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Quantum logic gates using coherent population trapping states

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Abstract. A scheme is proposed for achieving a controlled phase gate using interaction between atomic spin dipoles. Further, the spin states are prepared in coherent population trap states (CPTs), which are robust against perturbations, laser fluctuations etc. We show that one-qubit and two-qubit operations can easily be obtained in this scheme. The scheme is also robust against decoherences due to spontaneous emissions as the CPT states used are dressed states formed out of Zeeman sublevels of ground states of the bare atom. However, certain practical issues are of concern in actually obtaining the scheme, which are also discussed at the end of this paper.

Keywords. Coherent population trap; quantum computation; controlled phase gate.

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Conventional computers handle information in the form of bits – which take up values 0 or 1. Quantum computers on the other hand, use quantum bits (qubits), which can be prepared in states 0, 1 or any superposition of the two. Algorithms of quantum computation exploit this unique feature of quantum mechanical system to solve certain class of computational problems with lesser number of steps [1]. Hence there is a race to produce a reliable, robust and scalable quantum mechanical system which can be used as gates for quantum logic. There have been several attempts in the past to prepare such a system, using NMR of large molecules, quantum dot structures, ions in linear traps or neutral atoms in optical lattices [2,3], each system with its own benefits and drawbacks. One of the major requirement for designing a QC system is that they should be robust and reliable while interactions between any two of them should be on-demand. One such system is proposed here which involves neutral atoms prepared in coherent population trap (CPT) states. It is shown in this paper that such systems can be easily prepared and manipulated and it is possible to build one-qubit and two-qubit gates using them. Since CPT states are 'dark states' of the atom–light interaction, the atoms prepared in such states will not interact with the light any more [4,5]. They will not evolve in time also, since they are already eigenstates of the full Hamiltonian that consist of atomic as well as interaction terms.

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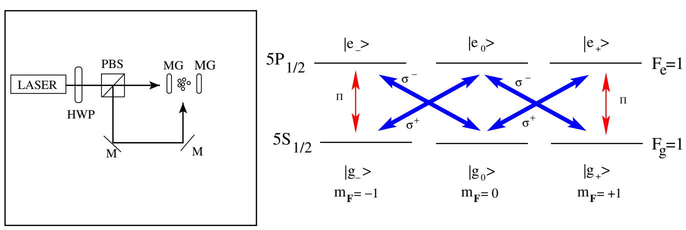
In this communication, a configuration involving Zeeman sublevels of ^{87}Rb atom is considered, which exhibits two different CPT states which can be mapped to two qubits 0 and 1. It is shown that robust states can be prepared and one-qubit and two-qubit operations can be performed using magnetic dipole interactions.

1. The configuration

We consider the transition between $|5S_{1/2}, F = 1\rangle$ and $|5P_{1/2}, F = 1\rangle$ of ^{87}Rb ($|3S_{1/2}, F = 1\rangle$ and $|3P_{1/2}, F = 1\rangle$ of Na is an equally valid set-up with equivalent configuration. We use the dipole–dipole interactions between one Na and one ^{87}Rb atom also in the later part of this paper.), coupled by two lasers, which are of the same frequency but polarized orthogonal to each other – one in plane containing quantization axis z and other in the xy plane. Following the selection rules [6] they both couple transitions between different Zeeman sublevels.

As shown in figure 1b, the beam $E_z = \mathcal{E}_z \exp[i(\omega t - k\{x, y\})]$ couples $\Delta m_{\mathrm{F}} = 0$ transitions between levels labelled $|g_+\rangle \leftrightarrow |e_+\rangle$ and $|g_-\rangle \leftrightarrow |e_-\rangle$. The other beam, with its plane of polarization in the xy plane can be considered as a combination of σ_+ and σ_- beams coupling $\Delta m_{\mathrm{F}} = \pm 1$ transitions $|g_\pm\rangle \leftrightarrow |e_0\rangle$ and $|g_0\rangle \leftrightarrow |e_\pm\rangle$. $|g_0\rangle \leftrightarrow |e_0\rangle$ is not coupled by the E_z laser due to the vanishing Clebsch–Gordon coefficients. Both E_z and E_p beams can be derived from a same laser source using a half-wave plate and a polarizing beam splitter as shown in figure 1a. The ratio of values of $E_{p,z}$ can be controlled by rotating the half-wave plate (HWP).

When only the E_p beam is present, the configuration is the well-known Λ system made up of $|g_-\rangle \leftrightarrow |e_0\rangle \leftrightarrow |g_+\rangle$. The steady-state solution of this situation is the coherent population trapping (CPT) state $|\psi_-\rangle = (1/\sqrt{2})[|g_-\rangle - |g_+\rangle]$ [5]. It is interesting to note that $|\psi_-\rangle$ is the CPT state even when there exists another CPT configuration – the V form of $|e_-\rangle \leftrightarrow |g_0\rangle \leftrightarrow |e_+\rangle$, and competes with the Λ. Our numerical results confirm this fact and it will be shown in a forthcoming communication. However, in the light of the argument presented in ref. [4], one can undertand this as a result of atoms trickling from one dressed state to the other, eventually reaching the state $|\psi_-\rangle$. On the other hand, when only E_z beam is present, then all the atoms in $|g_\pm\rangle$ will be optically pumped out and eventually



(a) (b)

Figure 1. (**a**) Schematic of laser arrangement. HWP is the half-wave plate, PBS is the polarizing beam splitter, M are mirrors and MG are magnets to provide the weak field. (**b**) Energy level configuration of the system used in the set-up. Details of the notations are given in the text.

reach $|g_0\rangle$. This is a trap state for the E_z beam. The two trap states $|\psi_0\rangle = |g_0\rangle$ and $|\psi_-\rangle$ can now be mapped to the qubit states $|\psi_0\rangle = |0\rangle$ and $|\psi_-\rangle = |1\rangle$.

More interestingly, if both E_p and E_z beams are present together, the steady-state solution is not a statistical mixture of the two trap states $|\psi_0\rangle$ and $|\psi_-\rangle$, but a three-component CPT states [7]

$$|\psi\rangle = \frac{\left(\Omega_p/\Omega_z\right)|g_0\rangle - |g_-\rangle + |g_+\rangle}{\sqrt{2 + |(\Omega_p/\Omega_z)|^2}}, \tag{1}$$

which can be rewritten as

$$|\psi\rangle = \sin(\theta/2)|\psi_0\rangle + \exp(i\phi)\cos(\theta/2)|\psi_-\rangle \tag{2}$$

or

$$|\psi\rangle = \sin(\theta/2)|0\rangle + \exp(i\phi)\cos(\theta/2)|1\rangle, \tag{3}$$

where

$$\sin(\theta/2) = \frac{\Omega_p}{\sqrt{2|\Omega_z|^2 + |\Omega_p|^2}} \quad \text{and} \quad \cos(\theta/2) = \frac{(\sqrt{2}\Omega_z)}{\sqrt{2|\Omega_z|^2 + |\Omega_p|^2}} \tag{4}$$

and $(\theta/2) = \tan^{-1}(\Omega_p/\sqrt{2}\Omega_z)$. Any desired value of θ can be obtained by varying the ratio of $(\Omega_p/\sqrt{2}\Omega_z)$, where $\Omega_{p,z} = dE_{p,z}/2\hbar$. The phase factor ϕ in (3) can also be obtained by controlling the phase between the two beams $E_{p,z} = \mathcal{E}_{p,z}\exp[i(\omega t - kx - \phi_{p,z})]$. If the set-up is as in figure 1a, then rotating the HWP will distribute the intensity between E_p and E_z and positioning it appropriately will produce any desired θ. Keeping a variable retarder at one of the output ports of PBS will also control the phase ϕ.

The operation then can be mathematically expressed by

$$H(\theta) = \begin{pmatrix} \sin(\theta/2) & e^{i\phi}\cos(\theta/2) \\ -e^{-i\phi}\cos(\theta/2) & \sin(\theta/2) \end{pmatrix}, \tag{5}$$

which, acting on the basis vectors

$$|0\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |1\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{6}$$

leads to dressed state vectors

$$|\Phi_-\rangle \equiv \begin{pmatrix} \sin(\theta/2) \\ -e^{-i\phi}\cos(\theta/2) \end{pmatrix} \quad \text{and} \quad |\Phi_+\rangle \equiv \begin{pmatrix} e^{i\phi}\cos(\theta/2) \\ \sin(\theta/2) \end{pmatrix}. \tag{7}$$

Operator $H(\theta)$ reduces to a Hadamard when $\theta/2$ is set to 45° and $\phi = 1$, which is the equivalent of setting the half-wave plate of figure 1a to 45°. The state Φ_+ is a CPT state, as given in eq. (3).

This suggested scheme to prepare the atoms in state (3), has several distinct advantages. (i) The qubit states represented by (6) as well as state Φ_+ are CPT states. CPT states are end points of atom–laser interaction and the atoms eventually reach CPT states via non-CPT states as shown by Cohen-Tannoudji and Reynaud [4]. This means that the state preparation is reliable and the desired state is always prepared. (ii) Once the states are prepared, the atoms in this state no longer interact with the laser that prepares them. This eliminates the need for precise time control of the lasers. The state preparation is therefore robust and certain. (iii) The state preparation involves only cw beams and does not require any complex pulse shaping schemes. (iv) Since it does not involve single photon processes, lasers with nominally high intensity can be used. This would allow very precise control of phase ϕ while allowing fluctuations in the intensity. (v) Any desired superposition corresponding to any desired Bloch vector can be prepared by simply varying the intensity ratio between two laser beams. Due to all these, the configuration allows a robust and reliable preparation of two qubit states and its superposition and also the method of state preparation is very easy. In the following sections, methods of performing one-qubit and two-qubit operations are discussed.

2. Operations of logic gates

2.1 *One-qubit operations*

Setting $\theta/2 = 0$ in (5) will result in a rotation, which is the NOT operation

$$H(\theta = 0) = \begin{pmatrix} 0 & e^{i\phi} \\ -e^{-i\phi} & 0 \end{pmatrix}, \tag{8}$$

which converts $|\psi\rangle = \sin(\theta/2)|0\rangle + \exp(i\phi)\cos(\theta/2)|1\rangle$ to $|\psi\rangle = \cos(\theta/2)|0\rangle + \exp(i\phi)\sin(\theta/2)|1\rangle$, for any value of the existing $\theta/2$.

This is an intriguing situation since setting $\theta/2 = 0$ in eq. (5) is equivalent to setting $\Omega_p = 0$ in (4), which is equivalent to switching off E_p beam and thus always creating the atoms in state $|1\rangle$, no matter what the original state is. This discrepancy can be understood in the manner that the NOT operation always operates on the full dressed state $|\Phi_+\rangle$ and hence valid.

2.2 *Two-qubit operation: C-phase gate*

Two-qubit opeations can be obtained in a manner similar to the earlier works that exploited the dipole–dipole interaction [3,8], except using magnetic dipole–dipole interaction between spin states instead of electric dipoles.

In an external magnetic field $\vec{B}$, the spin vectors align at an angle that depends on their m_{F} value and also makes a Larmor precision about $\vec{B}$, with a frequency $\omega_{\mathrm{L}} = \gamma_{\mathrm{L}}|B|$. γ_{L} is the gyromagnetic ratio of the atom and $|B|$ is the value of the magnetic field [9]. The atom can now be flipped from one m_{F} state to the other by applying an oscillatory magnetic field perpendicular to $\vec{B}$, and at a frequency equal to the difference between the two corresponding Larmor frequenices. The dipole–dipole interaction between the spins now manifest as a shift in the Larmor frequencies and hence the resonance frequency for the oscillatory magnetic field also shifts as shown in figure 2 [10,11].

As in case of electric dipoles, the spin dipole interaction is also inversely dependent on the cube of the distance between them, given by

$$V_{dd} = \frac{\mu_0}{4\pi}\frac{\gamma_{\mathrm{L}}^2}{r^3}\left[S_1 S_2 - 3(S_1 n)(S_2 n)\right] \tag{9}$$

which will be reduced to $V_{dd} = \frac{\mu_0}{4\pi}\frac{\gamma_{\mathrm{L}}^2}{r^3}\left(3\cos^2\theta_{\mathrm{s}} - 1\right)$, for two degenerate m_{F} levels [11].

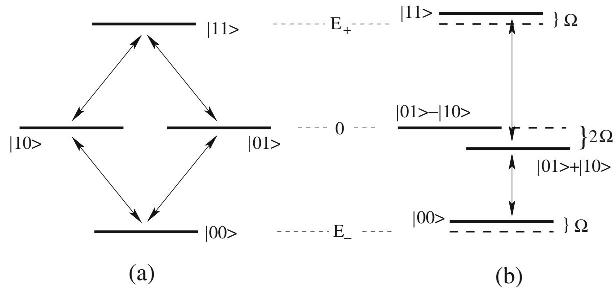


Figure 2. (**a**) Energy diagram of the two-atom system. (**b**) shows the effect of dipole–dipole interaction. The state $|10\rangle - |01\rangle$ is not coupled by radiofrequency transition either to $|00\rangle$ or to $|11\rangle$. The energy difference in non-interaction situation is equal to $\hbar\omega_{\mathrm{L}}$ where ω_{L} is the Larmor frequency. See text for the amount of shift in case of (**b**).

Here r is the normal distance between the two atoms, θ_{s} is the angle between the spin directions and $\vec{r}$, μ_0 is the permittivity of free space and the ratio $\mu_0/4\pi$ is a scaling factor for MKS units. The energy levels of the state atom pairs can be shown as in figure 2a. This interaction V causes a mixing of the pair states $|01\rangle$ and $|10\rangle$ as well as a shift in the energies as shown in figure 2b. The energy for the transition $|00\rangle \leftrightarrow |10\rangle + |01\rangle$ is shifted by $\Omega_m = 2(\mu_0/(4\pi))(\gamma_{\mathrm{L}}^2/r^3)$.

An RF field of frequency $\omega_{\mathrm{L}} + 2\Omega$, incident on this system will be absorbed by the atoms, if and only if the atom pair is in the state $|01\rangle + |10\rangle$, not otherwise. If now this field is in the shape of a pulse with a McCall–Hahn area 2π, then it will take the atom through $|11\rangle$ and back to $|01\rangle + |10\rangle$, but with an extra phase of π [12]. If the atom pair is in state $|01\rangle - |10\rangle$ instead, the phase factor already exists for $|01\rangle$ state. Therefore, if the atoms are brought together, the RF pulse applied and then taken apart, only the atoms in state $|01\rangle$ will return to $-|01\rangle$.

Another option, following Ryabstev *et al* [8] is to bring the atoms together and hold them close for a specific period. Since the dipole–dipole interaction causes a mixing of the states $|01\rangle$ and $|10\rangle$ to form a time-dependent superposition

$$|\psi_{dd}(t)\rangle = \cos(V_{dd}t/\hbar)|10\rangle - i\ \sin(V_{dd}t/\hbar)|01\rangle,$$

the atom pair oscillates between $|01\rangle$ and $|10\rangle$ with a half-period $T = 1/2(\hbar\pi)/V_{dd}$. The scheme then involves holding the atoms together for T and taking apart, which gives a control-swap gate or for $2T$, which will result in a controlled swap gate.

The serious drawback of this scheme is that one cannot distinguish *a priori* between control atom and the logic atom since they are both identical. An option then is to use two different atoms, with the same level configuraiton.

2.3 *Heterogeneous atoms*

Sodium, with $3S_{1/2}$, $F = 1$ and $3P_{1/2}$, $F = 1$ triplets, shows an identical behaviour of state preparation and qubit operations, but the corresponding Larmor frequency is different. The dipole–dipole interaction between spins of sodium atom and ^{87}Rb will cause only a level shift instead of a mixing states $|01\rangle$ and $|10\rangle$ as shown in figure 3 [11]. The amount of shift

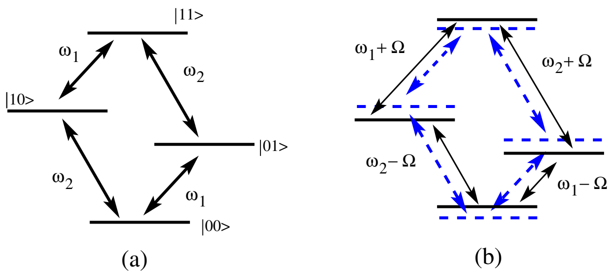


Figure 3. Energy diagram of the two-atom system for two different atoms, (**a**) without the spin dipole–dipole interaction and (**b**) energy shifts due to dipole interaction. Note that there is no mixing. Dashed lines in (**b**) are positions of unshifted energy levels.

$\Omega = (\mu_0/4\pi)(\gamma_1\gamma_2/r^3)(3\cos^2\theta_s - 1)$, where γ_1 and γ_2 are gyromagnetic ratios of sodium and ^{87}Rb atoms respectively.

Now a controlled NOT gate can be obtained using a RF pulse with a frequency $\omega_2 + \Omega$ with a McCall–Hahn area of 2π, or a pair of pulses with frequencies $\omega_2 + \Omega$ and $\omega_1 + \Omega$ times in a STIRAP-like fashion to obtain a controlled swap gate.

3. Conclusion

It is shown that a system that exhibits two trapping states can be obtained in ^{87}Rb and sodium atom interacting with two lasers that couple its $F=1 \leftrightarrow F=1$ transition. They can be mapped to the qubit states $|1\rangle$ and $|0\rangle$ and can be used for quantum computation. Any required superposition state $|\psi\rangle = \sin(\theta/2)|0\rangle + \exp(i\phi)\cos(\theta/2)|1\rangle$ can be prepared. Since this involves CPT states and also ground levels, it is very robust against decoherences. One-qubit and two-qubit operations are described with these states. However, the dipole–dipole interaction between spin states is weaker than that between electric dipole states and hence the shift is small. But the typical value of Larmor frquency for most alkali atoms are about 100 kHz and measuring a small shift in the radiofrequency domain is technologically feasible. The major technical difficulty with this scheme may be that one has to move the atoms nearer and apart as and when required for the two-qubit operation.

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References

[1] M A Neilson and I L Chuang, *Quantum computation and quantum information* (Cambridge, 2000)
Joachim Stolze and Dieter Suter, *Quantum computing* (Wiley-VCH, Weinheim, 2004)

[2] A K Khitrin and B M Fung, *Phys. Rev.* **A64**, 32306 (2001)
Ranbir Das and Anil Kumar, *Phys. Rev.* **A68**, 032304 (2003) and references therein
J I Cirac and P Zoller, *Phys. Rev. Lett.* **74**, 4091 (1995)
Ferdinand Schmidt-Kaler, Hartmut Häffner, Mark Riebe, Stephan Gulde, Gavin P T Lancaster, Thomas Deuschle, Christoph Becher, Christian F Roos, Jürgen Eschner and Rainer Blatt, *Nature* **422**, 408 (2003)
Daniel Loss and David diVincenzo, *Phys. Rev.* **A57**, 120 (1998)
[3] G K Brennen, C M Caves, Poul S Jessen and Ivan H Deutsch, *Phys. Rev. Lett.* **82**, 1060 (1999)
[4] C Cohen-Tannoudji and S Reynaud, *J. Phys.* **B10**, 345 (1977); *ibid*, 2311
[5] E Arimondo, *Progress in optics* edited by E Wolf (North Holland, 1996) Vol. XXXV, p. 257
[6] Alan Corney, *Atomic and laser spectroscopy* (Clarendon, Oxford, 1977)
[7] Ashok Vudayagiri and Surya P Tewari, *J. Phys.* **B39**, 3919 (2006)
[8] I I Ryabtsev *et al*, *J. Phys.* **B38**, 421 (2005)
[9] Dmitry Budker, Derek F Kimball and David P DeMille, *Atomic physics* (Oxford University Press, 2004) section 2.6
[10] J D Jackson, *Classical electrodynamics* (Wiley Eastern, New Delhi, 1989)
[11] C Cohen-Tannoudji, B Diu and F Laloë, *Quantum mechanics* (John Wiley, 1977) (see section ‘Complement B_{XI}’ on page 1120)
[12] S L McCall and E L Hahn, *Phys. Rev. Lett.* **18**, 908 (1967)
L Allen and J H Eberly, *Optical resonance and two level atoms* (Dover, New York, 1987)