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Delay or anticipatory synchronization in one-way coupled systems using variable delay with reset

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Abstract. We present a mechanism for the synchronization of one-way coupled nonlinear systems in which the coupling uses a variable delay, that is reset at finite intervals. Here the delay varies in the same way as the system in time and so the coupling function remains constant for the reset interval at the end of which it is reset to the value at that time. This leads to a novel and discrete error dynamics and the resulting general stability analysis is applicable to chaotic or hyperchaotic systems. We apply this method to standard chaotic systems and hyperchaotic time delay systems. The results of the detailed numerical analysis agree with the results from stability analysis in both cases. This method has the advantage that it is cost-effective since information from the driving system is needed only at intervals of reset. Further, in the context of time delay systems, optimization among the different time-scales depending upon the application is possible due to the flexibility among the four different time-scales in our method, viz. delay in the driving system, anticipation in the response system, system delay time and reset time. We suggest a bi-channel scheme for implementing this method in communication field with enhanced security.

Keywords. Synchronization; variable time delay; time delay systems; secure communication.

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1. Introduction

Synchronizing chaotic systems is a topic of great interest and relevance because of the basic challenges involved and the potential applications in biological, optical, electrical and communication systems. In such systems, synchronization emerges when two or more systems adjust a given property of their motion to a common behaviour due to coupling or forcing. The various types of synchronization that are studied and implemented in several applications are complete or identical, phase, lag, anti-phase and generalized synchronizations [1–3]. In all these cases, synchronization is achieved by different types of coupling schemes, like linear difference or diffusive, replacement coupling or nonlinear coupling and can also be induced by external forcing or noise. Using coupling with time delay, it is possible to achieve synchronization with delay or anticipation between the states of the systems [4–7]. Most of the cases studied with delay in the coupling involve a constant time delay. However, recently, variable delay and delay with modulation are reported that increase the richness of the dynamics and have possible applications to communication [8–11]. Moreover, projective synchronization in which the systems are synchronized up to a scaling factor [12–15] or scaling function [16,17] have been introduced recently. So it has been established that complete, generalized and lag synchronizations can be considered as manifestations of a common type called time-scale synchronization [18].

The fact that chaotic output can serve as broadband carriers with a high level of robustness and privacy in data transmission, makes synchronization of chaotic systems a relevant requirement for such applications. A practical realization of this scheme with the signal masked in optical chaos, has demonstrated its feasibility over a commercial fibre-optic channel at high bit rates over a distance of 120 km [19]. In this context, low-dimensional chaos is being replaced recently by hyperchaos from time delay systems since they ensure better security [20–24]. But in such systems also, there are ways of hacking the message by reconstructing the transmitter dynamics once the time delay is identified from the transmitted text [25–30].

In this paper, we describe a scheme of synchronizing one-way coupled nonlinear systems, recently introduced by us [31]. In this scheme, delay or anticipatory synchronization is realized with only intermittent information from the driver system and hence is both cost-effective and secure. This is achieved by using a coupling with an initial delay that varies at the same scale as the system dynamics so that the coupling function remains constant for a certain interval of time called the reset interval. After each reset interval, the delay is set back to its initial value. Unlike the case of fixed delay, this resetting mechanism makes the error dynamics discrete and therefore a detailed and general stability analysis can be carried out to isolate the region of stability of the synchronized state in parameter plane.

We apply this scheme to standard chaotic systems like Rössler and to hyperchaotic time delay systems like Mackey Glass system. In the latter case, the synchronization involves four flexible time-scales, viz. delay, anticipation, system delay and reset times that can be optimized based on the requirement. A detailed numerical analysis is given in all these cases, that support the results from the stability analysis well. We suggest a bi-channel scheme for implementing this scheme in communication using time delay systems with enhanced security. The synchronizing channel carries information about the transmitter dynamics only at intervals of reset time and hence is not susceptible to reconstruction easily. The message channel being separate can be made complex enough by adding nonlinear combination of the transmitter variable at different time delays. The delay and anticipatory times can function as additional basic keys in this context. We establish numerically the security of our scheme when subject to some recently reported methods of hacking.

2. Synchronization of chaotic systems using variable delay in coupling with reset

We consider a dynamical system x that drives an identical system y with a coupling of the linear difference type where the drive variable is delayed by τ_1 and the driven variable delayed by τ_2. Then as given in our recent work [31], we represent the dynamics of such a coupled system as

$$\begin{aligned} \dot{x} &= f(x) \\ \dot{y} &= f(y) + \epsilon \sum_{m=0}^{\infty} \Gamma(x_{t_1} - y_{t_2}) \chi_{(m\tau,(m+1)\tau)}, \end{aligned} \tag{1}$$

where $x_{t_1} = x(t - t_1)$, $y_{t_2} = y(t - t_2)$. $\chi_{(t',t'')}$ is an indicator function such that $\chi_{(t',t'')} = 1$ for $t' \leq t \leq t''$ and zero otherwise. τ is the reset time and the matrix Γ gives the coupling between the components of x and y. Both the delays t_1 and t_2 increase linearly in time from their initial values τ_1 and τ_2 within each reset interval. At the end of the interval τ, they are reset to their starting values. So we have always $t_1 - t_2 = \tau_1 - \tau_2$ and moreover in this method the coupling term uses the same value for both variables x_{t_1} and y_{t_2} during each interval τ. The synchronization manifold for the coupled systems evolving under this scheme can be defined as $y(t - \tau_2) = x(t - \tau_1)$ or $y(t) = x(t - \tau_1 + \tau_2)$. This means that for $\tau_2 - \tau_1 < 0$, we get delay or lag synchronization, $\tau_2 - \tau_1 > 0$ results in anticipatory synchronization and $\tau_2 - \tau_1 = 0$ corresponds to equal time or isochronous synchronization.

We apply the scheme to two identical Rössler systems and their equations are given as in eq. (1) by

$$\begin{aligned} \dot{x_1} &= -x_2 - x_3 \\ \dot{x_2} &= x_1 + ax_2 \\ \dot{x_3} &= b + x_3(x_1 - c) \\ \dot{y_1} &= -y_2 - y_3 + \epsilon \sum_{m=0}^{\infty} (x_{1t_1} - y_{1t_2}) \chi_{(m\tau,(m+1)\tau)} \\ \dot{y_2} &= y_1 + ay_2 \\ \dot{y_3} &= b + y_3(y_1 - c). \end{aligned} \tag{2}$$

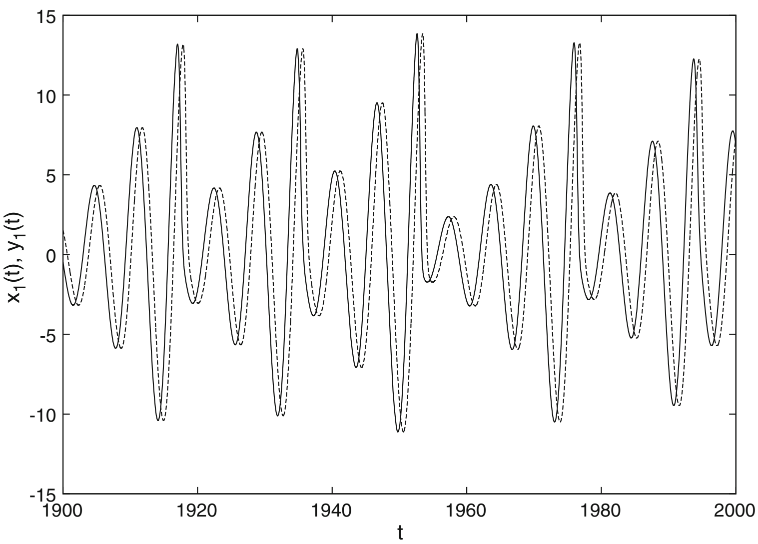

Figure 1. The time series of two Rössler systems coupled using the scheme in eq. (1). The delay synchronization is shown with a delay of 0.82 units between $x_1(t)$ (solid line) and $y_1(t)$ (dashed line).

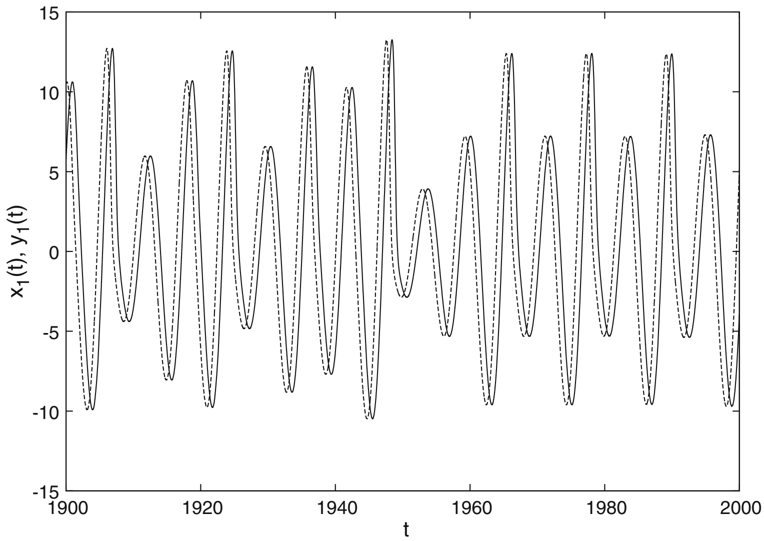

Figure 2. The time series of two Rössler systems coupled using the scheme in eq. (1). The anticipatory synchronization with $y_1(t)$ (dashed line) anticipating $x_1(t)$ (solid line) by 0.82 units is shown.

We take the parameter values $a = 0.15$, $b = 0.2$ and $c = 10$ that correspond to the chaotic state and both the systems are evolved from random initial conditions. With $\tau = 0.10$ and the coupling strength $\epsilon = 0.4$, the resulting time series obtained for $\tau_1 = 0.84$ and $\tau_2 = 0.02$ is plotted in figure 1. Here the response system $y(t)$ (dashed line) lags behind the driver $x(t)$ (solid line) by $\tau_2 - \tau_1$. Figure 2 shows the same for $\tau_1 = 0.02$ and $\tau_2 = 0.84$ such that $y(t)$ anticipates $x(t)$ with the same time shift.

3. Stability of the synchronized state

In this section, we discuss the stability analysis for the method of coupling given in the previous section. We find that the resetting of the variable delay times in the coupling function makes the error dynamics discrete and a detailed analysis of the stability of the resulting synchronized state has been reported by us [31]. We present the salient features of this analysis here. We define the transverse system by the variable $\Delta = y - x_{\tau_1-\tau_2}$ and its dynamics in linear approximation is obtained from eq. (1) as

$$\dot{\Delta} = f'(x_{\tau_1-\tau_2})\Delta - \epsilon \sum_{m=0}^{\infty} \chi_{(m\tau,(m+1)\tau)}\Delta_m, \tag{3}$$

where $\Delta_m = \Delta(t-t_2) = \Delta(m\tau-\tau_2)$ and we take coupling in all components of x and y, i.e. $\Gamma_i = 1,\ \forall i$. We note that Δ_m is a constant in each time interval $m\tau \le t < (m+1)\tau$. Also τ_1 can be eliminated by shifting the time-scale of the drive system linearly and redefining τ_2 as $\tau_2-\tau_1$. Then the fixed point $\Delta = 0$ corresponds to the lag (anticipatory) synchronized state corresponding to $\tau_2 < 0$ ($\tau_2 > 0$). In general, it is difficult to solve eq. (3) analytically.

We find that it is useful to do an approximate analysis by assuming that the Jacobian f' can be replaced by some effective time average Lyapunov exponent λ. In this approximation we treat f' as scalar so that the problem gets simplified retaining only the relevant features. But this then gives only the features near the transition. This type of approximation was used in [31,32] and we found that it describes the overall features of the phase diagram reasonably well. With this approximation, eq. (3) gives

$$\dot{\Delta} = \lambda\Delta - \epsilon \sum_{m=0}^{\infty} \chi_{(m\tau,(m+1)\tau)} \Delta_m. \tag{4}$$

In the interval $m\tau \leq t < (m+1)\tau$, we take the solution of eq. (4) as

$$\Delta = \alpha\Delta_m + C_m e^{\lambda t}. \tag{5}$$

Here $\alpha = \epsilon/\lambda$ is the normalized dimensionless coupling constant and C_m is an integration constant. The integration constant can be eliminated from eq. (5) to get

$$\Delta = \alpha\Delta_m + (\Delta_{m+1} - \alpha\Delta_m)e^{\lambda(t-(m+1)\tau+\tau_2)}. \tag{6}$$

Similarly in the interval $(m-1)\tau \leq t \leq m\tau$, we have

$$\Delta = \alpha\Delta_{m-1} + (\Delta_m - \alpha\Delta_{m-1})e^{\lambda(t-m\tau+\tau_2)}. \tag{7}$$

Matching the solutions (6) and (7) at $t = m\tau$, we get a recursion relation for Δ_m, which can be written as a 2D map in matrix form as

$$\begin{pmatrix} \Delta_{m+1} \\ \Delta_m \end{pmatrix} = \begin{pmatrix} a & -b \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \Delta_m \\ \Delta_{m-1} \end{pmatrix}, \tag{8}$$

where

$$a = \alpha(1 - e^{\lambda(\tau-\tau_2)}) + e^{\lambda\tau}, \tag{9a}$$

$$b = \alpha e^{\lambda\tau}(1 - e^{-\lambda\tau_2}). \tag{9b}$$

The eigenvalue equation for the Jacobian matrix is then

$$\mu^2 - a\mu + b = 0 \tag{10}$$

with the solutions

$$\mu_\pm = \frac{1}{2}(a \pm \sqrt{a^2 - 4b}). \tag{11}$$

The synchronized state, $\Delta = 0$, is stable if both the solutions satisfy $|\mu_\pm| < 1$. For $\tau_2 \leq \tau$, as shown in [31], we find that the lower limit of stability is given by $\alpha_l = 1$ while the upper limit is to be considered for the cases $\mu = 1$ ($a^2 - 4b > 0, a > 0$), $\mu = -1$ ($a^2 - 4b > 0, a < 0$) and $|\mu| = 1$ ($a^2 - 4b < 0$, μ complex). The results of this analysis show that the upper limit of stability in the parameter plane $\tau_2/\tau - \alpha$ has a peak at τ_{2p}. In the regime $\tau_2 \leq \tau_{2p}$, this is given by

$$\alpha_u = \frac{e^{\lambda\tau} + 1}{2e^{\lambda(\tau-\tau_2)} - e^{\lambda\tau} - 1} \tag{12}$$

while for larger values of τ_2 ($\tau_{2p} \leq \tau_2 \leq \tau$) it is given by

$$\alpha_u = \frac{e^{-\lambda\tau}}{1 - e^{-\lambda\tau_2}}. \tag{13}$$

For $\tau_2 > \tau$, the analysis is more complicated but can be carried out by taking $\tau_2 = k\tau + \tau_2'$, $k = 0, 1, \ldots$ and $\tau_2' < \tau$. Thus for $k = 1$, the upper limit of stability is obtained as [31]

$$\alpha_u = \frac{1}{2a_2}(-a_1 + \sqrt{a_1^2 + 4a_2}), \tag{14}$$

where

$$a_1 = e^{2\lambda\tau}(1 - e^{-\lambda\tau_2'}) + e^{\lambda(\tau-\tau_2')} - 1$$

and

$$a_2 = e^{2\lambda\tau}(1 - e^{-\lambda\tau_2'})^2.$$

A similar analysis is possible for negative λ also and the details are given in [31]. We note that this approximate analysis helps to locate the stability region in the parameter space (τ_2–ϵ). A detailed numerical study is carried out for two Rössler systems coupled

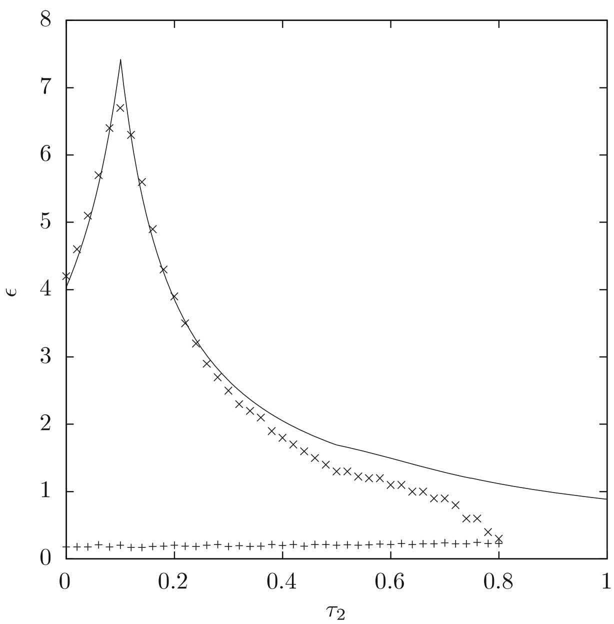

Figure 3. The limits of stability of the synchronized state of two chaotic Rössler systems in the parameter plane τ_2–ϵ. The solid line is the transition curve obtained from the stability analysis for a typical value of λ, while points correspond to values obtained from direct numerical simulations using correlation as the index. Here $\tau = 0.50$ and $\tau_1 = 0.02$ (for details, see [31]).

using our scheme. Here the asymptotic correlation coefficient $C = \langle y_1(t)x_1(t+\tau_2)\rangle / \sqrt{\langle x_1^2(t)\rangle\langle y_1^2(t)\rangle}$ between $x_1(t)$ and $y_1(t)$ shifted by the effective $\tau_2 = |\tau_2 - \tau_1|$ is used as the index for synchronized state. C is computed by direct numerical integration of the system in (2). The region of stability of the synchronized state is isolated as the region where $C = 0.99$ and boundaries of stability fixed when C goes below this value. The results obtained from this analysis are shown in figure 3. The upper limits of stability obtained from the analysis for different ranges of τ_2 are calculated using eqs (12)–(14) for a typical value of $\lambda = 0.65$. The solid line curve in the figure corresponds to the numerical fit thus obtained. We see that the overall behaviour agrees with the theoretical stability analysis given above.

4. Synchronization of time delay systems with variable delay and reset

In this section we extend the above method to systems with inherent time delay whose intrinsic dynamics follows a time delay equation of the type,

$$\dot{x} = -\kappa x + f(x_{\tau_s}), \tag{15}$$

where $x_{\tau_s} = x(t - \tau_s)$; τ_s being the time delay in the system. The present scheme of synchronization applied to two such systems gives the dynamics of the coupled systems as

$$\begin{aligned} \dot{x} &= -\kappa x + f(x_{\tau_s}) \\ \dot{y} &= -\kappa y + f(y_{\tau_s}) + \epsilon \sum_{m=0}^{\infty} (x_{t_1} - y_{t_2}) \chi_{(m\tau,(m+1)\tau)}. \end{aligned} \tag{16}$$

To determine the stability of the synchronized state, we consider the dynamics of the transverse system and using the approximation given in the previous section, we represent the time average of f' by ν. Then the transverse dynamics is given by

$$\dot{\Delta} = -\kappa\Delta + \nu\Delta_{\tau_s} - \epsilon \sum_{m=0}^{\infty} \chi_{(m\tau,(m+1)\tau)} \Delta_m. \tag{17}$$

Assuming a solution $\Delta = \mathrm{e}^{\lambda t}$ for the equation,

$$\dot{\Delta} = -\kappa\Delta + \nu\Delta_{\tau_s} \tag{18}$$

we get

$$\lambda = -\kappa + \nu \mathrm{e}^{-\lambda\tau_s}. \tag{19}$$

The solution of eq. (17) in any reset interval, $m\tau \leq t < (m+1)\tau$, can be written in the same way as eq. (5). Then the stability analysis detailed in §3 can be continued analogously for time delay systems also, the only difference being that here λ depends on the system time delay τ_s and has to be evaluated for each case using eq. (19). We restrict to real solutions of this transcendental equation since we are interested in the Lyapunov exponent which is the real part of the eigenvalue. We note that complex solution will change the values to some extent but we do not expect the general behaviour to change at least for a small imaginary part. In the context of a different problem, a calculation with complex eigenvalue reported in [33] shows that a small imaginary part changes the Lyapunov exponent but the general behaviour does not change. Thus by considering the real solutions which is adequate for getting the stability regions in the parameter space, we find that for $\nu > 0$ and $\kappa < \nu$, λ is positive and as τ_s increases λ decreases, while for $\kappa > \nu$, λ is negative and as τ_s increases λ increases.

We apply the scheme to a typical time delay system, the Mackey Glass (MG) system, which is a well studied model exhibiting hyperchaos [34]. The synchronization in two such coupled systems has been reported for various types of coupling [9]. It has been applied extensively in the context of communications, especially with its circuit equivalents [35]. The dynamics of a single MG system is

$$\dot{x} = -ax + \frac{bx(t-\tau_s)}{1+x(t-\tau_s)^c}. \tag{20}$$

As reported in [34], for $a = 1$, $b = 2$ and $c = 10$, with τ_s between 2.5 and 5 the system exhibits hyperchaotic behaviour. We couple two such systems using our scheme given in eq. (16) as

$$\begin{aligned} \dot{x} &= -x + \frac{2x(t-\tau_s)}{1+x(t-\tau_s)^{10}} \\ \dot{y} &= -y + \frac{2y(t-\tau_s)}{1+y(t-\tau_s)^{10}} + \epsilon \sum_{m=0}^{\infty} (x_{t_1} - y_{t_2}) \chi_{(m\tau,(m+1)\tau)}. \end{aligned} \tag{21}$$

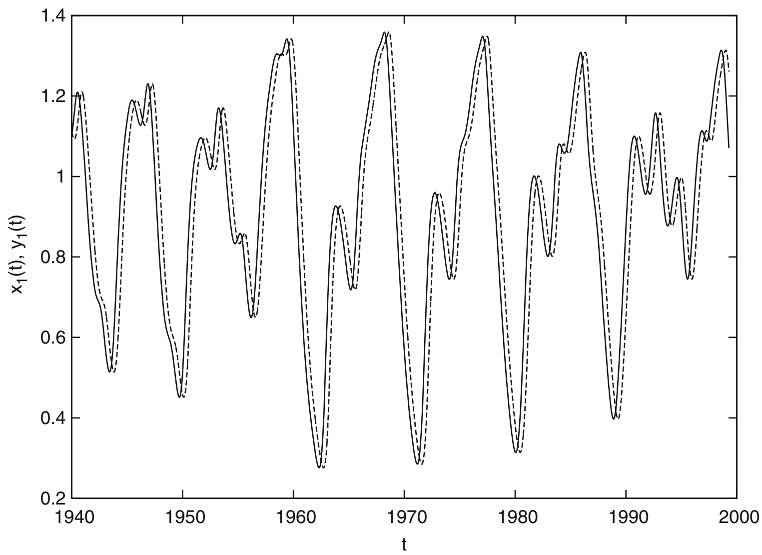

Figure 4. The delay synchronization in two MG systems coupled using the scheme in eq. (16) is shown with a delay of 0.4 units between $x_1(t)$ (solid line) and $y_1(t)$ (dashed line). Here $\tau_s = 2.5$, $\tau = 0.5$ and $\epsilon = 2$.

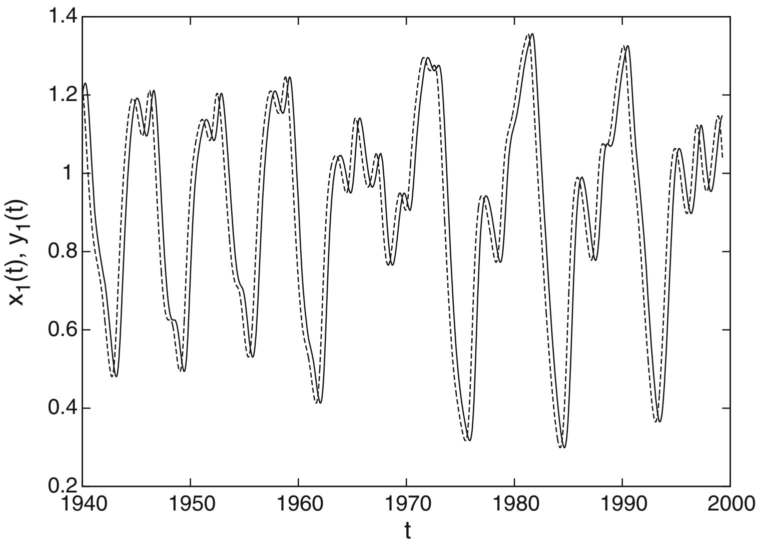

Figure 5. The anticipatory synchronization in two coupled MG systems with $y_1(t)$ (dashed line) anticipating $x_1(t)$ (solid line) by 0.4 units.

For a typical choice of parameters, $\tau_s = 2.5$, $\tau = 0.5$ and coupling strength, $\epsilon = 2$, we integrate the systems numerically and the resulting time series showing delay and anticipatory synchronous behaviour is shown in figures 4 and 5. By computing the asymptotic correlation index from the time series, the stability region is obtained in the parameter plane τ_2–ϵ

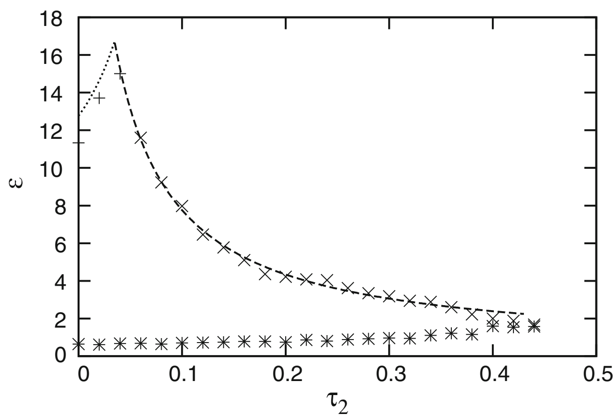

Figure 6. The limits of stability of the synchronized state for two coupled Mackey Glass systems in the parameter plane τ_2–ϵ. The dotted and dashed curves for the upper boundary correspond to the curves obtained by fitting the limits of stability obtained from the stability analysis, while the points correspond to the values obtained by direct numerical simulations using correlation as the index. Here $\tau_s = 2.5$, $\tau = 0.2$ and $\tau_1 = 0.02$.

(figure 6). The dotted and dashed curves for the upper boundary correspond to the curves obtained from the stability analysis, while the points correspond to the values obtained by direct numerical simulations.

5. Application to secure communication

In this section we propose a scheme for secure communication using hyperchaotic systems coupled by the method of synchronization discussed in this paper. We also show how the method can lead to enhanced security. We follow the chaotic masking method as applied to a bi-channel scheme. The transmitter and receiver dynamics in our scheme are given by eq. (21). The synchronizing signal $S(t)$ is the intermittent values of $x(t-\tau_1)$ sent at intervals of the reset time τ. The message to be transmitted is assumed to be a sine signal given by

$$m(t) = 0.01\sin(\omega t). \tag{22}$$

This message is encrypted using the method given in [36] for a bi-channel scheme. The carrier is generated by combining the output of the transmitter at eleven different delay times using a nonlinear function given below.

$$CW(t) = \sum_{i=0}^{10} \frac{a_i x(t-\tau_{s_i})}{1+x(t-\tau_{s_i})^{10}}. \tag{23}$$

Here the values of (a_i, τ_{s_i}) used are (4.0, 2.0), (5.0, 1.9), (6.0, 1.8), (7.0, 1.7), (8.0, 1.6), (9.0, 1.5), (10.0, 1.4), (11.0, 1.3), (12.0, 1.2), (13.0, 1.1), (14.0, 1.0). This carrier is used to mask the message and the cipher text $C(t)$ thus obtained is

$$C(t) = m(t) + \sum_{i=0}^{10} \frac{a_i x(t-\tau_{s_i})}{1+x(t-\tau_{s_i})^{10}}. \tag{24}$$

This cipher text is communicated to the receiver along one channel, while the synchronizing signal $S(t)$ is transmitted through another channel. Once the systems at both ends are synchronized, the signal is recovered at the receiving end as plain text.

$$PT(t) = C(t) - \sum_{i=0}^{10} \frac{a_i y(t-\tau_{s_i})}{1+y(t-\tau_{s_i})^{10}}. \tag{25}$$

We display the cipher text $C(t)$ and the recovered plain text $PT(t)$ computed using the method in figures 7 and 8 respectively. It is seen that after the transients, the error decreases to zero and the message can be recovered truthfully. To illustrate the enhanced security of the method, we note that $S(t)$, derived using the present scheme, has information about the dynamics of the transmitter only at intervals of reset time τ_r and as such cannot be used to derive the dynamics using conventional methods of cracking reported so far. We show this explicitly by considering the method used in [25,26], where the time delay in the dynamics

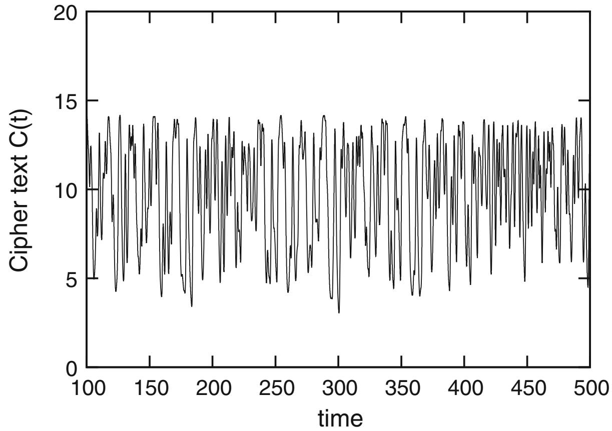

Figure 7. The cipher text $C(t)$ given in eq. (24) generated by masking the signal $m(t)$ using the carrier wave given in eq. (23).

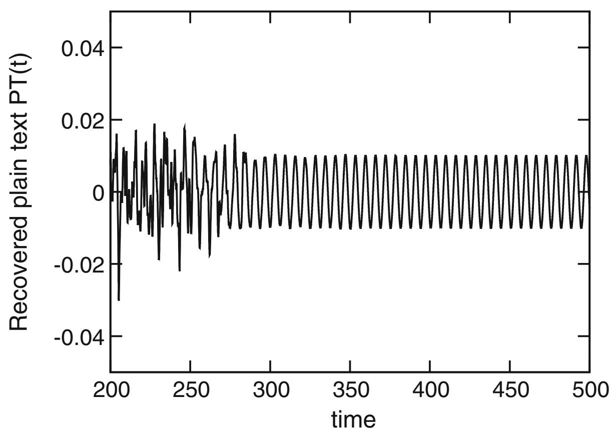

Figure 8. The plain text $PT(t)$, recovered from the transmitted signal using eq. (25).

of the systems is to be extracted first to reconstruct the transmitter system. For this the number of pairs of extrema separated in time by different τ in the time series obtained from the channel is taken and the absolute minimum gives a clue to the time delay in the system τ_s. Once this is obtained the functions in the transmitter dynamics can be derived [26]. For applying this to our method, we reconstruct the time series from the synchronizing channel by removing the reset intervals and use this collapsed time series to take the count of extrema for different separations. This is given in figure 9. It is clear that as the minima happen at a large number of points due to finite reset times, an estimate of the time delay τ_s cannot be obtained. Moreover, the eleven different sets of parameters (b_i, τ_{s_i}) used in constructing the cipher text give added security, by providing extra keys. In addition to this, in our method, the two time delays τ_1 and τ_2 serve as additional keys. Hence our method

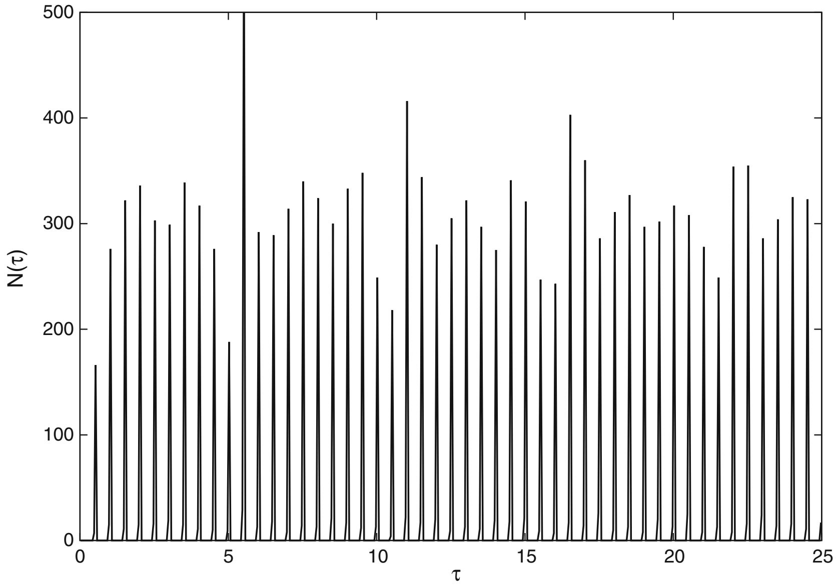

Figure 9. The method of reconstruction in [26] applied to the synchronizing signal to extract the system delay τ_s. It is clear that there is no single absolute minimum and hence extracting τ_s is difficult.

of synchronization can be easily implemented with the bi-channel scheme proposed above for communication with enhanced security.

6. Conclusion

We present a coupling scheme for synchronizing one-way coupled chaotic systems where the delay in coupling varies with system dynamics within intervals of the reset time with the obvious advantage that synchronization can be achieved with intermittent information from the driver in intervals of reset that can be pre-fixed. So also an approximate analytic stability analysis is possible since the error dynamics is discrete. The stability region and limits of stability in the parameters of coupling strength and anticipation time are worked out by direct numerical analysis for a standard system. The general features of the stability region in parameter space match with the theoretical stability analysis and the numerical fit of the limits of stability from the analysis agrees well with those from direct numerical values for small values of λ_2 and above a certain value of ϵ.

We further extend this scheme to include time delay systems where the synchronization can be obtained as delay or anticipatory, independent of the system delay. The stability analysis is adapted for such systems and applied to a standard time delay system like Mackey Glass system. Moreover, we note that the transition curves in the parameter plane τ_2–ϵ are of the same general type for both the cases studied, chaotic systems and hyperchaotic time delay systems, with the critical coupling strength increasing with anticipation time till a maximum and then decreasing. This results in a peak value for the critical coupling strength.

We suggest the possibility of applying this method in communication with enhanced security with bi-channel method of implementation. Our method has the advantage that synchronizing channel carries only intermittent information from the driver. We show that this enhances the security of transmission as extracting system delay from this channel is not possible by the conventional methods reported so far. The message channel can be made complex resulting in extra keys. We illustrate this by using a carrier wave constructed with a Mackey Glass system as the driver. Such bi-channel schemes have been reported earlier also. But our scheme promises extra security in the synchronizing channel due to the intermittent transmission and the delay and anticipation time in the coupling can function as additional keys that are independent of system delay.

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