

Non-minimal quintessence: Dynamics and coincidence problem

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Abstract. Brans–Dicke scalar–tensor theory provides a conformal coupling of the scalar field with gravity in Einstein’s frame. This model is equivalent to an interacting quintessence in which dark matter is coupled to dark energy. This provides a natural mechanism to alleviate the coincidence problem. We investigate the dynamics of this model and show that it leads to comparable dark energy and dark matter densities today.

Keywords. Non-minimal quintessence; coincidence problem.

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1. Introduction

According to Einstein’s theory of relativity, the observed late time acceleration [1] of the Universe can be successfully explained by introducing the dark energy component [2] on the right-hand side of the Einstein’s equations. Two main scenarios are used to explain the nature of dark energy: a cosmological constant Λ [3] and a scalar field, usually called quintessence [4]. The cosmological model consisting of a mixture of cosmological constant and cold dark matter is called Λ CDM and the other possibility leads to QCDM cosmological model. Although Λ CDM model opens the possibility to fit it with the observational data, two questions arise: (1) why is the value of the cosmological constant so small and (2) why does its energy density begin to dominate now. This is known as the coincidence problem [5].

Quintessential models of dark energy also provide an adequate fit to the cosmological data. Moreover, the existence of stable tracking solutions of the quintessence field shows the importance of this model in the investigation of the fine-tuning problem. But it cannot explain the present ratio of energy densities in the Universe, i.e. the coincidence problem [6]. One possible way to make a model to overcome the coincidence and fine-tuning problems, is to introduce a proper coupling term between the dark energy and dark matter. The form of the interaction term is motivated by the phenomenological consideration or by

requiring that the ratio of dark energy to dark matter densities, $r = \rho_\phi / \rho_c$, be stable under perturbations [6,7].

Introducing the interaction in QCDM model, can lead to a finite r at present or can cause this ratio to vary slowly now, which is called soft coincidence [8]. Moreover, a smooth transition from matter-dominated Universe to the accelerated expanding Universe is required. Denoting the coupling term by Q in the conservation equations of the quintessence and cosmic dark fluid, one has

$$\dot{\rho}_c + 3H\rho_c = Q, \tag{1}$$

$$\dot{\rho}_\phi + 3H(1 + \omega)\rho_\phi = -Q, \tag{2}$$

where $\rho_c, \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ are the energy density of cosmic dark fluid, the energy density and pressure density of the scalar field respectively. $\omega = p_\phi / \rho_\phi$ is the equation of state parameter for the scalar field and a dot over any quantity denotes derivation with respect to the cosmic time.

A variety of interaction terms have been suggested in [6–20]. One of the most important ones is $Q = \lambda_1 H\rho_c$ [8,10–12]. Inserting it into the above equations, together with the Friedmann equations, yields a monotonic decreasing function of the scale factor for r which changes slowly at the present time, i.e. $\dot{r}/r|_{t_0} \ll H_0$ [10]. Furthermore, assuming an accelerated Universe and negative \dot{r} , together with satisfying the soft coincidence criterion, the functions $H(a)$, $V(\phi)$ and the allowed range of λ_1 are determined. The more general interaction term $Q = \lambda_1 H\rho_c + \lambda_2 H\rho_\phi$ is investigated in [9,10]. This yields the evolution from a stable value r_+ at early times to an unstable value r_- at late times [10]. Also the constants λ_1 and λ_2 are constrained by the evolution of the background energy density.

It must be noted that the observational data are unable to discriminate between this interaction and the previous one. Also these two models are investigated in a more general manner in which the transfer rate of energy densities is determined by the properties of dark sector and in general is different from the expansion rate of the Universe [13].

In [12], it is assumed that the scaling solutions (having a constant r at all times) exist for the generic form of a non-minimally scalar-field Lagrangian. This leads to the coupling $Q = \beta\rho_c\dot{\phi}$ in the Einstein’s frame. It can be shown [15] that the proper sequence of cosmological epochs in the form of radiation-dominated, ϕ -matter-dominated and accelerated epoch is followed in this model. Also the cosmological consequences of this kind of coupling are studied in [14–17] assuming an exponential potential for the quintessence field.

The simplest model which can provide this particular interaction is a non-minimally coupled gravity in which the scalar field couples minimally to gravity but couples conformally to the matter fields [15,18]. This can be obtained from the following action [21]:

$$S_E = \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) + \mathcal{L}_c(e^{-2\beta\phi} g_{\mu\nu}) + \mathcal{L}_b(g_{\mu\nu}) \right]. \tag{3}$$

It is obvious that this model is conformally related to a non-minimal gravity in the form of Brans–Dicke-type Lagrangian. The cosmological properties of this model is investigated in the literature. In [22] an observational analysis of non-minimally coupled theory and it’s differences from general relativity is presented. Also the authors of [23] discuss the attractive aspect of non-minimal quintessence and show that after the tracking era, the

quintessence-dominated epoch is started. The particle production of this model in inflationary epoch is studied by Sahni and Habib [24]. Their analysis leads to a spatially flat Universe consisting of dark energy (in the form of quanta of the non-minimally coupled scalar field created during inflation) and dark matter. In this way it presents a new way of generating dark energy quantum mechanically [25].

For the latter coupling, a more general form of the potential and allowing β to be a function $\beta(\varphi)$ can be found in [19].

In summary, the coupling of dark energy to dark matter shows that the dark energy can affect the expansion history of the Universe and the change of the growth rate of matter perturbations in the structure formation stage. Therefore, this makes tight bounds on the strength of coupling terms [20]. Also in [11], the different forms of coupling and their aspects have been investigated and compared with the observational data. The result indicates that the parameter space, which includes the arbitrary coupling constants and the equation of state parameter, is greatly limited.

This work rests on the latter coupling pointed above. Moreover we use new variables, proposed in [10], to simplify the analysis of the critical points of the non-minimal scalar field model.

The outline of this paper is as follows. In the next section we shall study the critical points of the autonomous system of equations associated with the non-minimally coupled quintessence model. Then in §3 we discuss the dependence of dark energy–dark matter ratio on the redshift. Finally in §4 we shall give some conclusions.

2. The model and its critical points

Here we consider that the quintessence field decays into the pressureless CDM according to the latter kind of coupling term discussed in the introduction, i.e. $Q = \beta\rho_c\dot{\varphi}$. We also include baryons' density as an independent conserved component such that

$$\dot{\rho}_b + 3H\rho_b = 0. \quad (4)$$

The conservation equations (1), (2) and (4) can be obtained from the action (3). It has to be noted that similar models are discussed in [15] (in which baryons plus CDM couples to dark energy component) and in [16] (neglecting baryons). Both these works assume an exponential potential for dark energy. Here we shall use a more direct way to investigate the model by assuming that the equation of state parameter, ω , is constant and then absorb it in the governing equations via introducing new variables, and we have not considered any assumption about the special form of the potential.

Now, consider a spatially flat FRW Universe occupied by the above three components. The cosmological equations of motion are (we have chosen the unit in which $8\pi G = c = 1$)

$$H^2 = \frac{1}{3}(\rho_\varphi + \rho_b + \rho_c) \quad (5)$$

$$\dot{H} = -\frac{1}{2}[\rho_\varphi(1 + \omega) + \rho_b + \rho_c]. \quad (6)$$

Defining the dimensionless variables:

$$x = \frac{\rho_\phi}{3H^2}, \quad y = \frac{\rho_c}{3H^2}, \quad u = \frac{Q}{3H^3} \tag{7}$$

and following the same method as in [10], the conservation relations of dark sector, eqs (1) and (2) can be written as

$$x' = 3x(1-x) - \tilde{u}, \tag{8}$$

$$y' = -3xy + \tilde{u} \tag{9}$$

in which $\tilde{u} = -u/\omega$ and the prime denotes derivation with respect to $N = -\omega \ln a$ (we assume the current value of the scale factor is one, $a_0 = 1$). Using the relation $\dot{\phi}^2 = (1 + \omega)\rho_\phi$, one obtains

$$\tilde{u} = \pm \tilde{\beta} y \sqrt{3(1 + \omega)x}, \tag{10}$$

where $\tilde{\beta} = \beta/|\omega|$ and the positive (negative) sign indicates that $\dot{\phi}$ and ω have the opposite (same) signs. Here we consider $\tilde{\beta}$ to be a positive constant. Inserting (10) in (8) and (9), the critical points of the above dynamical system together with their stability along the dynamical state of the Universe are analysed using eigenvalues method and the results are summarized in table 1.

The critical points are labelled by A, B and C and the type I (II) solution corresponds to the opposite (same) sign of $\dot{\phi}$ and ω . For the first critical point, A, we get $x = 0, y = y_0$ which results in $\tilde{u} = 0$ and thus there is neither scalar field nor interaction. This point represents the baryon-cold dark matter domination at early times, with no contribution from dark energy. For this point, the secular equation of the autonomous systems (8) and (9) is singular. A detailed study of the equations near this critical point shows that the system diverges exponentially from this critical point, and thus it is an unstable point.

For the second critical point, B, we get $x = 1, y = 0$ and again there is no interaction ($\tilde{u} = 0$). This corresponds to the pure dark energy-dominated era which can be accelerating if $\omega < -1/3$. Therefore depending on the initial conditions, approaching this point, the final state of the Universe can be a pure dark energy-dominated one in this model.

Finally the third point, C, corresponds to the dark energy-cold dark matter-dominated Universe. It is only of type I, so that the signs of ω and $\dot{\phi}$ are different. It can represent the late time Universe as the previous one. For the latter points, B and C, one of the eigenvalues is always negative. The second eigenvalue for B type II is negative, while for B types I and C an appropriate range for β can be chosen so that the eigenvalue becomes

Table 1. Critical points of the model and their properties.

Point	(x, y)	Existence	Stable?	Acceleration?	Ω_b	Q
A	(0, y_0)	All ω, β	No	No	$\neq 0$	0
B Type I	(1, 0)	All ω, β	$\beta < \sqrt{\frac{3}{1+\omega}} \omega $	$\omega < -\frac{1}{3}$	0	0
B Type II	(1, 0)	All ω, β	All β	$\omega < -\frac{1}{3}$	0	0
C Type I	$(\frac{(1+\omega)\tilde{\beta}^2}{3}, 1 - \frac{(1+\omega)\tilde{\beta}^2}{3})$	All ω, β	$\beta > \sqrt{\frac{3}{1+\omega}} \omega $	$ \omega < \frac{\beta^2}{1+\beta^2}$	0	$-3H^3\omega\tilde{\beta}^2(1 + \omega)$

negative. Therefore, B type II is stable, while B types I and C are stable or saddle depending upon the value of the coupling constant, β . On the other hand, the critical points should be such that $0 \leq x \leq 1$ and $0 \leq y \leq 1$ which for point C is satisfied provided that $\beta < \sqrt{3/(1+\omega)}|\omega|$. Therefore, point C is a saddle point. We see that there are only two critical points that admit accelerated solutions, B and C. These are in agreement with the solutions a and b_M of Amendola in [15]. The point C is the only accelerated solution in which the ratio of matter and dark energy densities is constant and therefore is a scaling solution. The existence of this critical point can provide the same order of energy densities for both interacting components independently of the initial conditions. This point is also in agreement with the solution D of Bohemer *et al* in [16] (see table I of [16]). Restricting ourselves to the positive value of Q , which means $\dot{\phi} > 0$, corresponds to the transfer of energy from quintessence to the dark matter, and this is compatible with the second law of thermodynamics [26]. In this case, allowing the observed acceleration expansion of the Universe, we have to select the type I solution from any critical point.

3. Dark energy–dark matter ratio

In what follows, we shall discuss the evolution of the ratio of dark energy to dark matter densities, r . To do this, from eqs (8) and (9), we obtain [10]

$$\frac{r'}{r} = 3 - \frac{(r+1)}{ry} \tilde{u}. \quad (11)$$

With the interaction term given by (10), eq. (11) becomes

$$\frac{r'}{r} = 3 \mp \tilde{\alpha} \sqrt{x} \left(1 + \frac{1}{r} \right), \quad (12)$$

where $\tilde{\alpha} = \tilde{\beta} \sqrt{3(1+\omega)}$. To solve this equation, one needs the present value of r as the initial condition. However, as the authors of [27] point out, it depends on the present value of Hubble parameter and on the other hand there are no model-independent observational data for $r(H)$ and $r(z)$. Thus, these authors propose to determine the dependence of the Hubble parameter in terms of redshift, z , to compare with observational data which are model-independent [27]. To do this, using relation (6), eq. (12) becomes

$$\frac{dr}{dH} = \frac{2\omega(3r \mp \tilde{\alpha} \sqrt{x}(r+1))}{3H(1+\omega x)}. \quad (13)$$

Now combining eq. (12) with the expression $H = -dz/[(1+z)dt]$, yields

$$\frac{dr}{dz} = \frac{\omega(3r \mp \tilde{\alpha} \sqrt{x}(r+1))}{(1+z)}. \quad (14)$$

Dividing eq. (14) by (13), gives

$$\frac{dH}{dz} = \frac{3H(1+\omega x)}{2(1+z)}. \quad (15)$$

This relation shows that $H(z)$ depends on the interaction term, Q , through the dark energy density in this model (see relation (8)).

Although we do not have an exact solution of eqs (14) and (15) in general, the behaviour of r can be studied for extreme limits of the early and the late times. We shall show that if ω and β are chosen appropriately, we have solutions that predict the correct behaviour of dark energy–dark matter ratio.

At early times (matter-dominated era), for which $x \rightarrow 0$, eq. (15) solves to $H \sim (1+z)^{3/2}$, and eq. (14) reads as

$$\frac{dr}{dz} = \frac{3\omega r}{1+z} \tag{16}$$

with the solution $r \sim (1+z)^{3\omega}$.

On the other hand, at late time, the baryonic matter density can be ignored and thus $x + y \rightarrow 1$. Then eq. (14) reads as

$$\frac{dr}{dz} = \frac{3\omega r}{1+z} \mp \frac{\tilde{\alpha}\omega}{1+z} \sqrt{r(1+r)} \tag{17}$$

which has the following exact solution:

$$1+z = \left(\frac{F_1(r)F_2(r)}{F_1(r_0)F_2(r_0)} \right)^{-1/(-9+\tilde{\alpha}^2)\omega}, \tag{18}$$

where

$$F_1(r) = (1+2r+2\sqrt{r(1+r)})^{\pm\tilde{\alpha}} \tag{19}$$

and

$$F_2(r) = (\tilde{\alpha}(-6\sqrt{r(1+r)} + \tilde{\alpha}) + r(9 + \tilde{\alpha}^2))^3 \tag{20}$$

and r_0 is the current value of r .

Now we shall set some assumptions on the asymptotic value and slope of dark energy–dark matter ratio. The late time value $r_\infty = R$ should be such that

$$R > r_0, \tag{21}$$

where r_0 is the present value of r . Moreover, to have a nearly constant dark energy–dark matter ratio at late times, we have the following restriction:

$$\left. \frac{dr}{dz} \right|_{r=R, z=Z} \ll 1, \tag{22}$$

where Z tends to -1 .

Using eq. (17), the conditions (21) and (22) lead to

$$\tilde{\alpha} < 3\sqrt{\frac{R}{1+R}} \tag{23}$$

and

$$\omega \ll \frac{1+Z}{3R - \tilde{\alpha}\sqrt{R(1+R)}}. \tag{24}$$

The evolution of dark energy–dark matter ratio with respect to the redshift is shown in figures 1 and 2. In figure 1 graphs for different values of β and a fixed ω are drawn while

Non-minimal quintessence

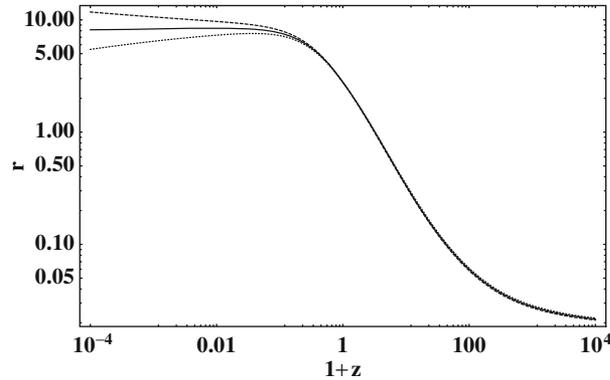


Figure 1. Dark energy–dark matter ratio as a function of redshift. ω is chosen to be -0.5 and $\beta = 1.14, 1.19$ and 1.16 for dashed, dotted and thick lines respectively. r at present time is chosen to be $7/2.5$.

in figure 2 graphs for different values of ω and a fixed β are considered. We see that this model leads to a constant unstable r at late times and to a lower constant stable r for early times. This behaviour is similar to what is obtained in [10,27] for two broad classes of interacting models of dark energy.

In figures 3 and 4 we have plotted the phase space diagrams of type I model (eqs (8)–(10)). Figure 5 is the corresponding plot for type II model.

In figure 3 the stability condition of BI point is satisfied and thus the point CI is a saddle point as it is expected from table 1. Changing the value of $\tilde{\beta}$ such that the stability condition of BI point is not satisfied leads to figure 4 and one can see that the critical point CI is out of the acceptable range of x and y .

Finally as can be seen in figure 5, for a type II solution, the point BII is an attractor point.

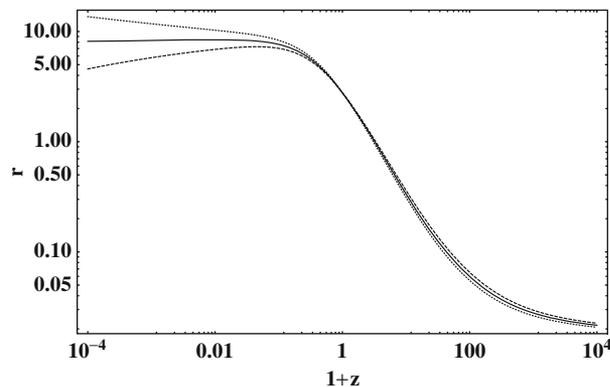


Figure 2. Dark energy–dark matter ratio as a function of redshift. β is chosen to be 1.16 and $\omega = -0.49, -0.51$ and -0.5 for dashed, dotted and thick lines respectively. r at present time is chosen to be $7/2.5$.

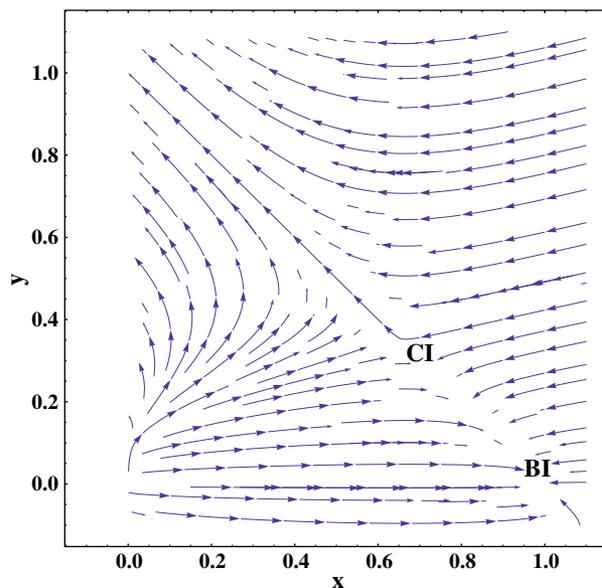


Figure 3. Phase diagram of the type I system for $\omega = -0.5$ and $\tilde{\beta} = 2$. BI and CI denote points B and C of type I.

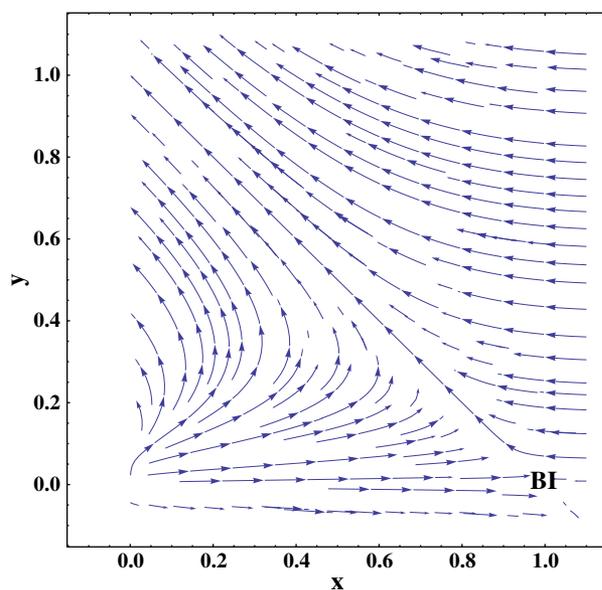


Figure 4. Phase diagram of the type I system for $\omega = -0.5$ and $\tilde{\beta} = 2.7$. BI denotes point B of type I. The point CI is now out of acceptable range of x and y .

Non-minimal quintessence

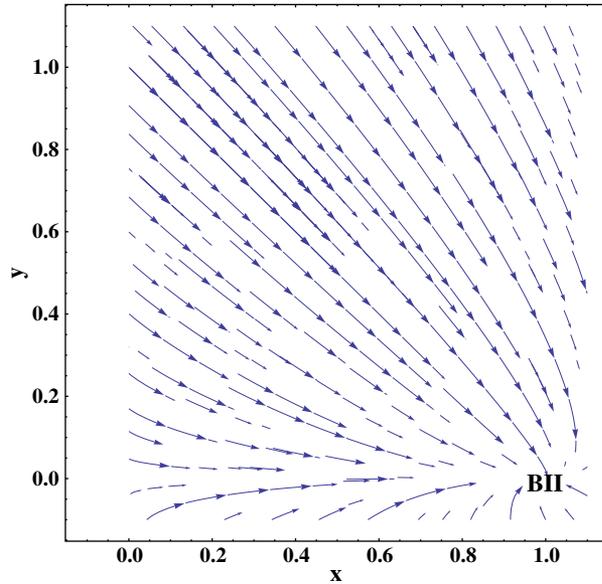


Figure 5. Phase diagram of the type II system for $\omega = -0.5$ and $\tilde{\beta} = 2$. BII denotes point B of type II.

A question that arises here is which kind of potential leads to the derived solution. To reconstruct the potential, one can use the relations $V = (1 - \omega)\rho_\phi/2$, $\dot{\phi} = \sqrt{(1 + \omega)\rho_\phi}$ and eq. (2) to obtain

$$\frac{dV}{d\phi} = -\frac{1 - \omega}{2}(3H\sqrt{(1 + \omega)\rho_\phi} + \beta\rho_c) \quad (25)$$

ρ_ϕ in the above equation can be substituted by integration eq. (1) leading to [14]

$$\rho_c = \rho_{c0} \left(\frac{a}{a_0}\right)^3 \exp(\beta(\phi - \phi_0)), \quad (26)$$

where the subscript 0 denotes the current values of quantities.

At late times, $z \sim -1$, $\rho_\phi/\rho_c \sim R$ and using eqs (1) and (2) we get

$$\rho_c = \rho_{c0} \left(\frac{a_0}{a}\right)^{3(1 + \frac{\omega R}{1 + R})}. \quad (27)$$

Inserting eq. (27) in eq. (5) leads to

$$H^2 = \frac{1 + R}{3} \rho_{c0} \exp\left(\beta \left(1 + \frac{1 + R}{\omega R}\right) (\phi - \phi_0)\right) \quad (28)$$

and thus eq. (25) can be integrated to

$$V \sim \exp\left(\beta \left(1 + \frac{1 + R}{\omega R}\right) (\phi - \phi_0)\right). \quad (29)$$

For early times, ρ_ϕ is small and thus $\dot{\phi} \sim 0$ and $\phi \sim \phi_0$ and therefore the above potential can be expanded up to first order. Using this approximate potential one can check that the Friedmann equations and the equations governing densities are satisfied.

4. Conclusion

In this paper we have investigated non-minimal quintessence model of dark energy. Following [10], we have defined a new variable, $N = -\omega \ln a$, which reduces the parameter space into a two-dimensional space. Our results are in agreement with those of [15,16]. We found some range of the interaction parameter β given by the stability criteria of the critical points, and we presented an exact solution of dark energy–dark matter ratio for the early and late times.

According to the early time behaviour ($r \sim (1+z)^{3\omega}$) the model behaves like Λ CDM model. On the other hand, it can provide a finite dark energy–dark matter ratio at late times and thus it can alleviate the coincidence problem. In fact it is shown here that if the strength of the interaction (β) is appropriately chosen, this interacting model can predict the expected behaviour for the dark energy–dark matter ratio with a constant ω .

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Non-minimal quintessence

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