological constant we have

$$\tau \dot{S} = 0, \tag{76}$$

from (75) we find

$$\dot{\rho} + \rho \left(\frac{\dot{B}}{B} + 2\frac{\dot{A}}{A} \right) - \lambda \frac{\dot{A}}{A} = 0. \tag{77}$$

As $\dot{\rho} < 0$, from (77) we obtain

$$3\rho H = \lambda \frac{\dot{A}}{A} + Q, \tag{78}$$

where $Q = -\dot{\rho}$ is positive.

Therefore, since in the case of Nambu string, $\lambda > 0$, we conclude that $3\rho H$ is always positive. But as we mentioned already, for massive string, λ is always negative. In this case, right-hand side of (73) may be negative. Hence

$$3\rho H < 0. \tag{79}$$

From eq. (79), we observe that $H < 0$, which gives the contradictory result and hence we conclude that cosmological constant plays an important role in the evolution of the Universe.

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