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Energy of vanishing flow in heavy-ion collisions: Role of mass asymmetry of a reaction

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Abstract. We aim to understand the role of Coulomb interactions as well as different equations of state on the disappearance of transverse flow for various asymmetric reactions leading to the same total mass. For the present study, the total mass of the system is kept constant ($A_{\mathrm{TOT}} = 152$) and mass asymmetry of the reaction is varied between 0.2 and 0.7. The Coulomb interactions as well as different equations of state are found to affect the balance energy significantly for larger asymmetric reactions.

Keywords. Mass asymmetry; balance energy; equations of state; Coulomb interactions.

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1. Introduction

The heavy-ion physics has attracted much attention during the last three decades [1–5]. The behaviour of nuclear matter under extreme conditions of temperature, density, angular momentum etc., is a very important aspect of heavy-ion physics. One important quantity which has been used extensively to study this hot and dense nuclear matter is the collective transverse in-plane flow [1,2,6]. This quantity vanishes at a certain incident energy. This energy is dubbed as balance energy (E_{bal}) or the energy of vanishing flow (EVF) [6,7]. This is due to the counterbalancing of attractive mean field at low incident energies and repulsive nucleon–nucleon (NN) collisions at higher incident energies. The balance energy of the masses ranging from $C^{12} + C^{12}$ to $U^{238} + U^{238}$ at different colliding geometries was studied experimentally and theoretically and found to be sensitive with the composite mass of the system [6,8] as well as with the impact parameter of a reaction [6,9,10].

With the passage of time, isospin degree of freedom in terms of symmetry energy and NN cross-section is found to affect the balance energy or energy of vanishing flow and related phenomenon in heavy-ion collisions [4,5,11,12].

Experimentally, Pak *et al*, studied the isospin effects on the collective flow and balance energy at central and peripheral collision geometries [11]. On the other hand, theoretically, this effect was studied using the isospin-dependent Boltzmann–Uehling–Uhlenbeck (IBUU) model [3,5], and isospin-dependent quantum molecular dynamics (IQMD) model [9,12–14].

As noted, balance energy is due to the counterbalancing of the attractive mean-field and repulsive nucleon–nucleon collisions. The Coulomb interaction in intermediate energy heavy-ion collisions is expected to play a dominant role in balance energy due to its repulsive nature. These effects are supposed to be more pronounced in the presence of isospin effects [15]. A comparative study which will show the shift in balance energy due to Coulomb interactions in the presence of isospin effects by taking into account the mass asymmetry of reaction in a controlled fashion is still missing in the literature. The second point is the mass asymmetry of the reaction. In some of the studies, the mass asymmetry of a reaction is taken into consideration but not in other studies, which is very important to study the isospin effects [15,16]. The mass asymmetry of the reaction can be defined by the parameter $\eta = |(A_\mathrm{T} - A_\mathrm{P})/(A_\mathrm{T} + A_\mathrm{P})|$, where A_T and A_P are the masses of the target and projectile. $\eta = 0$ corresponds to the symmetric reactions, whereas non-zero values of η correspond to different mass asymmetries of the reaction. It is worth mentioning that the reaction dynamics in a symmetric reaction ($\eta = 0$) can be quite different from the asymmetric reaction ($\eta \neq 0$) [17]. This is due to the deposition of excitation energy in the form of compressional energy and thermal energy in symmetric and asymmetric reactions, respectively. The effect of mass asymmetry of a reaction on multifragmentation was studied many times in the literature [15–17]. The relative multiplicity of various fragments follows the hyperbolic behaviour [17]. Unfortunately, very little study is available for the mass asymmetry of the reaction in terms of transverse in-plane flow.

In this paper, we shall perform the first ever study for the balance energy in terms of mass asymmetry of the reaction and then observe the effect of Coulomb interactions, symmetry energy and different equations of state. The IQMD model used for the present analysis is explained in §2. The results are presented in §3, leading to the conclusions in §4.

2. The model

The isospin-dependent quantum molecular dynamics (IQMD) [2,14] model treats different charge states of nucleons, deltas and pions explicitly, as inherited from the Vlasov–Uehling-Uhlenbeck (VUU) model. The IQMD model has been used successfully in analysing a large number of observables such as collective flow, multifragmentation and nuclear stopping etc. from low to relativistic energies.

In this model, baryons are represented by Gaussian-shaped density distributions as follows:

$$f_i(r, p, t) = \frac{1}{\pi^2\hbar^2} \mathrm{e}^{-(r-r_i(t))^2/2L} \mathrm{e}^{-(p-p_i(t))^2 2L/\hbar^2}. \tag{1}$$

Nucleons are initialized in a sphere with radius $R = 1.12A^{1/3}$ fm, in accordance with the liquid drop model. Each nucleon occupies a volume of $\hbar^3$ so that the phase space is uniformly filled. The initial momenta are randomly chosen between 0 and the Fermi momentum p_F. The nucleons of the target and projectile interact via two- and three-body

Skyrme forces and the Yukawa potential. The isospin degrees of freedom are treated explicitly by employing a symmetry potential and explicit Coulomb forces between the protons of the colliding target and the projectile. This helps us to achieve the correct distribution of protons and neutrons within the nucleus.

The hadrons propagate using the Hamilton equations of motion:

$$\frac{\mathrm{d}\vec{r}_i}{\mathrm{d}t} = \frac{\mathrm{d}\langle H\rangle}{\mathrm{d}\vec{p}_i}; \qquad \frac{\mathrm{d}\vec{p}_i}{\mathrm{d}t} = -\frac{\mathrm{d}\langle H\rangle}{\mathrm{d}\vec{r}_i}, \tag{2}$$

where

$\langle H\rangle = \langle T\rangle + \langle V\rangle$ is the Hamiltonian.

$$= \sum_i \frac{p_i^2}{2m_i} + \sum_i \sum_{j>i} \int f_i(\vec{r}, \vec{p}, t) V^{ij}(\vec{r}\,', \vec{r}) f_j(\vec{r}\,', \vec{p}\,', t)\mathrm{d}\vec{r}\,\mathrm{d}\vec{r}\,'\,\mathrm{d}\vec{p}\,\mathrm{d}\vec{p}\,'. \tag{3}$$

The baryon–baryon potential V^{ij} which is a term in the above relation reads as

$$\begin{aligned} V^{ij}(\vec{r}\,' - \vec{r}) &= V^{ij}_{\mathrm{Skyrme}} + V^{ij}_{\mathrm{Yukawa}} + V^{ij}_{\mathrm{Coul}} + V^{ij}_{\mathrm{Sym}} \\ &= t_1\delta(\vec{r}\,' - \vec{r}) + t_2\delta(\vec{r}\,' - \vec{r})\rho^{\gamma-1}\left(\frac{\vec{r}\,' + \vec{r}}{2}\right) \\ &+ t_3\frac{\exp(|\,\vec{r}\,' - \vec{r}\,|/\mu)}{(|\,\vec{r}\,' - \vec{r}\,|/\mu)} + \frac{Z_i Z_j e^2}{|\,\vec{r}\,' - \vec{r}\,|} \\ &+ t_4\frac{1}{\rho_0}T_3^i T_3^j \delta(\vec{r}_i\,' - \vec{r}_j). \end{aligned} \tag{4}$$

In the above equation, $\mu = 1.5$ fm, $t_3 = -6.66$ MeV and $t_4 = 100$ MeV. The values of t_1 and t_2 depend on the values of α, β and γ [2]. Here Z_i and Z_j denote the charges of the ith and jth baryon, and T_3^i, T_3^j are their respective T_3 components (i.e. we have $1/2$ for protons and $-1/2$ for neutrons). The parameters μ and $t_1, ..., t_6$ are adjusted to the real part of the nucleonic optical potential.

3. Results and discussion

As discussed earlier, mass asymmetry of a reaction is found to affect the phenomena of multifragmentation in intermediate energy heavy-ion collisions [16,17]. On the other hand, system mass dependence of balance energy was studied many times in the literature [6]. To check the effect of Coulomb interactions and mass asymmetry of a reaction on the balance energy, we have fixed $A_{\mathrm{TOT}} = A_{\mathrm{T}} + A_{\mathrm{P}} = 152$ and varied the mass asymmetry of the reaction just like this: ${}_{26}\mathrm{Fe}^{56} + {}_{44}\mathrm{Ru}^{96}$ ($\eta = 0.2$), ${}_{24}\mathrm{Cr}^{50} + {}_{44}\mathrm{Ru}^{102}$ ($\eta = 0.3$), ${}_{20}\mathrm{Ca}^{40} + {}_{50}\mathrm{Sn}^{112}$ ($\eta = 0.4$), ${}_{16}\mathrm{S}^{32} + {}_{50}\mathrm{Sn}^{120}$ ($\eta = 0.5$), ${}_{14}\mathrm{Si}^{28} + {}_{54}\mathrm{Xe}^{124}$ ($\eta = 0.6$), ${}_{8}\mathrm{O}^{16} + {}_{54}\mathrm{Xe}^{136}$ ($\eta = 0.7$). The mass asymmetry of a reaction with multifragmentation in this fashion is varied many times [16,17]. The whole reaction dynamics is studied at semicentral geometry ($\hat{b} = b/b_{\mathrm{max}}$, where b is a particular impact parameter in Fermi (fm) and $b_{\mathrm{max}} = 1.12(A_{\mathrm{T}}^{1/3} + A_{\mathrm{P}}^{1/3})$ by varying the incident energy between 50 and 250 MeV/nucleon with an increment of 50 MeV/nucleon by employing hard as well as soft equations of state. Our main purpose here is to understand the effect of equations of state and Coulomb interactions on the energy of vanishing flow or alternatively, on the balance energy by taking into account the mass asymmetry of a reaction.

The directed transverse flow is calculated using $\langle P_x^{\mathrm{dir}} \rangle$ [6]

$$\langle P_x^{\mathrm{dir}} \rangle = \frac{1}{A} \sum_i \mathrm{sgn}\{Y(i)\} P_x(i), \tag{5}$$

where $Y(i)$ and $P_x(i)$ are, respectively, the rapidity distribution and transverse momentum of the ith particle.

We display in figure 1, the variation of mass asymmetry η on directed flow $\langle P_x^{\mathrm{dir}} \rangle$ at an incident energy of $E = 50$ MeV/nucleon. Figure 1a represents the directed transverse flow while figure 1b represents the relative Coulomb effect with respect to the mass asymmetry of the reaction. This relative effect is calculated as

$$\langle \Delta P_x^{\mathrm{dir}} |_{\mathrm{Coul}} \rangle = \langle P_x^{\mathrm{dir}} \rangle_{\mathrm{Coul+Sym}} - \langle P_x^{\mathrm{dir}} \rangle_{\mathrm{No\,Coul+Sym}}. \tag{6}$$

As is evident from the figure, directed flow is found to increase in a systematic manner at $E = 50$ MeV/nucleon with mass asymmetry of the reaction. The inclusion of Coulomb interactions does not alter the conclusions. Note that the mass asymmetry of the reaction increases with N/Z, except for $\eta = 0.4$ and 0.6. Moreover, the increase in the mass asymmetry is governed by the increase in $A_{\mathrm{T}} - A_{\mathrm{P}}$, $Z_{\mathrm{T}} - Z_{\mathrm{P}}$ and $N_{\mathrm{T}} - N_{\mathrm{P}}$. The percentage increase of the neutron excess $N_{\mathrm{T}} - N_{\mathrm{P}}$ is increasing over the rest of the two factors because the symmetry potential for the neutron-rich systems is stronger than the neutron-poor systems due to the large relative neutron strength. Furthermore, the symmetry

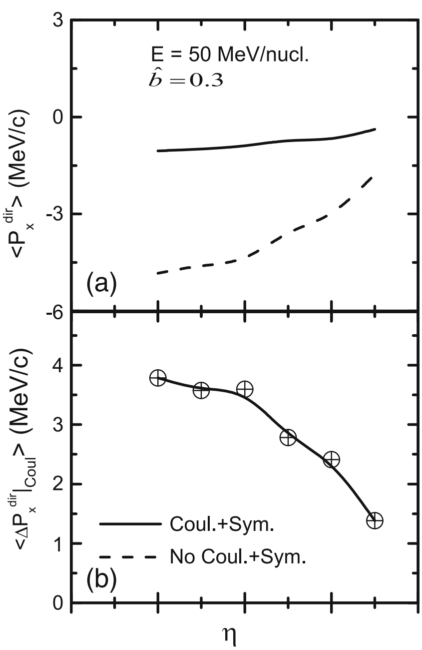

Figure 1. **(a)** The asymmetry dependence of directed flow using soft equation of state, **(b)** the relative effect of Coulomb interactions. The different lines in the figure are representing the effect of symmetry energy and Coulomb interactions.

potential is repulsive for neutrons and attractive for protons. On the other hand, more negative value of directed flow (dominating the mean field) is observed in the absence of Coulomb interactions. This is due to the enhancement of the chemical and mechanical instability domains in the absence of Coulomb interactions [18]. Similar type of study and conclusion was done for nuclear stopping by Liu using IQMD model [15]. To further strengthen our interpretation of the results, the relative effect of Coulomb interactions $\langle \Delta P_x^{\mathrm{dir}}|_{\mathrm{Coul}}\rangle$ is studied. The relative effect of the Coulomb interactions is found to decrease with increase in the mass asymmetry of a reaction. For further study, we have performed the simulations using different equations of state as well as different options of Coulomb (Coul-on and Coul-off) because we want to see the shift in the balance energy due to both these factors.

In figures 2 and 3, we display the excitation function of directed flow at different asymmetries from $\eta = 0.2$ to 0.7. The value of abscissa at zero value of $\langle P_x^{\mathrm{dir}}\rangle$ corresponds to the energy of vanishing flow (EVF) or alternatively, the balance energy (E_{bal}). Figure 2 shows the shift in the balance energy due to Coulomb interactions, while figure 3 represents the shift in the balance energy due to different equations of state. In figure 2, one sees a linear enhancement in the nuclear flow with increase in the incident energy. This increase in the transverse flow is sharp at smaller incident energies (upto 200 MeV/nucleon). If one

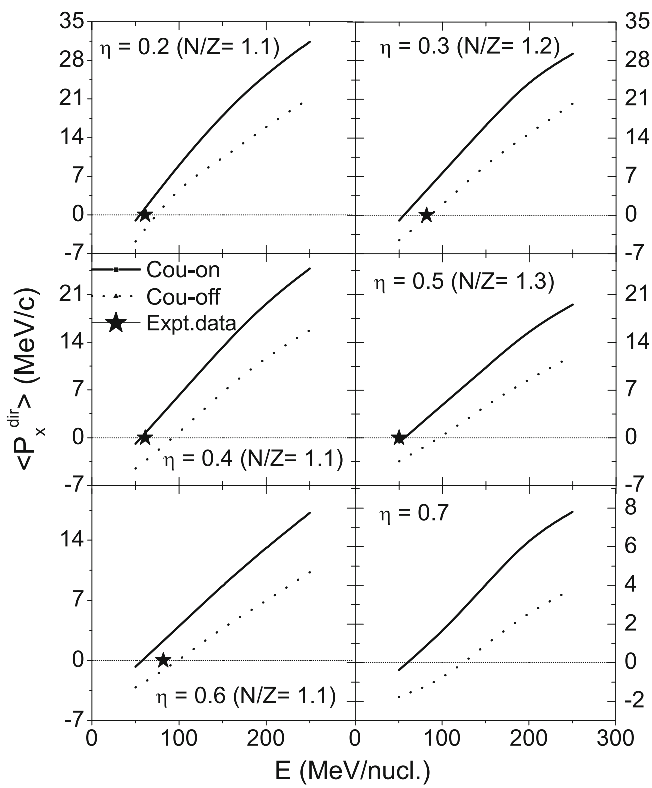

Figure 2. Excitation function of directed flow at different asymmetries with and without Coulomb interactions at semicentral geometry.

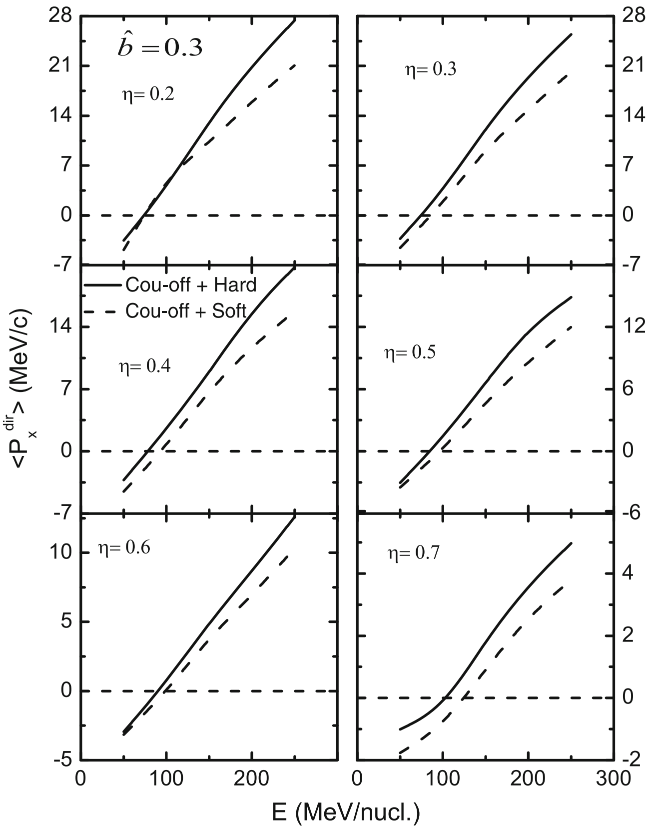

Figure 3. Excitation function of directed flow with hard and soft equations of state in the absence of Coulomb interactions for different asymmetries.

goes to higher incident energies, the value gets saturated as discussed in ref. [6]. We have displayed here the results upto 250 MeV/nucleon, since we are interested in and around the balance energy. In the presence of Coulomb interactions, a more positive value of the flow is obtained. This is due to the well-known repulsive nature of Coulomb interactions. At higher energies, the repulsion due to Coulomb interactions is stronger during the early phase of the reaction and transverse momentum increases sharply. The overall effect depends on the mass asymmetry of the reaction. If one looks at the balance energy, the shift in the incident energy towards the higher value is obtained at $\langle P_x^{\mathrm{dir}} \rangle = 0$ with the mass asymmetry of the reaction showing that with increase in mass asymmetry of the reaction and in the absence of Coulomb interactions, attractive mean field is dominating large region of incident energy. Here, we have displayed data points of the balance energy of various asymmetric reactions on the basis of N/Z ratios that are available experimentally [19]. The systematics of the balance energy with mass asymmetry of the reaction is discussed in figure 4.

The detailed analysis with soft (S) and hard (H) equations of state is displayed in figure 3. The excitation function of directed flow follows similar trend as explained in

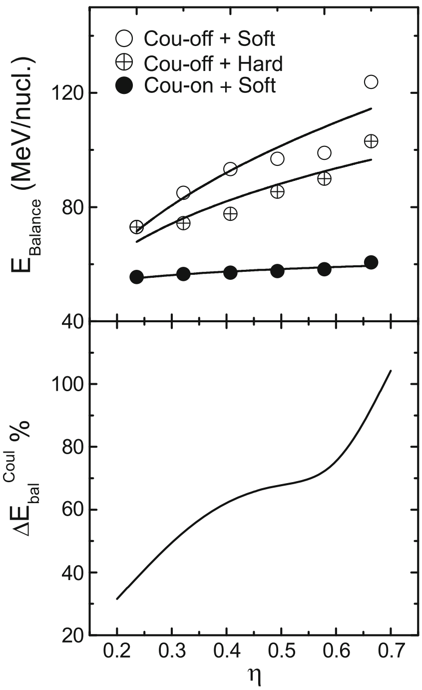

Figure 4. Power-law dependence of balance energy with mass asymmetry of the reaction. The lower panel represents the relative % effect of Coulomb interactions on the balance energy.

figure 2. For nearly symmetric systems ($\eta = 0.2$), the balance energy is found to be independent of the equations of state. However, reasonable differences are observed at higher incident energies. More positive values of directed flow are obtained with hard equation of state compared to soft equation of state. This is true at all asymmetries from $\eta = 0.3$ to 0.7. This is due to different compressibilities of hard (380 MeV) and soft (200 MeV) equations of state. With an increase in the mass asymmetry of a reaction, the shift in the balance energy towards higher values of incident energy takes place with soft equation of state compared to hard one. This is consistent with the experimental findings at National Superconducting Cyclotron Laboratory (NSCL) [11].

To sum up, in figure 4, we have displayed the mass asymmetry dependence of balance energy. We displayed the results for hard and soft equations of state by switching off the Coulomb interactions. In addition, for a comparative study, the results in the presence of Coulomb interactions with soft equation of state are also shown. All the lines are fitted with power law of the form $E_{\mathrm{bal}} = C(\eta)^{\tau}$, where C and τ are the constants. The values of τ in the absence of Coulomb interactions for soft and hard equations of state are 0.375 and 0.282, respectively, while in the presence of Coulomb interactions, for soft equation of state, $\tau = 0.06067$.

If we compare the mass asymmetry dependence of balance energy with mass dependence, the trend is opposite [6]. It is also clear from the figure that shift in the balance energy is due to Coulomb interactions as well as equations of state with mass asymmetry of the reaction. The shift is more due to Coulomb interactions than due to equations of state, indicating the importance of Coulomb interactions in intermediate energy heavy-ion collisions. The higher balance energy is obtained with Cou-off + soft equation of state followed by Cou-off + hard equation of state and finally Cou-on + soft equation of state.

For the further understanding, the relative percentage difference in the balance energy is plotted in the lower panel denoted by the quantity $\Delta E_{\text{bal}}^{\text{Coul}}\%$ given by

$$\Delta E_{\text{bal}}^{\text{Coul}}\% = \left[\frac{E_{\text{bal}}^{\text{Cou-off+soft}} - E_{\text{bal}}^{\text{Cou-on+soft}}}{E_{\text{bal}}^{\text{Cou-on+soft}}}\right]\times 100. \tag{7}$$

$\Delta E_{\text{bal}}^{\text{Coul}}\%$ is found to increase with increase in mass asymmetry of a reaction. This indicates shift of the nuclear matter towards the attractive mean-field region in the absence of Coulomb interactions with the mass asymmetry of the reaction. This difference $\Delta E_{\text{bal}}^{\text{Coul}} =$ 30 at $\eta = 0.2$, while it is 115 at $\eta = 0.7$. The difference of 90 in the shift of balance energy with mass asymmetry cannot be ignored. This is the first ever study which shows that the balance energy is affected by both Coulomb interactions and equations of state.

4. Summary

Our aim was to understand the influence of Coulomb interactions as well as equations of state on the dynamics of large asymmetric reactions in semicentral heavy-ion collisions. Both these factors are found to affect the balance energy significantly for larger asymmetric reactions. The balance energy is found to increase with the increase in the mass asymmetry of the reaction which is further parametrized in terms of mass power law.

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