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The second Born approximation of electron–argon elastic scattering in a bichromatic laser field

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Abstract. We study the elastic scattering of atomic argon by electron in the presence of a bichromatic laser field in the second Born approximation. The target atom is approximated by a simple screening potential and the continuum states of the impinging and emitting electrons are described as Volkov states. We evaluate the S-matrix elements numerically. The dependence of differential cross-section on the relative phase between the two laser components is presented. The results obtained in the first and second Born approximations are compared and analysed.

Keywords. Second Born approximation; free–free transition; bichromatic laser field; multiphoton process.

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The multiphoton free–free transitions (MFFT) first studied by Bunkin and Fedorov [1], has attracted much attention in physical community. Summaries of these investigations were presented in the book written by Mittleman [2] and some reviews [3,4]. The results obtained in theoretical work treating the laser radiation classically with a single frequency ω and some narrow band multimode approximation are in perfect agreement with the results from experiments by Weingartshofer *et al* [5]. With the development of laser technology, the atomic and molecular processes assisted or induced by powerful and new kinds of laser fields have been researched, especially for the case by multicolour laser. Free–free transitions in a powerful bichromatic laser field has become feasible experimentally to coherently control the phase between the two components of the radiation field. A considerable body of research work was done on the coherent phase control (CPC) of the elastic stattering processes [6–18]. In the above-mentioned papers, the multiphoton processes were treated in the first Born approximation (FBA), whereas in this

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paper, we carried out the calculation in the second-order Born approximation (SBA) and compared the results with that in the FBA formation. Atomic units $\hbar = m = e = 1$ are used throughout.

If, in a laser beam, the density of radiation quanta is so large that the depletion of this beam by emitting or absorbing quanta from it is negligible, then the laser field is treated classically. Hence, in our model, the bichromatic laser field is described as a classical electromagnetic field with ω as the fundamental frequency and 2ω as its second harmonic, i.e., $\boldsymbol{\mathcal{E}}(t) = \boldsymbol{\mathcal{E}}_0[\sin\omega t + \sin(2\omega t + \varphi)]$, where $\boldsymbol{\mathcal{E}}_0$ is the electric field amplitude and the relative phase φ can be arbitrarily changed.

The target atom is described by a screening potential [19]:

$$V(r) = -\frac{Z}{r}\sum_{i=1}^{3} A_i \exp(-\alpha_i r), \tag{1}$$

where r is the position of the electron with respect to the nucleus and Z is the nuclear charge number. For argon, $A_1 = 2.1912$, $A_2 = -2.8252$, $A_3 = 1 - A_1 - A_2$, $\alpha_1 = 5.5470$, $\alpha_2 = 4.5687$ and $\alpha_3 = 2.0446$.

The scattering matrix for the laser-assisted free–free transition in the second Born approximation reads as

$$S_{fi}^{\langle 2\rangle} = S_{fi}^{(1)} + S_{fi}^{(2)}, \tag{2}$$

where

$$S_{fi}^{(1)} = -i\langle \chi_{\mathbf{k}_f} | V | \chi_{\mathbf{k}_i}\rangle. \tag{3}$$

Here $\chi_{\mathbf{k}_i}$ and $\chi_{\mathbf{k}_f}$ are the states of the electrons in the initial and final channels, described by the Volkov wave function:

$$\begin{aligned}\chi_{\mathbf{k}_{i,f}} &= \exp(i\mathbf{k}_{i,f}\cdot r)\exp\left[-iE_{i,f}t - \frac{i}{\omega^2}\mathbf{k}_{i,f}\cdot\boldsymbol{\mathcal{E}}_0\sin\omega t\right] \\ &\quad\times \exp\left[-\frac{i}{4\omega^2}\mathbf{k}_{i,f}\cdot\boldsymbol{\mathcal{E}}_0\sin(2\omega t + \varphi)\right],\end{aligned} \tag{4}$$

where $\mathbf{k}_{i,f}$ are the wave vectors of the incident and scattered electrons, and $E_{i,f}$ are the corresponding kinetic energies.

Using the potential of eq. (1) and the wave functions in eq. (4), the first term of the $S_{fi}^{\langle 2\rangle}$ can be recast as

$$S_{fi}^{(1)} = -2\pi i\sum_l T_{fi}^{(1)}(l)\delta(E_f - E_i + l\omega). \tag{5}$$

$T_{fi}^{(1)}(l)$ is the ionization amplitude accompanying the exchange of l photons with the laser field ($l > 0$ for emission and $l < 0$ for absorption) and is given as

$$T_{fi}^{(1)}(l) = B_l\left(\lambda, \frac{1}{4}\lambda, \varphi\right)V(\mathbf{k}_{f,i}), \tag{6}$$

in which

$$V(\mathbf{k}_{f,i}) = \int \mathrm{d}\mathbf{r}\, \mathrm{e}^{-\mathrm{i}(\mathbf{k}_f - \mathbf{k}_i)\cdot\mathbf{r}}V(r). \tag{7}$$

Clearly it is just the Fourier transformation of the potential. The other term has the form:

$$B_l\left(\lambda, \frac{1}{4}\lambda, \varphi\right) = \sum_{n=-\infty}^{\infty} J_{l-2n}(\lambda) J_n\left(\frac{1}{4}\lambda\right) \exp(-\mathrm{i}n\varphi), \tag{8}$$

which is the generalized Bessel function with $\lambda = (\mathbf{k}_f - \mathbf{k}_i) \cdot \boldsymbol{\mathcal{E}}_0/\omega^2$. J_n is the ordinary Bessel function.

The second term of SBA matrix is:

$$\begin{aligned} S_{fi}^{(2)} &= -i \int \mathrm{d}\mathbf{r} \int \mathrm{d}\mathbf{r}' \int_{-\infty}^{\infty} \mathrm{d}t \int_{-\infty}^{\infty} \mathrm{d}t' \chi_{\mathbf{k}_f}^*(\mathbf{r}, t) \\ &\quad \times V(r) G(\mathbf{r}, t; \mathbf{r}', t') V(r') \chi_{\mathbf{k}_i}(\mathbf{r}', t), \end{aligned}$$

where $G(\mathbf{r}, t; \mathbf{r}', t')$ is the Green function which is given as

$$G(\mathbf{r}, t; \mathbf{r}', t') = -\frac{i}{(2\pi)^3} \int \mathrm{d}\mathbf{k} \chi_{\mathbf{k}}(\mathbf{r}, t) \chi_{\mathbf{k}}^*(\mathbf{r}, t) u(t - t'). \tag{9}$$

Here $u(t - t')$ is the step function.

Using eqs. (4) and (9), we get

$$S_{fi}^{(2)} = -2\pi i \sum_l T_{fi}^{(2)}(l) \delta(E_f - E_i + l\omega), \tag{10}$$

where the photon-number-resolved transition amplitude can be expressed as

$$\begin{aligned} T_{fi}^{(2)}(l) &= \frac{1}{(2\pi)^3} \sum_m \int \mathrm{d}\mathbf{k} \frac{1}{E_i - E - m\omega + i\eta} \\ &\quad \times V(\mathbf{k}_f, \mathbf{k}) V(\mathbf{k}, \mathbf{k}_i) B_m\left(\lambda_1, \frac{1}{4}\lambda_1, \varphi\right) \\ &\quad \times B_{l-m}\left(\lambda_2, \frac{1}{4}\lambda_2, \varphi\right), \end{aligned} \tag{11}$$

where η is a small positive quantity.

So we can obtain the SBA scattering amplitude as

$$T_{fi}^{(2)}(l) = T_{fi}^{(1)}(l) + T_{fi}^{(2)}(l). \tag{12}$$

The differential cross-sections (DCS) for the net exchange of l photons between the colliding system and the bichromatic laser field can be described as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{2\pi}\right)^2 \frac{k_f}{k_i} |T_{fi}^{(2)}(l)|^2. \tag{13}$$

For numerical calculation, we studied the dependence of differential cross-section for electron–argon atom scattering on the relative phase angle φ between the two laser components under the geometry of the experiment by Weingartshofer *et al* [5]. The angle between the polarization vector $\boldsymbol{\mathcal{E}}$ and the momentum of the incident electron $\mathbf{k}_i$ is $\phi = 38°$, the momentum $\mathbf{k}_f$ of the scattered electron is in the plane defined by the polarization vector $\boldsymbol{\mathcal{E}}$ and $\mathbf{k}_i$. The bichromatic laser parameters $\omega = 0.117$ eV and $\mathcal{E}_0 = 2.7 \times 10^8$ V/cm.

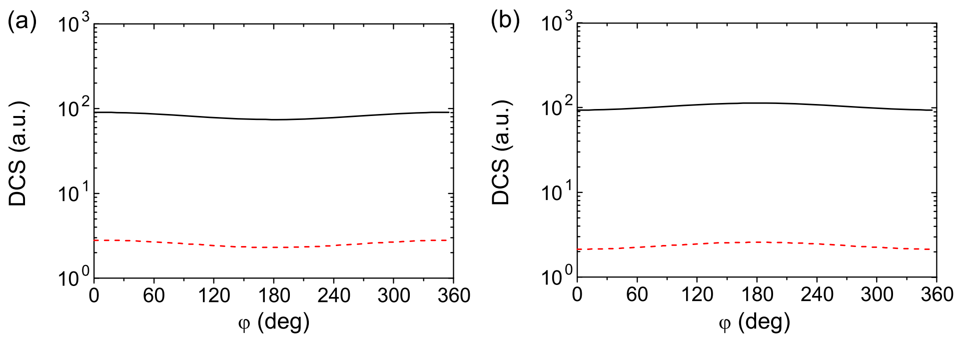

Figure 1. (**a**) DCS vs. φ for elastic electron–argon scattering with the emission of one photon ($l = 1$). The scattered angle of emitting electron $\theta = 13°$. The kinetic energies of the incident electron $E_i = 9.5$ eV. (—) SBA results; (- - -) FBA results. (**b**) The same as (a), but for one-photon absorption ($l = -1$).

In figure 1, we show the DCS vs. the phase angle φ. The CPC effects are not very apparent for one-photon exchange process. The distribution of the controlling effect is centred on $\varphi = 180°$. Such a phenomenon is more distinct for the processes with more than one photons transfer which could be seen clearly in figures 2 and 3. This may be due to the fact that the generalized Bessel function satisfies the relationship [4]: $B_{-l}(a, b, \varphi) = (-1)^l B_l^*(a, b, \varphi - \pi)$. Moreover, the results have some prominent improvements in the second Born approximation. This indicates that the intermediate states have significant contribution to the collision processes.

Figure 2 displays DCS vs. φ for two-photons transfer. As depicted in the figure, DCS for $l = \pm 2$ are much smaller than for $l = \pm 1$. This shows that the probability of two-photons transfer between laser field and the colliding system is small. With the increase of $|l|$ (not presented here), the DCS becomes smaller and smaller. In two-photons emission ($l = 2$), more electrons are emitted at $\theta = 13°$ when the phase difference between the laser components is near $\varphi = 180°$ where the CPC effect reaches the maximum; while

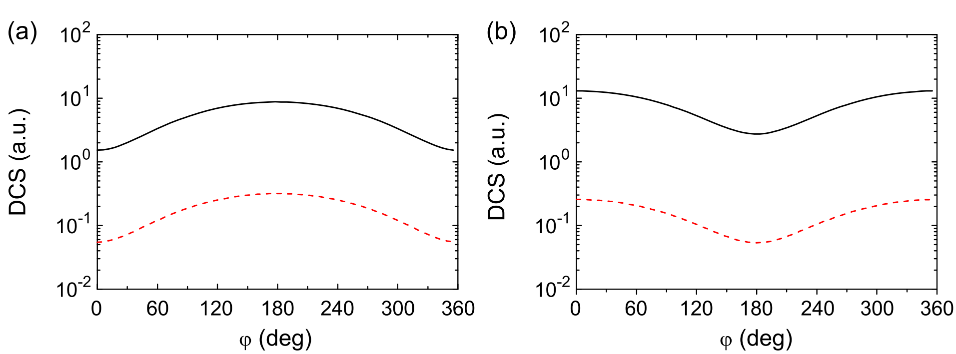

Figure 2. (**a**) The parameters are the same as for figure 1 except for the number of the exchanged photons. For (**a**) $l = 2$ and for (**b**) $l = -2$. (—) SBA results; (- - -) FBA results.

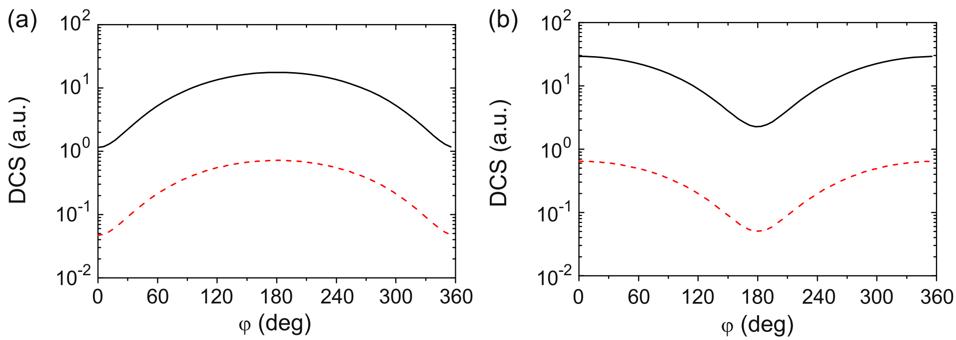

Figure 3. The parameters are the same as for figure 2 except for the impact energy. In this figure, the energy of the incoming electron $E_i = 19.5$ eV. (—) SBA results; (- - -) FBA results.

for $l = -2$, the situation is inverse. The second-order corrections are still distinct, and for different φ, the contribution of the intermediate states are almost the same.

In figure 3, we show the influence of the impact energy on the collision process. It is evident that the DCS becomes larger when the energy of the incident electron increases and the CPC effects are more prominent.

In summary, the electron–argon elastic scattering in a bichromatic laser field is investigated in the second Born approximation. The dependence of DCS on the relative phase is analysed. The DCS are significantly improved in the second Born approximation. We attribute it to the contributions of the intermediate states.

Acknowledgements

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References

[1] F V Bunkin and M V Fedorov, *Sov. Phys. Rep.* **22**, 844 (1996)
[2] M H Mittleman, *Introduction to the theory for laser–atom interactions* (Plenum, New York, 1982)
[3] C J Joachain, M Dörr and N J Kylstra, *Adv. At. Mol. Opt. Phys.* **42**, 225 (2000)
[4] F Ehlotzky, *Phys. Rep.* **345**, 175 (2001)
[5] A Weingartshofer, J K Holemes, J Sabbagh and S L Chin, *J. Phys.* **B16**, 1805 (1983)
[6] J F Sun, M C Liang and Z L Zhu, *Chin. Phys. Lett.* **24**, 2572 (2007)
[7] D Nehari, J Holmes, K M Dunseath and M Terao-Dunseath, *J. Phys.* **B43**, 025203 (2010)
[8] B A Harak, L Ladino and N L S Martin, *Bull. Am. Phys. Soc.* **55**, 5 (2010)
[9] M Ghalim and F Mastour, *J. Phys.* **B32**, 3783 (1999)
[10] S T Zhang, J Chen and S M Li, *Can. J. Phys.* **80**, 969 (2002)

[11] M Protopapas, P L Knight and K Burnett, *Phys. Rev.* **A49**, 1945 (1994)
[12] J Z Kamiński and F Ehlotzky, *Phys. Rev.* **A50**, 4404 (1994)
[13] S Varró and F Ehlotzky, *J. Phys.* **B28**, 1613 (1995)
[14] S T Zhang and J Chen, *J. Phys.* **B34**, 2201 (2001)
[15] S Varró and F Ehlotzky, *J. Phys.* **B30**, 1061 (1995)
[16] S Varró and F Ehlotzky, *Phys. Rev.* **A47**, 715 (1993)
[17] A Cionga, F Ehlotzky and G Zloh, *J. Phys.* **B34**, 2057 (2001)
[18] D B Miloševic and F Ehlotzky, *Phys. Rev.* **A56**, 3879 (1997)
[19] F Salvat, J D Martínez, R Mayol and J Parellada, *Phys. Rev.* **A36**, 467 (1987)