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# Analysis of shape isomer yields of $^{237}$Pu in the framework of dynamical–statistical model

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**Abstract.** Data on shape isomer yield for $\alpha+^{235}$U reaction at $E_\alpha^{\text{lab}} = 20$–29 MeV are analysed in the framework of a combined dynamical–statistical model. From this analysis, information on the double humped fission barrier parameters for some Pu isotopes has been obtained and it is shown that the depth of the second potential well should be less than the results of statistical model calculations.

**Keywords.** Fission; fission barrier; isomer yield.

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## 1. Introduction

The second minimum in the potential energy surface of heavy nuclei was established by Strutinsky's shell correction method. Therefore, for heavy nuclei which have double-humped fission barrier, two classes of excited states, in the first and second potential wells, can be considered. The existence of a second well in the fission barrier explains the nature of spontaneously fissionable isomers (shape isomers), the resonance effects in sub-barrier fission and other phenomena. In the present work, we use the dynamical–statistical model of induced fission [1] to analyse experimental fission probabilities of some Pu isotopes and shape isomer yields for the $\alpha+^{235}$U reaction at $E_\alpha^{\text{lab}} = 20$–29 MeV. It should be stressed that so far experimental data on shape isomer yields for $^{237}$Pu have only been analysed on the basis of the statistical model [2]. This model has been applied under the assumption of the complete damping of collective motion in the second potential well.

## 2. Details of the model and analysis of the experimental data

We combine a dynamical (Langevin) and a statistical description of heavy ion-induced fission, so that in the first potential well we use the statistical model and at each time step $\hbar/\Gamma_{\text{tot}}$, calculate the decay widths for emissions of $n$, $p$, $\alpha$, $\gamma$ and the width of the

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decay channel related to passing the inner fission barrier. The probability of decay of a fissionable nucleus via different channels can be calculated using a standard Monte Carlo cascade procedure where the kind of decay is selected with the weights $\Gamma_i/\Gamma_{\rm tot}$ ($i = n, p, \alpha, \gamma$, fissions) and $\Gamma_{\rm tot} = \sum_i \Gamma_i$. If a random choice of decay channel leads to the transition of the nucleus from the first potential well to the second one, further evolution of the nucleus is simulated in terms of the coupled Langevin equations. It should be noted that simulation of the fission process of the nucleus in terms of Langevin equations also allows for the emission of $n$, $p$, $\alpha$ and $\gamma$ quanta. The result of simulation of nucleus evolution in the second potential well can be classified as follows: (1) overcoming the second barrier and reaching the scission point; (2) population of the second potential well and cooling via particle or $\gamma$ emission (this event is interpreted as the formation of shape isomers); (3) returning of the system into the first potential well.

In order to specify the shape collective coordinates for a dynamical description of nuclear fission, we use the shape parameters $r$, $h$, $\alpha$ as suggested by Brack *et al* [3]. However, we simplify the calculation by considering only symmetric fission ($\alpha = 0$) and neglect the neck degree of freedom ($h = 0$). Consequently, in terms of the one-dimensional potential, $V(r)$, the coupled Langevin equations in one dimension take the form [4]

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{1}{2}\left(\frac{p}{m(r)}\right)^2 \frac{\mathrm{d}m(r)}{\mathrm{d}r} - \frac{\mathrm{d}V}{\mathrm{d}r} - \beta(r)p + f(t),$$
$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{p}{m(r)}. \tag{1}$$

Here, $f$ is a random force with an amplitude $\eta(Tm\beta)^{1/2}$ and $\eta$ is a random number with the properties $\langle\eta(t)\rangle = 0$ and $\langle\eta(t)\eta(t')\rangle = 2\delta(t - t')$.

The temperature dependence of the fission mode damping parameter, $\beta$, is taken to be of the form $\beta \approx 0.6T^2/(1 + T^2/40)$. It is also a good approximation of the microscopic calculations carried out in terms of the linear response theory. The initial conditions for Langevin equations can be chosen by the Neumann method with the generating function

$$\Phi(r, p, J) \propto \frac{\rho_{1f}(E^* - B_f^1 - \varepsilon, J)}{1 + \exp(-2\pi\varepsilon/\hbar\omega_{1f})}\delta(r - r_{1f}), \tag{2}$$

where $E^*$ is the excitation energy of the fissioning nucleus, $\varepsilon = p^2/(2m)$ is the kinetic energy of the collective motion, $\rho_{1f}(E)$ is the level density at the first saddle point, $B_f^1$ is the height of the inner fission barrier, and $\hbar\omega_{1f}$ and $r_{1f}$ are the barrier width and the collective coordinate at the first saddle point, respectively.

The collective inertia, $m$, is calculated in the frame of the Werner–Wheeler approach and the nuclear temperature is defined as

$$T = \left(\frac{E_{\rm int}}{a(r)}\right)^2 \tag{3}$$

with

$$E_{\rm int} = E^* - p^2/(2m) - V(r, T, J) - E_{\rm rot}(J), \tag{4}$$

where $E_{\rm rot}(J)$ and $a(r)$ are the rotational energy and the level density parameter, respectively.

The potential energy in eq. (4) for a fissionable nucleus can be calculated as the sum of the liquid drop potential energy $V_{\rm ld}(r, J)$ of a rotating nucleus with an angular momentum $J$ and the shell correction $\delta w$

$$V(r, J, T) = V_{\rm ld}(r, J) + \delta\, w\,(r) \times \left[1 + \exp\left(\frac{T - T_0}{d}\right)\right]^{-1}, \tag{5}$$

where $r$ is the distance between the centres of mass of the fission fragments formed and the bracketed expression describes the damping of the shell effects with the temperature $T$. The values of the parameters $T_0 = 1.75$ MeV and $d = 0.2$ MeV are taken from ref. [5].

In the zero temperature limit, $\delta w(r)$ can be taken to be equal to the difference between the above approximation and $V_{\rm ld}(r, J=0)$. Therefore, if we approximate $V(r, J=0, T=0)$ in terms of another method, then we can calculate $\delta w(r)$ by eq. (5). The double-humped fission barrier, $V(r, J = 0, T = 0)$, can be approximated by smoothly joined parabolas [6]. The value of equilibrium deformation, corresponding shell correction and the position of the inner-fission barrier are taken from [7,8]. Other parameters of the double-humped fission barriers (the second potential well depth, inner barrier height, outer barrier height, first barrier width and the second barrier width) are taken as free parameters.

In the calculation, the value of the curvature energy, $\hbar\omega_{1w}$, of the primary potential well is assumed to be 1 MeV for all the nuclei considered in the present work. This is consistent with its values used by other authors [9–11] in the past in the fission literature. The value of the curvature energy, $\hbar\omega_{2w}$, in the second-well region is also taken as 1 MeV for all the nuclei considered in the present work. Such a value was found to reproduce reasonably well the observed ground-state spontaneous fission half-lives as well as the isomeric half-lives of a wide variety of actinide nuclei [12–14].

Figure 1 shows the results of calculation of the double-humped fission barrier and the shell correction for $^{239}$Pu, $^{238}$Pu and $^{237}$Pu.

The emission of light particles $n$, $p$, $\alpha$ and $\gamma$ quanta are calculated using the Hauser–Fechbach approach [15]. The widths of the decay channels related to passing the inner and outer fission barriers are calculated by the Bohr–Wheeler relations using Kramer's

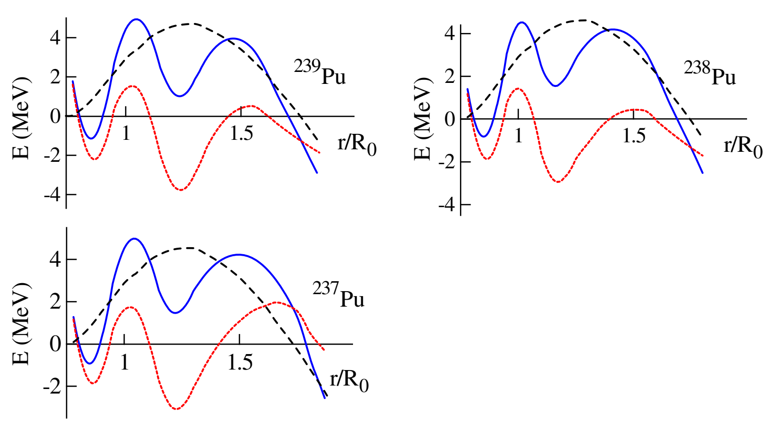


**Figure 1.** Calculated double-humped fission barrier (solid curve), liquid-drop fission barrier (dashed curve) and shell correction (dotted curve) for $^{239}$Pu, $^{238}$Pu and $^{237}$Pu. $R_0$ is the radius of the spherical nucleus.

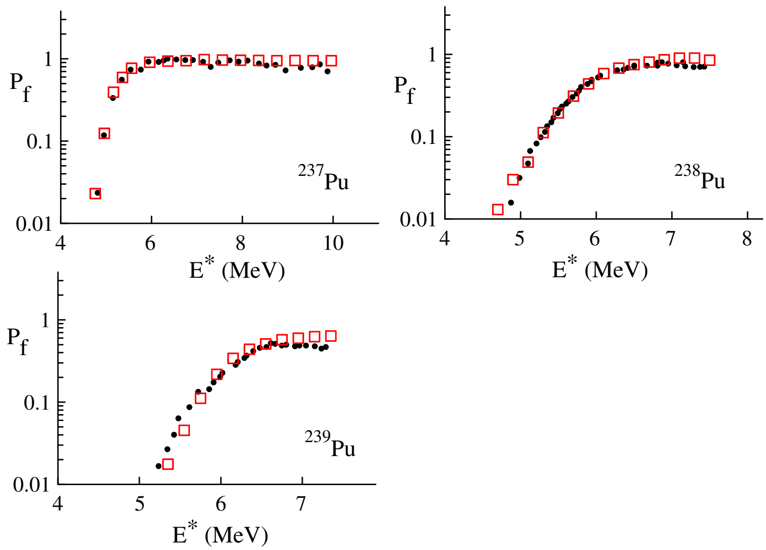


**Figure 2.** Fission probabilities of $^{239}$Pu, $^{238}$Pu and $^{237}$Pu. (□) Calculation, (•) experimental data [19–21].

correction [16]. Level densities are calculated by considering the paring correlations, collective vibrations and rotations in the nuclei in the adiabatic approximation as in refs [17,18]. It should be stressed that level density is a key physical quantity in the calculations because all the disintegration widths, i.e. evaporation, fission and gamma emission depend on this.

The height and barrier width of the inner fission barrier can be determined from the condition of the best fit to the experimental values of the fission probabilities. Figure 2 shows the results of calculation of the fission probabilities for $^{239}$Pu, $^{238}$Pu and $^{237}$Pu isotopes.

A number of interesting points can be noted while comparing the calculated values of the fission probability with the experimental data. First of all, the shape for $^{237}$Pu is rather different compared to $^{238}$Pu and $^{239}$Pu, because $^{237}$Pu is more unstable than $^{238}$Pu and

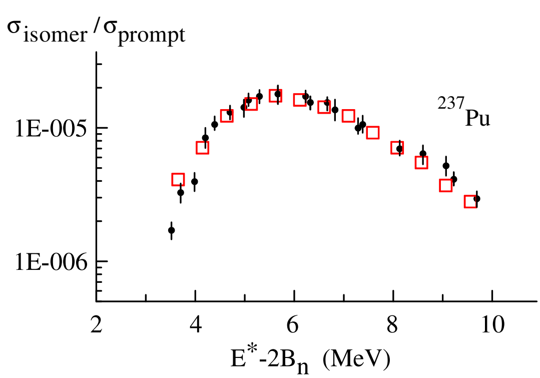


**Figure 3.** Yield of the $^{237}$Pu shape isomer for the $\alpha+^{235}$U reaction. (□) Calculation, (•) experimental data [2].

**Table 1.** Parameters of the double-humped fission barrier for Pu isotopes.

| Nucleus | $B_f^{\rm I}$ (MeV) | | $B_f^{\rm II}$ (MeV) | | $\Delta E$ (MeV) | | $\hbar\omega_f^{\rm I}$ (MeV) | | $\hbar\omega_f^{\rm II}$ (MeV) | |
|---|---|---|---|---|---|---|---|---|---|---|
| $^{239}$Pu | 6.12[a] | 6.20±0.20[b] | 5.12[a] | 5.50±0.20[b] | 2.20[a] | 2.60±0.20[b] | 0.9[a] | 0.80[b] | 0.75[a] | 0.52[b] |
| $^{238}$Pu | 5.40[a] | 5.50±0.20[b] | 5.10[a] | 5.00±0.20[b] | 2.40[a] | 2.70±0.20[b] | 1.05[a] | 1.04[b] | 0.61[a] | 0.60[b] |
| $^{237}$Pu | 6.02[a] | 5.90[b] | 5.30[a] | 5.20[b] | 2.40[a] | 2.80±0.20[b] | 0.82[a] | 0.80[b] | 0.53[a] | 0.52[b] |

[a]Present study.
[b]Reference values [22].

$^{239}$Pu. The starting points of the curves are located at different $E^*$ values. This is due to the differences in the heights and the widths of the first fission barrier of $^{239}$Pu, $^{238}$Pu and $^{237}$Pu.

It should be noted that the height of the first fission barrier of Pu isotopes is higher than the second one; therefore the first fission barrier is used to calculate the fission probability. On the other hand, when one of the peaks of the fission barrier is much higher than the other, then the experiments give only information on the height and fission width of the highest peak.

The depth of the second potential well and the height of the outer fission barrier are found from the condition of the best fit to the experimental values of the $^{237}$Pu shape isomer yield. Figure 3 shows the results of calculation of the shape isomer yield of $^{237}$Pu.

In table 1, the parameters calculated in the present work for the double-humped fission barriers are compared with well-known reference values [22]. It is clear that the results calculated for the depth of the second potential well in terms of the combined dynamical statistical model is somewhat reduced for the $^{239}$Pu, $^{238}$Pu and $^{237}$Pu isotopes.

To show that the lowering of the second well depth could not be because of the differences in statistical model parameters (level density prescription, shell correction, etc.) between [22] and the present work, we run our code after switching off the dynamical branch of the model and analysed the above-mentioned experimental data.

Figure 4 shows the results of calculation of the double-humped fission barrier for $^{238}$Pu on the basis of the combined dynamical statistical model and the statistical model.

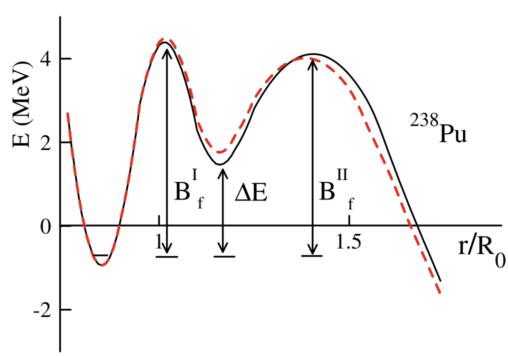


**Figure 4.** Calculated double-humped fission barrier on the basis of the combined dynamical–statistical model (solid curve) and the statistical model (dashed curve) for $^{238}$Pu.

From a physical point of view, it is reasonable that the fission probability of the excited nuclei is reduced by considering the dynamical effects. Therefore, it can be expected that in the analysis with the combined dynamical–statistical model, the depth of the second potential well is lower than that for the statistical model.

## 3. Conclusions

A dynamical–statistical model was used to analyse the experimental shape isomer yield data in the reaction $\alpha + ^{235}\text{U}$ at $E = 20$–$29$ MeV. In terms of this analysis, information on fission barrier parameters of several Pu isotopes has been obtained.

It should be stressed that so far experimental data on shape isomer yields for $^{237}$Pu have only been analysed within the statistical theory of nuclear reactions, but in this work the effect of collective motion is considered. The main conclusion is that while the heights of the two barriers are close to those obtained from statistical model calculations (ref. [22]), in the dynamical calculations the depth of the second potential well is somewhat reduced for all the $^{239}$Pu, $^{238}$Pu and $^{237}$Pu isotopes.

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