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Quark model for kaon nucleon scattering

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Abstract. Kaon nucleon elastic scattering is studied using chiral $SU(3)$ quark model including antiquarks. Parameters of the present model are essentially based on nucleon–nucleon and nucleon–hyperon interactions. The mass of the scalar meson σ is taken as 635 MeV. Using this model, the phase shifts of the S and P partial waves of the kaon nucleon elastic scattering are investigated for isospins 0 and 1. The results of the numerical calculations of different partial waves are in good agreement with experimental data.

Keywords. Kaon nucleon scattering; quark model; calculated phase shifts.

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1. Introduction

In the last few years, chiral $SU(3)$ quark models have been found [1] useful for studying light quark systems by solving the nonperturbative quantum chromodynamics problem. The nucleon–nucleon and hyperon–nucleon bound and scattering states are calculated for different partial waves. The calculated binding energy of the nucleon–nucleon particle using the quark model is in agreement with the experimental value for double Λ hypernucleus. The baryon–meson interactions are studied using the quark model after including antiquark to the systems, with which the annihilation will be difficult. However, the annihilation of the kaon nucleon system to gluons and vacuum is not allowed, so that the system will be annihilated to kaon mesons only.

Also, from recent experimental measurements [2–6], the existence of the pentaquark state $\Theta^+(1540)$ from the exotic K^+n or K^0p resonance has been observed. This requires deep understanding of the kaon–nucleon interaction to investigate the different phase shifts which had been calculated [7,8] using quark model including gluon, pion and sigma exchanges in the quark–quark interactions, to obtain the ground state energies of mesons. A chiral $SU(3)$ quark model is used [1,9] to study the partial wave elastic scattering phase shifts using resonating group method calculations. The obtained results [10] are useful to study the structure of the pentaquark state $\Theta^+(1540)$.

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In the present work, the kaon nucleon elastic scattering is studied using a quark model. A chiral $SU(3)$ quark model including antiquarks is used. This quark model is calculated by solving the nonperturbative quantum chromodynamics problem. The input model parameters are taken as those fitting the nucleon–nucleon and hyperon–nucleon experimental data. The mass of the scalar meson σ is taken as 635 MeV in the present work. This choice reasonably reduces the attraction of σ meson in the kaon nucleon S_{01} partial wave. Numerical calculations are carried out for different partial waves leading to reasonable results which are in agreement with the experimental data, as well as with previous theoretical phase shifts [7,9,10]. Therefore, the addition of antiquark to the kaon nucleon system enables the extraction of two quark interactions which in turn can be used to study the pentaquark state $\Theta^+(1540)$.

In §2, the quark model for the kaon nucleon system is introduced. Numerical calculations of the kaon nucleon elastic scattering phase shifts and results are given in §3. Section 4 is devoted to the discussion and conclusions.

2. Quark model for the kaon nucleon system

The interaction Lagrangian of the quark-chiral $SU(3)$ field can be given in terms of the quark chiral field coupling constant $g_{\rm ch}$, the quark-left and quark-right spinors ($\Psi_{\rm L}$ and $\Psi_{\rm R}$), the Goldstone boson field π_a, the flavour $SU(3)$ group Gell–Mann matrix λ_a, the scalar nonet fields σ_a and the pseudoscalar nonet fields π_a. Hence, the interactive Hamiltonian can be written as

$$H_{\rm ch} = g_{\rm ch} F(\mathbf{q}^2)\bar{\psi}\left(\sum_{a=0}^{8}\sigma_a\lambda_a + i\sum_{a=0}^{8}\pi_a\lambda_a\gamma_5\right)\psi, \tag{1}$$

where the chiral field from the factor $F(\mathbf{q}^2)$ is given as

$$F(\mathbf{q}^2) = \Lambda/(\Lambda^2+\mathbf{q}^2)^{1/2} \tag{2}$$

with the chiral symmetry breaking scale is identified by the cut-off mass Λ. Then, from eqs (1) and (2), the $SU(3)$ chiral field-induced quark–quark potentials can be given by the expressions

$$\begin{aligned} V_{\sigma_a}(\mathbf{r}_{ij}) &= [g_{\rm ch}^2/4\pi]F^2(-m_{\sigma_a}^2)(\Lambda - m_{\sigma_a}) \\ &\quad \times W(m_{\sigma_a}r_{ij})[\lambda_a(i)\lambda_a(j)] + V_{\sigma_a}^{l.s}(\mathbf{r}_{ij}), \end{aligned} \tag{3}$$

where

$$\begin{aligned} V_{\sigma_a}^{l.s} &= [g_{\rm ch}^2/(16\pi\, m_{q_i}m_{q_j}r_{ij}^2)]F^2(-m_{\sigma_a}^2) \\ &\quad \times [\Lambda(1+\Lambda r_{ij})W(\Lambda r_{ij}) - m_{\sigma_a} \\ &\quad \times (1+m_{\sigma_a}r_{ij})W(m_{\sigma_a}r_{ij})] \\ &\quad \times [\mathbf{L}\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)][\lambda_a(i)\lambda_a(j)] \end{aligned} \tag{4}$$

and

$$\begin{aligned} V_{\pi_a}(\mathbf{r}_{ij}) &= [g_{\rm ch}^2/(48\pi m_{q_i}m_{q_j})]F^2(-m_{\pi_a}^2) \\ &\quad \times [m_{\pi_a}^3 W(m_{\pi_a}r_{ij}) - \Lambda^3 W(\Lambda r_{ij})] \\ &\quad \times (\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)[\lambda_a(i)\lambda_a(j)] \end{aligned} \tag{5}$$

where

$$W(x) = (1/x)[\exp(-x)], \tag{6}$$

m_{σ_a} and m_{π_a} are the masses of the scalar and pseudoscalar mesons, respectively.

From the chiral $SU(3)$ quark model, we find that the interaction induced by the coupling of chiral field describes the nonperturbative quantum chromodynamics effect of the low-momentum medium-distance range. Then, it is required to have an effective one-gluon-exchange interaction which describes the short-range perturbative quantum chromodynamics behaviour, to study the hadron structure and hadron–hadron dynamics.

The effective one-gluon-exchange (OGE) interaction expressing the short-range perturbative quantum chromodynamics is given as

$$\begin{aligned} V_{ij}^{\mathrm{OGE}} &= (1/4) g_i g_j (\lambda_i^c \lambda_j^c) \\ &\quad \times \{(1/\mathbf{r}_{ij}) - (\pi/2)\delta(\mathbf{r}_{ij})[(1/m_{q_i}^2) + (1/m_{q_j}^2) \\ &\qquad + (4/3)(m_{q_i}^{-1} m_{q_j}^{-1})(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)]\} + V_{\mathrm{OGE}}^{\boldsymbol{l.s}}, \end{aligned} \tag{7}$$

where

$$V_{\mathrm{OGE}}^{\boldsymbol{l.s}} = -(3/16) g_i g_j (\lambda_i^c \lambda_j^c)\, m_{q_i}^{-1} m_{q_j}^{-1} r_{ij}^{-3} \mathbf{L}(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j). \tag{8}$$

The nonperturbative quantum chromodynamics effect in long distance is given by the confinement potential as

$$V_{ij}^{\mathrm{conf}} = -a_{ij}^c (\lambda_i^c \lambda_j^c) r_{ij}^2 - a_{ij}^{c0} (\lambda_i^c \lambda_j^c). \tag{9}$$

The total Hamiltonian of the kaon nucleon system is obtained by introducing an antiquark in the chiral $SU(3)$ quark model and is given as

$$H = \sum_{i=1}^{5} H_0^i + H_{\mathrm{c.m.}} + \sum_{i<j=1}^{4} V_{ij} + \sum_{i=1}^{4} V_{i\bar{5}}. \tag{10}$$

H_0 is the kinetic energy operator and $H_{\mathrm{c.m.}}$ is the centre of mass motion kinetic operator. V_{ij} is the quark–quark interaction and $V_{i\bar{5}}$ is the quark–antiquark interaction. The quark–quark interaction is given as

$$V_{ij} = V_{ij}^{\mathrm{OGE}} + V_{ij}^{\mathrm{conf}} + V_{ij}^{\mathrm{ch}}, \tag{11}$$

where V_{ij}^{OGE} and V_{ij}^{conf} are given by eqs (7) and (9), and V_{ij}^{ch} is expressed as

$$V_{ij}^{\mathrm{ch}} = \sum_{a=0}^{8} V_{\sigma_a}(\mathbf{r}_{ij}) + \sum_{a=0}^{8} V_{\pi_a}(\mathbf{r}_{ij}). \tag{12}$$

The interaction between the up (down) quark and the strange quark can be written [11] as the sum of direct interaction and annihilation as

$$V_{i\bar{5}} = V_{i\bar{5}}^{\mathrm{dir}} + V_{i\bar{5}}^{\mathrm{ann}}. \tag{13}$$

$V_{i\bar{5}}^{\rm dir}$ in eq. (13) can be calculated using the equation

$$V_{i\bar{5}}^{\rm dir} = V_{i\bar{5}}^{\rm conf} + V_{i\bar{5}}^{\rm OGE} + V_{i\bar{5}}^{\rm ch}, \tag{14}$$

where

$$V_{i\bar{5}}^{\rm conf} = -a_{i5}^{c}(-\lambda_i^c \lambda_5^{c*}) - a_{i5}^{c0}(-\lambda_i^c \lambda_5^{c*}) \tag{15}$$

and

$$\begin{aligned} V_{i\bar{5}}^{\rm conf} =\ & (1/4) g_i\, g_s (-\lambda_i^c \lambda_5^{c*}) \\ & \times \{(1/r_{i5}) - (\pi/2)\delta(\mathbf{r}_{i5})[(1/m_{q_i}^2) + (1/m_S^2) \\ & \quad + (4/3)\, m_{q_i}^{-1}\, m_S^{-1} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)]\} - (3/16) \\ & \times g_i\, g_j (-\lambda_i^c \lambda_5^{c*}) m_{q_i}^{-1} m_{q_5}^{-1} r_{i5}^{-3} \mathbf{L}(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), \end{aligned} \tag{16}$$

where

$$V_{i\bar{5}}^{\rm ch} = \sum_i (-1)^{G_j}\, V_{i5}^{{\rm ch},j}. \tag{17}$$

The G parity of the j meson is described by the factor $(-1)^{G_j}$. The annihilation part of the interaction between the up (down) quark and the strange quark $V_{i\bar{5}}^{\rm ann}$ appeared in eq. (13), annihilates only into a kaon meson for the kaon nucleon system. Thus, for the present case of kaon nucleon system

$$\begin{aligned} V_{i\bar{5}}^{\rm ann} =\ & -(\tilde{g}_{\rm ch}^2/4\pi)[m_k^2 - (\tilde{m} + \tilde{m}_s)^2]^{-1} \\ & \times [(1 - \boldsymbol{\sigma}_q \cdot \boldsymbol{\sigma}_{\bar{q}})/2]_{\rm spin} [(2 + 3\lambda_q \lambda_{\bar{q}}^*)/6]_{\rm colour} \\ & \times [(38 + 3\lambda_q \lambda_{\bar{q}}^*)/18]_{\rm flavour} (\Lambda^2/r)[\exp(-\Lambda r)]. \end{aligned} \tag{18}$$

In eq. (18) $\tilde{g}_{\rm ch}^2$ and $\tilde{m}$ are the effective coupling constant of the chiral field in the annihilation case and the effective quark mass, respectively. In eq. (18), $\tilde{m}$ is treated as the effective

Table 1. Meson masses and cut-off masses.

Meson	Meson masses (MeV)	Cut-off masses (MeV)
$m_{\sigma'}$	969	
m_{κ}	1438	
m_{ϵ}	969	
m_{σ}	635	
m_{π}	134	
$m_{\rm K}$	489	
m_{η}	552	
$m_{\eta'}$	963	
Λ		1280

Table 2. Model parameters.

Parameters	Values	Units
m_{u}	311	MeV
m_{s}	473	MeV
b_{u}	0.49	fm
g_{u}	0.878	
g_{s}	0.762	
a^{c}_{uu}	51.24	$\mathrm{MeV/fm^2}$
a^{c}_{us}	77.18	$\mathrm{MeV/fm^2}$
a^{c0}_{uu}	−52.17	MeV
a^{c0}_{us}	−64.93	MeV

quark mass, while it is actually quark momentum-dependent. A form factor $F(q2)$ given by eq. (2) is also used in the vertex of the quark chiral field coupling and inserted in the present form of the annihilation interaction $V^{\mathrm{ann}}_{i\bar{5}}$ to flatten the sharp behaviour of the δ function. Presently, the first part of the right-hand side of eq. (18) is treated as a parameter and is adjusted to fit the mass of kaon meson.

3. Numerical calculations and results

Presently, we are interested to study the partial wave phase shifts of the kaon nucleon scattering. The phase shifts are calculated using the Hamiltonian given by eq. (10). The

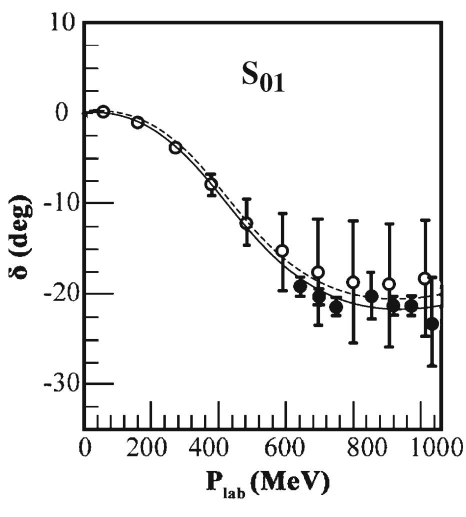

Figure 1. Phase shifts of the kaon nucleon S_{01} wave as a function of the laboratory momentum of the kaon meson. The dashed curve gives the data of previous calculations [9]. The solid curve represents our present theoretical calculations. The empty circles and filled circles represent data taken from refs [12] and [13], respectively.

theoretical formalism of the kaon nucleon system is expressed by the mathematical expressions given in §2. The meson masses and cut-off masses are given in table 1. The masses of the mesons are taken from experimental data, while the mass of the σ meson is taken in the present work as 635 MeV to be an adjustable parameter. However, the value of the cut-off radius Λ^{-1} is taken to be close to the chiral symmetry breaking scale. The input parameters are taken such that the harmonic oscillator width parameter b_{u}, the up (down) quark mass $m_{\mathrm{u(d)}}$ and the strange quark mass m_{s} together with the other parameters that the chiral coupling constant g_{ch} is consistent with the experimental value as $g^2_{NN\pi}/4\pi = 13.79$, where

$$g^2_{\mathrm{ch}}/4\pi = (3/5)^2(g^2_{NN\pi}/4\pi)(m^2_{\mathrm{u}}/M^2_N). \tag{19}$$

The one-gluon-exchange coupling constants g_{u} and g_{s} are determined as the mass splits between octet and decuplet baryons. The confinement strengths a_{ij} and the zero point energies a^{c0}_{ij} are fixed by fitting masses of the octet and decuplet baryons. The values of these parameters are given in table 2.

Mixing between the flavour singlet and octet mesons is taken into account. The mixing for the η_0 and η_8 mesons are between σ_0 and σ_8. Ideal mixing between σ_0 and σ_8 is introduced for the σ and ϵ mesons which restricts the σ meson to act only on the strange s quark. This means that the scalar meson exchange interactions between the up (down) and the strange antiquark $\bar{s}$ do not exist, which reduce very much the attraction force of the scalar meson between the kaon and the nucleon.

The resonating group method is applied to the present kaon nucleon system, with total S spin and T isospin. The total wave function of the system is constructed following the cluster model calculations. This wave function is divided into the trial wave function of the relative motion between the interacting kaon and nucleon clusters $\chi^{\mathrm{L}}_{\mathrm{rel}}(R_{Kn})$ and the

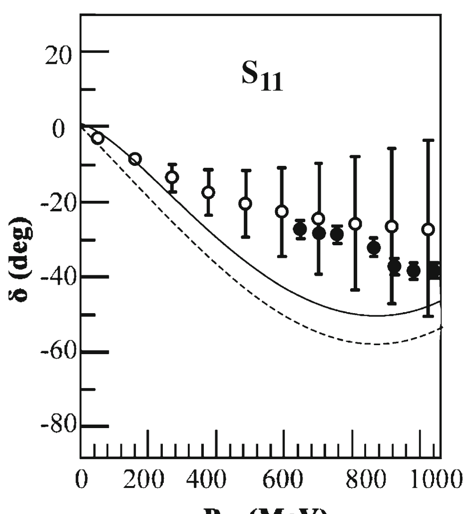

Figure 2. Phase shifts of the kaon nucleon S_{11} wave as a function of the laboratory momentum of the kaon meson. The meaning of the symbols are the same as in figure 1.

wave function of the total centre of mass motion of the centre of mass of the whole system $\Psi(R_{\text{c.m.}})$. Then, the resonating group method equation is obtained with the Hamiltonian and normalization kernels. The relative wave function of the present scattering problem is expanded in partial wave expansion in terms of the spherical Hankel functions $h_{\text{L}}^{\pm}$ as

$$\chi_{\text{rel}}^{\text{L}}(R_K n) = \sum_{i=1}^{n} c_i u^{\text{L}}\,(R_{\text{KN}}, S_i) \tag{20}$$

with

$$u^{\text{L}}(R_{\text{KN}}, S_i) = \begin{cases} \alpha_i u^{\text{L}}(R_{\text{KN}}, S_i)\,, & R_{\text{KN}} \leq R_{\text{C}} \\ \left[h_{\text{L}}^{-}(k_{\text{KN}} R_{\text{KN}}) - h_{\text{L}}^{+}\,(k_{\text{KN}} R_{\text{KN}})\right] R_{\text{KN}}, & R_{\text{KN}} \geq R_{\text{C}} \end{cases}, \tag{21}$$

where S_i is the generate coordinate, k_{KN} is the momentum of the relative motion and R_{C} is the cut-off radius beyond which all the strong interactions can be neglected. The smoothness condition at $R_{\text{KN}} = R_{\text{C}}$ is used to determine the complex parameters α_i and s_i, and c_i's satisfy $\sum_{i=1}^{n}, c_i = 1$. Then, the Lth partial wave equation for the scattering problem can be deduced by performing variational procedure. This partial wave equation is solved by calculating the kernel and using the asymptotic form of the spherical Hankel function. Solving this partial wave equation, the S-matrix element S^{L} and the phase shifts δ_{L} are given by

$$S^{\text{L}} = \exp\,(2i\delta_{\text{L}}) = \sum_{i=1}^{n} c_i s_i\,. \tag{22}$$

Numerical calculations of the phase shifts of the partial waves of the kaon nucleon scattering are carried out by solving the mathematical theoretical expression of the Hamiltonian

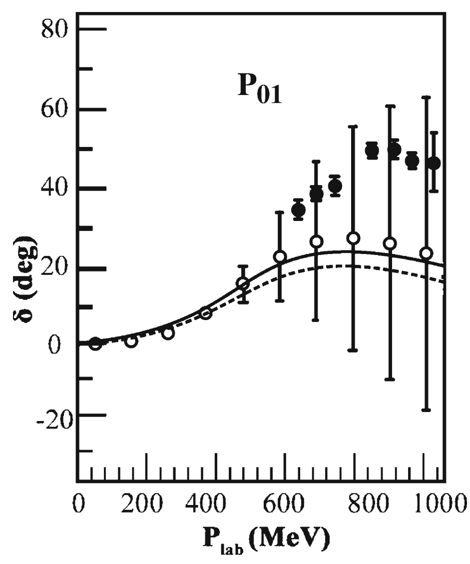

Figure 3. Phase shifts of the kaon nucleon P_{01} wave as a function of the laboratory momentum of the kaon meson. The meaning of the symbols are the same as in figure 1.

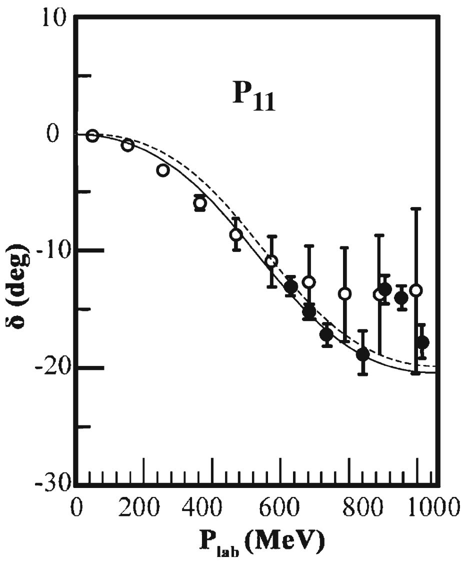

Figure 4. Phase shifts of the kaon nucleon P_{11} wave as a function of the laboratory momentum of the kaon meson. The meaning of the symbols are the same as in figure 1.

given by eq. (10). The calculations of the phase shifts are done for the S and P waves with isospins 0 and 1. The results of the present theoretical numerical calculations are introduced in figures 1–6 by the solid curves, for S_{01}, S_{11}, P_{01}, P_{11}, P_{03} and P_{13} respectively. The first subscript for the partial waves stands for the isospin quantum number, while the

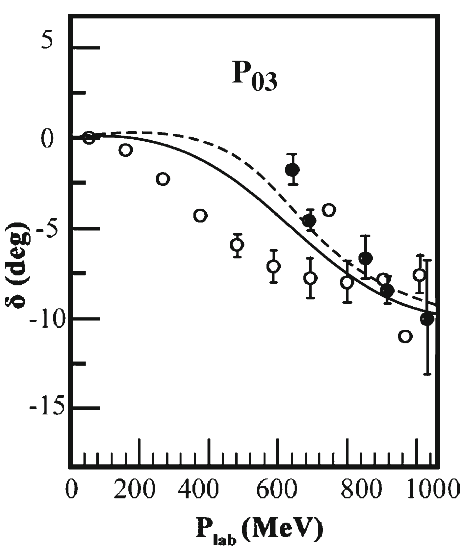

Figure 5. Phase shifts of the kaon nucleon P_{03} wave as a function of the laboratory momentum of the kaon meson. The meaning of the symbols are the same as in figure 1.

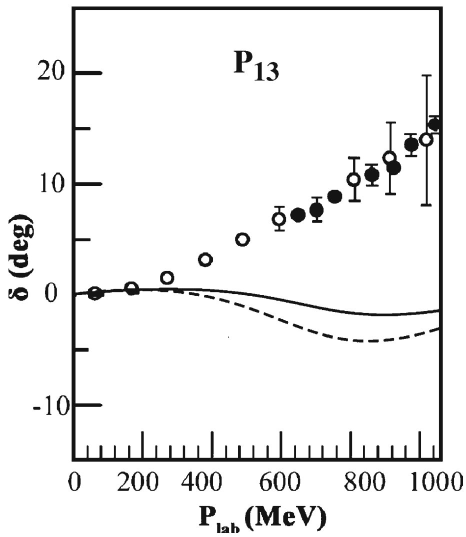

Figure 6. Phase shifts of the kaon nucleon P_{13} wave as a function of the laboratory momentum of the kaon meson. The meaning of the symbols are the same as in figure 1.

second subscript refers to twice the total angular momentum of the system. The experimental data are shown in figures 1–6 as the empty circles and the filled circles, taken from refs [12] and [13], respectively. Also, previous calculations [9] are given in figures 1–6 as dashed curves for comparison. From figures 1–6, we see that our present theoretical calculations as the solid curves are in good agreement with the experimental data [12,13]. The present calculations reproduce the experimental data better than the previous calculations. This means that the interaction between the kaon and the nucleon obtained from the chiral $SU(3)$ quark model is more reasonable. Thus, our present calculations considerably improve the theoretical phase shifts in the magnitude and are in reasonable, better and good agreement with the experimental data.

4. Discussion and conclusions

In the present work, the kaon nucleon scattering is studied using chiral $SU(3)$ quark model including antiquark. The values of different parameters are taken to reproduce the nucleon–nucleon and hyperon–nucleon experimental data. However, in the present work, the mass of the σ scalar meson is taken to be 635 MeV, and ideal mixing of σ_0 and σ_8 also is taken into account. The present results of the theoretical and numerical calculations of the phase shifts are in a good agreement with the experimental data. Comparison with previous calculations shows that the present results extract the phase shifts considerably better. So, the parameters used in the present formalism together with the large error bars on the data show that the present theoretical calculations extract better reproductions of the phase shift data.

Therefore, we can conclude that the inclusion of antiquark $\bar{s}$ besides the four up (down) quarks in the kaon nucleon system, achieves and improves the experimental phase shifts.

Moreover, due to the absence of signal for kaon nucleon resonance in the S and P waves in the range of the momentum of the kaon meson up to 1000 MeV, the observed exotic baryon Θ^+ is hardly regarded as a kaon resonance state. Also, the inclusion of antiquark in the kaon nucleon system helps very much in studying the structure of the $\Theta^+(1540)$ pentaquark state. Although the exotic state of $\Theta^+(1540)$ was first found by the LPES Collaborations [2], its existence is in doubt now. Most of the recent experimental studies have failed to find any signal for $\Theta^+(1540)$. Recently [14,15] a serious doubt has been raised on the last analysis of the LEPS data.

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