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Free convection effects and radiative heat transfer in MHD Stokes problem for the flow of dusty conducting fluid through porous medium

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Abstract. The present note deals with the effects of radiative heat transfer and free convection in MHD for a flow of an electrically conducting, incompressible, dusty viscous fluid past an impulsively started vertical non-conducting plate, under the influence of transversely applied magnetic field. The heat due to viscous dissipation and induced magnetic field is assumed to be negligible. The governing linear partial differential equations are solved by finite difference technique. The effects of various parameters (like radiation parameter N, Prandtl number Pr, porosity parameter K) entering into the MHD Stokes problem for flow of dusty conducting fluid have been examined on the temperature field and velocity profile for both the dusty fluid and dust particles.

Keywords. MHD flow; finite difference technique; porosity; radiative heat transfer; free convection.

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1. Introduction

The flow of a binary mixture of viscous fluid and solid particles has been a subject of great interest to engineers and scientists because such flows occur in powder technology, transport of liquid, slurries in chemical and nuclear processing and in different geophysical situations. Such a binary mixture is called a dusty fluid.

The study of dusty fluids has a significant role in the area of fluidization, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, polymer technology and fluid droplet sprays. Radiative heat transfer is important for systems such as a porous solid fuel bed. The knowledge of the radiative heat transfer is useful for predicting the heat feedback to the burning surface which controls the gasification rate of the energetic material for estimating heat transfer. Radiation effect on flow and heat transfer is important in the context of space technology and processes involving high temperature.

In recent years, the problems of free convective and heat transfer flows through a porous medium under the influence of a magnetic field have attracted the attention of a number of researchers because of their possible applications in many branches of science and technology, such as its applications in transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vapourization in combustion chambers. On the other hand, flow through a porous medium has numerous engineering and geophysical applications. For example, in chemical engineering for filtration and purification process; in agriculture engineering to study the underground water resources; in petroleum technology, to study the movement of natural gas, oil and water through the oil reservoirs. Saffman [1] proposed a mathematical model for dusty fluid flow which was also considered by various authors. Micheal and Norey [2] have studied the motion of a sphere in a dusty fluid. Dennis [3] discussed the steady flow about a sphere at low moderate Reynolds number and studied the Stokes problem for a semi-infinite plate by finite difference method. Soundalgeker *et al* [4] analysed the effect of free convection on MHD Stokes problem for a vertical plate and free convection currents on the velocity field by analytical method. Bestman and Adjepong [5] discussed the buoyancy-induced flow of an electrically conducting fluid with radiative heat transfer past a vertical plate of infinite length. Sharma and Rajput [6] have discussed thermal convection in porous medium in the presence of suspended particles. Mansour [7] studied the effect of variable viscosity on the combined radiation and forced convection on the flow over a flat plate submerged in a porous medium of variable viscosity. Various scientists have discussed steady two-dimensional free convection flows through a porous medium bounded by a vertical infinite porous plate in the presence of radiation [8–10]. Bakier [11] investigated the effect of radiation on mixed convection from a vertical plate in a saturated porous medium. Raptis and Perdikis [12] have studied the unsteady flow through a highly porous medium in the presence of radiation. Kumar and Srivastava [13] have studied two-phase model by introducing the fluid as binary mixture of fluid and suspended particles. Kumar and Srivastava carried out a numerical study of free convection effects on the MHD Stokes problem for the flow of dusty conducting fluid.

The present investigation is an extension of a recent study of Kumar and Srivastava [13] through porous medium with radiative heat transfer. Here we are keenly interested to see the variations in velocities of fluid and dust particles under the stated geometries in the presence of radiative heat transfer and porous medium.

2. Mathematical formulation

We consider the flow of an electrically conducting, incompressible, dusty viscous fluid (e.g. sea water, rain water, and sewage) past an impulsively started infinite vertical non-conducting plate, moving in its own plane. The x^*-axis is taken along the plate in the vertical direction and the y^*-axis is normal to it. A uniformly transverse magnetic field is applied along the axis and induced magnetic field is negligible. The density of fluid is associated with temperature, so it is considered in body force term. N' is assumed to be the number density of particles. i.e. $N' = \rho_p/m$ and $\tau_m = m/k$ where $k = 3mr_p\mu$ is the Stokes resistance coefficient, m is the mass of each particle, r_p is the particle diameter and τ_m is the relaxation time of the particle phase. In most studies of fluid flows, certain simplifying assumptions are usually made for dilute suspension according to Dalal *et al* [14]. The present study of free convection effects on MHD Stokes problems for the flow of dusty conducting fluid is under the following assumptions:

(1) Boussinesq's approximation is valid.
(2) The number density of suspended particles is constant within the fluid, i.e. fluid is spatially homogeneous.
(3) The solid particles are uniformly distributed and they are non-interacting. So only the fluid pressure acts for velocity vector and temperature. Due to this assumption of deficiency of randomness in particle motion, the pressure associated with the particle cloud is negligible. Hence the fluid pressure will be the same as the total pressure of the mixture.
(4) In the modelling of the Stokes resistance coefficient, electromagnetic field is considered such that the field affects only the fluid, not the particles.

Saffman [1] has given the model for the stability of dusty gases. Here MHD Stokes problem for the flow of dusty conducting fluid through porous medium is considered by neglecting the rate of work done by the particles due to the force of interaction with the fluid and the viscous dissipation of the fluid under the influence of radiative heat transfer is given by

$$\frac{\partial u^*}{\partial t^*} = g\beta\left(T^* - T^*_\infty\right) + \nu\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{kN'}{\rho}(u^* - u^*_p) - \sigma B_0^2 u^* - \frac{\nu}{K_1^*}u^*, \tag{1}$$

$$m\frac{\partial u^*_p}{\partial t^*} = k(u^* - u^*_p), \tag{2}$$

$$\rho C_p\frac{\partial T^*}{\partial t^*} = k_T\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{\partial q}{\partial y}. \tag{3}$$

Here, u^* is the velocity of the fluid, u^*_p is the velocity of the fluid particles, T^* is the temperature of the fluid near the plate, T^*_∞ is the temperature of the fluid far away from the plate, g is the acceleration due to gravity, β is the coefficient of volume expansion, ν is the kinematic viscosity, α is the electrical conductivity, B_0 is the applied uniform magnetic field, ρ is the density of the fluid, k_T is the thermal conductivity, t^* is the time, m is the mass of the dust particle, K_1^* is the porosity parameter, N' is the number density of the dust particles and k is the Stokes resistance coefficient. In eq. (3) heat due to viscous dissipation is assumed to be negligible. This is possible when the velocity is small.

Last term on the right-hand side of eq. (3) represents the radiative heat transfer. It is assumed that both walls temperature T_p, T_∞ are high enough to induce radiative heat transfer. Following Cogley *et al* [15], it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by

$$\frac{\partial q}{\partial y} = 4\alpha^2(T_p - T_\infty).$$

The boundary conditions are:
For $t^* = 0$

$$T^* = T_\mathrm{p}, \quad u = 0, \quad u_\mathrm{p} = 0.$$

For $t^* > 0$

$$u^* = u_\mathrm{p}^* = U_0, \quad T^* = T_\mathrm{p} \quad \text{at } y^* = 0$$
$$u^* = u_\mathrm{p}^* \to 0, \quad T^* = T_\infty \quad \text{at } y^* \to \infty. \tag{4}$$

We now introduce the following non-dimensional variables:

$$y = \frac{y^* U_0}{\nu}, \quad t = \frac{t^* U_0^2}{\nu}, \quad u = \frac{u^*}{U_0}, \quad u_\mathrm{p} = \frac{u_\mathrm{p}^*}{U_0}, \quad K_1 = \frac{K_*^1}{U_0},$$
$$T = \frac{T^* - T_\infty}{T_\mathrm{p} - T_\infty}$$

$$\text{Pr (Prandtl Number)} = \frac{\mu C_\mathrm{p}}{k}, \quad \text{Gr (Grashof Number)} = \left(\frac{\nu\beta\left(T_\mathrm{p} - T_\infty\right)}{U_0^3}\right)$$

$$\text{Magnetic field parameter } M^2 = \frac{\sigma\beta_0^2\nu}{\rho U_0^2}$$

$$\text{Porosity parameter } K = \frac{K_1 U_0^4}{\nu^2}, \quad \text{Radiation parameter } N^2 = \frac{4\alpha^2}{k_\mathrm{T} U_0^2}. \tag{5}$$

Then by introducing the relations (5) in eqs (1)–(4), we obtain the following dimensionless partial differential equations:

$$\frac{\partial u}{\partial t} = \mathrm{Gr}\, T + \frac{\partial^2 u}{\partial y^2} + \frac{l}{w}\left(u - u_\mathrm{p}\right) - \left(M^2 + \frac{1}{K}\right) u, \tag{6}$$

$$w\frac{\partial u_\mathrm{p}}{\partial t} = \left(u - u_\mathrm{p}\right), \tag{7}$$

$$\mathrm{Pr}\,\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} + N^2 T. \tag{8}$$

For $t = 0$

$$T(y, 0) = 0, \quad u(y, 0) = 0, \quad u_\mathrm{p}(y, 0) = 0.$$

For $t > 0$

$$u = u_\mathrm{p} = 1, \quad T = 1 \quad \text{at } y = 0$$
$$u = u_\mathrm{p} \to 0, \quad T = 0 \quad \text{at } y \to \infty. \tag{9}$$

Equations (6) and (7) represent a system of simultaneous partial differential equations. Equations (6)–(8) are solved numerically under the initial and boundary conditions (9) using finite difference approximations. The finite difference solutions for eqs (6)–(8) are obtained by writing the equations at midpoint of the computational cell and then replacing the different terms by their second-order central difference approximations in

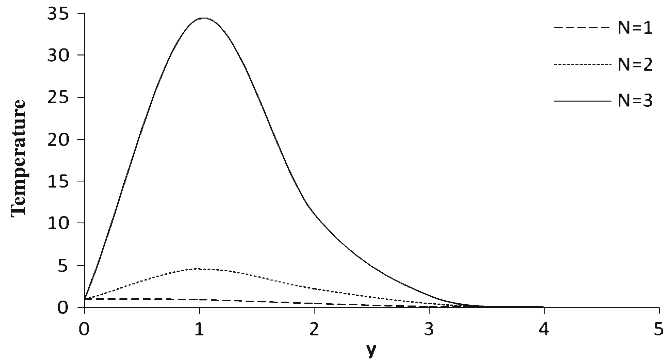

Figure 1. Temperature profile with y for different values of N.

the y-directions. An iterative scheme is used at every time-step to solve the system of difference equations.

3. Graphical results and discussion

Numerical calculations have been carried out for different values of N, Pr, K and for fixed values of M, Gr, l and w. The mesh spacing for y is 1 and time increament for t is 0.2 for finite difference technique. In the following discussion, Gr $= 5$, $M = 0.5$, $l = 1$, $w = 0.5$. These small values like Gr are taken in view of laminar flow.

Figures 1 and 2 respectively represent the variations of temperature with distance y for different values of radiation parameter N and Prandtl number Pr. We can observe that as N increases the temperature increases for some distance y and then decreases as y increases. For $N = 1, 2$ the temperature increases slowly while for $N = 3$ it increases faster near the initial point of the channel. The radiation parameter N is more effective near the initial point ($y = 1$) of the channel than away from the plate or we can say that the

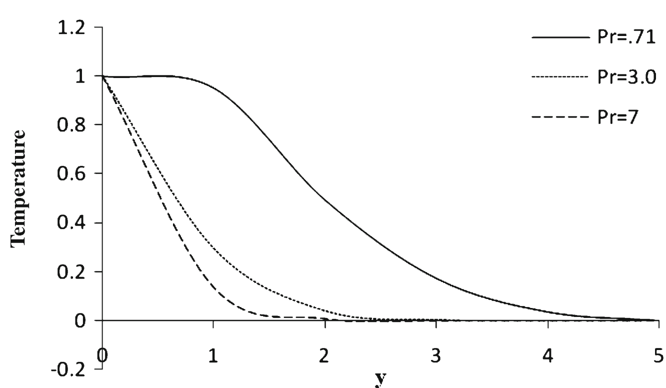

Figure 2. Temperature profile with y for different values of Pr.

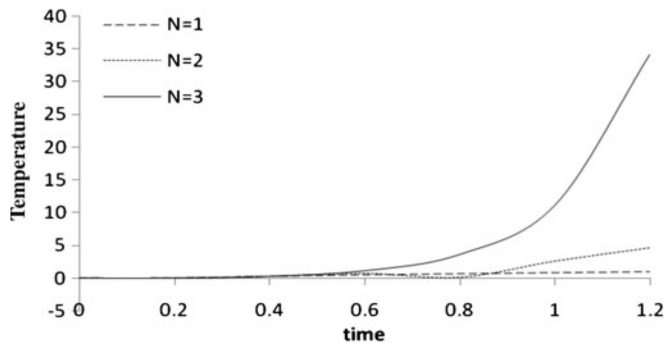

Figure 3. Temperature profile with time for different values of N.

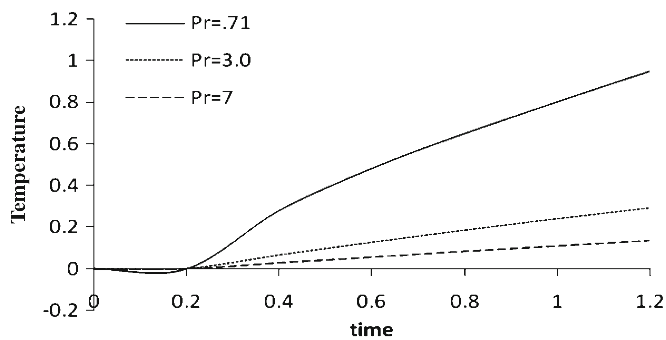

Figure 4. Temperature profile with time for different values of Pr.

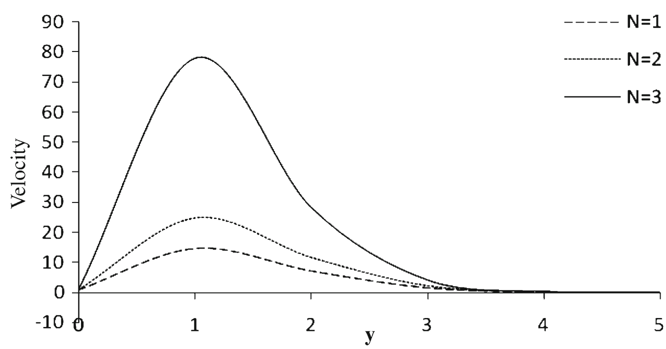

Figure 5. Velocity profile for the fluid with y for different values of N.

effect of N is negligible at the other end of the channel. As Pr increases, the temperature decreases and the temperature tends to zero faster against y for increasing Pr.

Figures 3 and 4 indicate the variations of temperature at $y = 1$ near the initial point of the channel with time for different values of N and Pr respectively. The temperature decreases with increasing values of Pr and it takes some time to show the effect. Increasing values of N increases the temperature. Temperature increases against time for $N = 1$, 2 slowly while for $N = 3$ it increases very fast.

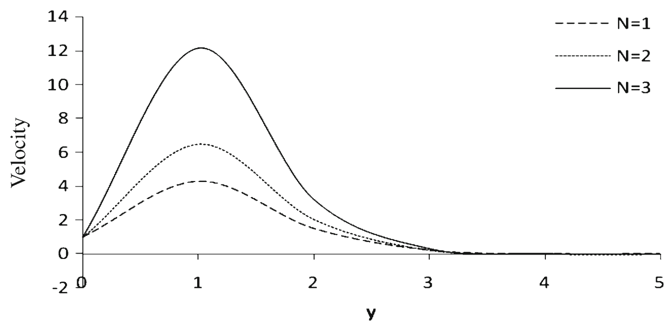

Figure 6. Velocity profile for the dust particles with y for different values of N.

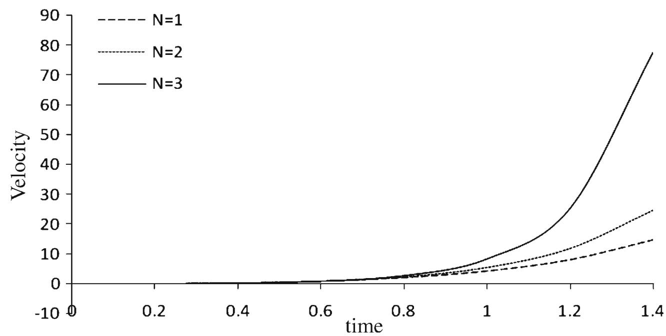

Figure 7. Velocity profile for the fluid with time for different values of N.

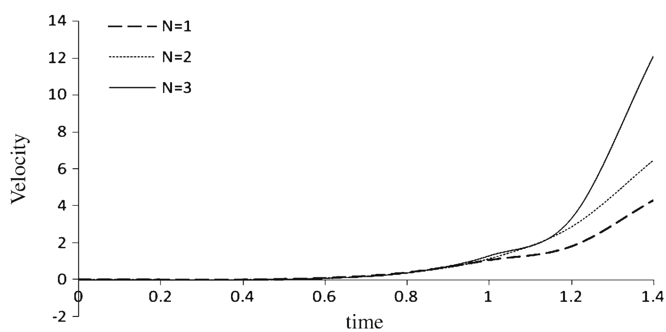

Figure 8. Velocity profile for the dust particles with time for different values of N.

The velocity profiles for both the fluid and dust particles against y for various values of radiation parameter N are displayed in figures 5 and 6. It is observed that increasing radiation parameter N leads to an increase in the velocity of fluid and dust particles. It is evident that at fixed time the velocity of the fluid is always greater than the velocity of the dust particles and velocity of the dust particles tends to zero before the velocity of fluid tends to zero. This is because the fluid velocity is the source for the dust particles velocity. The effect of $N = 3$ on velocity of both fluid and dust particles is very high when compared to the effect when $N = 1, 2$.

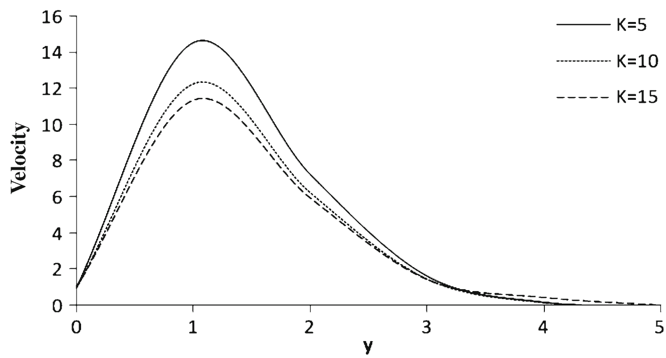

Figure 9. Velocity profile for the fluid with y for different values of K.

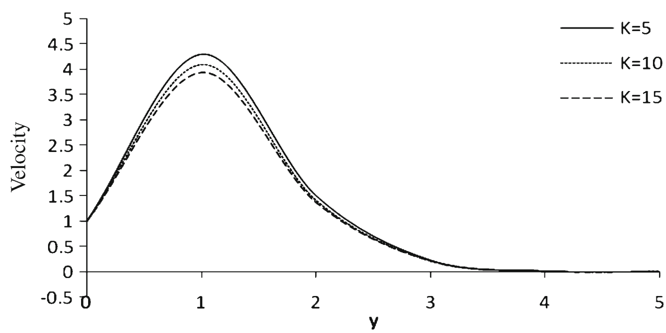

Figure 10. Velocity profile for the dust particles with y for different values of K.

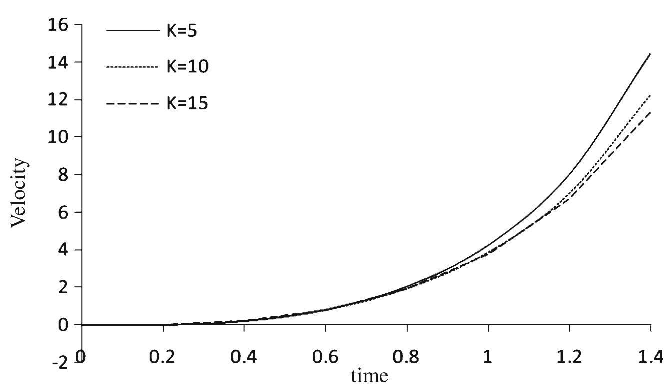

Figure 11. Velocity profile for the fluid with time for different values of K.

Figures 7 and 8 show the velocity profiles for the fluid and the dust particles near the initial point of the plate ($y = 1$) against the time. Velocities of the fluid and dust particles increase with increasing values of radiation parameter and time. We observe from the figures that the radiation parameter takes some time to show its effect, i.e. velocity of the fluid and the dust particles increases after some time.

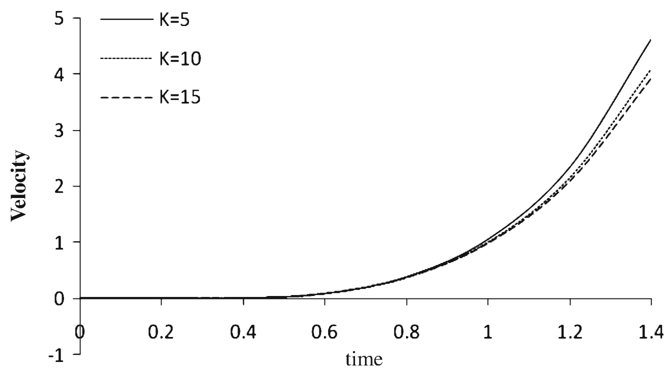

Figure 12. Velocity profile for the dust particles with time for different values of K.

Figures 9 and 10 present the variations of velocity for the fluid and the dust particles for different values of K against y. As K increases, the velocity decreases for the fluid and the dust particles. Effect of porous parameter K on velocity is negligible for its larger values. Velocity of the fluid is greater than the velocity of the dust particles for fixed time. Velocity of the dust particles tends to zero faster than velocity of fluid. This result of porous parameter K gives good agreement with radiation parameter N. Effects of porous parameter K on velocity of the fluid and the dust particles near the initial point of the plate ($y = 1$) are shown in figures 11 and 12. From figures 11 and 12, it is clear that velocities for both the fluid and the dust particles increase with time.

4. Conclusion

The present analysis deals with the effects of radiative heat transfer and free convection in MHD for a flow of an electrically conducting, incompressible, dusty viscous fluid past an impulsively started vertical non-conducting plate, under the influence of transversely applied magnetic field. Our observations are:

(1) Rheological properties of fluids often change with the geometry. When the fluid flows through porous media, pressure difference reduces with increasing porous parameter which implies a reduction in mass of the fluid that reduces the velocities of the fluid and the dust particles.
(2) The radiative heat transfer N reduces the viscosity of the fluid which is the cause of increase in velocity of the dust particles and the dusty fluid.
(3) Our theoretical study finds that increasing Prandtl number decreases the temperature of the fluid.
(4) Velocity is maximum at $y = 1$ due to damping effects of the applied magnetic field, which reduces to zero near the plate and increases when the magnetic field is very large. Thus the presence of magnetic field results in a reduction in velocities. Increase in the value of magnetic field reduces the velocity of fluid as well as particles.

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