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Estimate of stellar masses from their QPO frequencies

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Abstract. Kilohertz quasiperiodic oscillations (kHz QPOs) are observed in binary stellar systems. For such a system, the stellar radius is very close to the marginally stable orbit R_{ms} as predicted by Einstein's general relativity. Many models have been proposed to explain the origin of the kHz QPO features in the binaries. Here we start from the work of Li *et al* (*Phys. Rev. Lett.* **83**, 3776 (1999)) who in 1999, from the unique millisecond X-ray pulsations, suggested SAX J1808.4−3658 to be a strange star, from an accurate determination of its rotation period. It showed kHz QPOs eight years ago and so far it is the only set that has been observed. We suggest that the mass of four compact stars SAX J1808.4−3658, KS 1731−260, SAX J1750.8−2900 and IGR J17191−2821 can be determined from the difference in the observed kHz QPOs of these stars. It is exciting to be able to give an estimate of the mass of the star and three other compact stars in low-mass X-ray binaries using their observed kHz QPOs.

Keywords. Khz QPOs; strange stars.

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1. Introduction

The phenomenon of quasiperiodic oscillations was discovered in 1985 by EXOSAT in the bright galactic-bulge sources GX 5-1, Cygnus X-2 (Cyg X-2) and Scorpius X-1 (Sco X-l). QPOs are revealed in a power–density spectrum as a broad peak covering many frequencies rather than a sharp spike at one frequency. Moreover, in the bulge sources the position of this broad peak is seen to vary with time, and the changes seem to be correlated with changes in the source intensity. The power spectrum in low mass X-ray binaries (LMXBs) provided evidence of QPOs. The brightest part of the system is the accretion disk around the compact object. The large amplitudes and short time-scales of the observed intensity variations suggest that they are associated with an accretion process in the innermost region of LMXBs, close to compact objects. X-ray spectral studies provide insights to the evolutionary connection between LMXBs and millisecond radio pulsars.

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The kHz QPOs with strong variability on millisecond time-scales, observed in the LMXBs, was first reported by van der Klis *et al* way back in 1996 [1], with observations from the Rossi X-ray Timing Explorer (RXTE). Ever since, this has led to many theoretical models and predictions of their origin, and to the determination of the correct equation of state (EOS) of the compact objects as stated in a recent review by van der Klis [2]. In earlier stages of detection, kHz QPOs were found in the atoll and z class of sources of the LMXBs, and later, they were also observed in the millisecond X-ray pulsars and a few uncategorized sources.

There were a lot of efforts in the recent past to model the QPO behaviour of the compact binaries. The most popular model explaining the difference of kHz QPOs was the beat frequency model by Strohmayer *et al* [3] which assumed the upper QPO ($\nu_{\rm up}$) as the Keplerian orbital frequency $\nu_{\rm k}$ of the innermost orbit in the accretion disk around the star, the separation between the two peaks, i.e. $\delta\nu_{\rm peak} = \nu_{\rm up} - \nu_{\rm low}$ as $\nu_{\rm spin}$ and the lower QPO ($\nu_{\rm low}$) as the beat frequency of $\nu_{\rm k}$ and $\nu_{\rm spin}$. This model had suggested that for a particular star $\delta\nu = \nu_{\rm spin}$ will remain constant. However, it has been observed that in many sources the $\delta\nu$ is not related to $\nu_{\rm spin}$. Also $\delta\nu$ changes with time. Kluzniak and Abramowicz [4,5] has put forward the QPO resonance model but this model has its own set of shortcomings. It states that the QPOs could be explained by nonlinear resonant motions of accreting fluid in strong gravity. The two QPO frequencies depend here on the amplitude of the oscillatory motion and are approximately in the ratio 3 : 2. However, the difficulties mainly arise because QPO frequencies are not consistently equal to this ratio, and the twin peaks shift their positions [6]. Although all the present QPO models come with some conceptual difficulty or another, for our present work, we shall adhere to the QPO model of Titarchuk and Osherovich [7] which allowed us in an earlier paper to determine the mass and limits to the radius of the accreting star (see [8]).

2. kHz QPOs observations and modelling

Typically, the characteristic frequencies of the kHz QPOs have two peaks, the lower ($\nu_{\rm low}$) one ranges from a few hundreds of hertz and the upper ($\nu_{\rm up}$) one often goes beyond a kilohertz. In their earlier stages of detection, two simultaneous kHz peaks (~200–1200 Hz) were observed in the power spectra of the X-ray count rate variations, with the separation frequency $\delta\nu$ roughly constant (e.g., Strohmayer *et al* [3]; Ford *et al* [9]; Wijnands *et al* [10]). Sometimes a third kHz peak has been detected in a few atoll sources during type-I X-ray bursts at a frequency $\nu_{\rm b}$ equal to the separation frequency $\delta\nu$ of the two peaks or twice that value (see Strohmayer *et al* [11] for a review). These two results suggest that a beat-frequency mechanism is at work, with the upper kHz peak at the Keplerian orbital frequency at the inner edge of the accretion disk around the neutron star, the third peak at the neutron star spin frequency (or twice that), and the lower kHz peak at the difference frequency between them (Strohmayer *et al* [3]; Miller *et al* [12]).

Within the beat-frequency model, the burst oscillations $\nu_{\rm b}$ and the kHz peak separations $\delta\nu$ were both believed to be close to the neutron star spin frequency and thus $\delta\nu$ should remain constant. However, the following observations have challenged this interpretation: in Sco X-1 (van der Klis *et al* [13]), 4U 1608-52 (Méndez *et al* [14]) and 4U 1735-44 (Ford *et al* [15]), $\delta\nu$ slowly decreased as the frequencies of both QPOs increased. Moreover, in 4U 1636-53 the frequency $\nu_{\rm b}$ of the burst oscillations (or half this value) did not match with the frequency difference $\delta\nu$ between the kHz QPOs (Méndez *et al* [16]). In the atoll source 4U 1728-34, Méndez and van der Klis [17] found that $\delta\nu$ was always significantly smaller than $\nu_{\rm b}$, even at the lowest inferred mass accretion rate, when $\delta\nu$ seemed to reach its maximum value, and that $\delta\nu$ decreased significantly as the frequency of the lower kHz QPO increased.

A different approach was invoked by Osherovich and Titarchuk [18] and Titarchuk and Osherovich [7], who proposed a unified classification of kHz QPOs and the related low-frequency phenomena. In this model, kHz QPOs were modelled as Keplerian oscillations under the influence of the Coriolis force in a rotating frame of reference (magnetosphere). The frequencies $\nu_{\rm up}$ of the upper kHz QPO branch held a hybrid frequency relation with the Keplerian frequencies $\nu_{\rm k}$ (referred to as the lower kHz QPO branch, the $\nu_{\rm low}$): $\nu_{\rm u}^2 = \nu_{\rm k}^2 + (2\nu_{\rm m})^2$, where $\nu_{\rm m}$ is the rotational frequency of the star's magnetosphere. For Sco X-1, the QPOs with frequencies $\sim$45 and 90 Hz were interpreted as the 1st and 2nd harmonics of the lower branch of the Keplerian oscillations (Osherovich and Titarchuk [18]). The same interpretation was applied to the $\sim$35 Hz QPOs in the atoll source 4U 1702-42 (Osherovich and Titarchuk [19]). The observed low Lorentzian frequency in 4U 1728-34 was suggested to be associated with radial oscillations in the boundary layer of the disk, whereas the observed break frequency was determined by the characteristic diffusion time of the inward motion of the matter in the accretion flow (Titarchuk and Osherovich [7]). Predictions of this model regarding relations between the QPO frequencies mentioned above compare favourably with the observations of Sco X-1, 4U 1608-52, 4U 1702-429 and 4U 1728-34.

Although some QPOs with both the frequencies falling much below the kilohertz range have been observed in some black hole candidates, for the present study we shall not consider them.

Here we begin with a recent controversy of whether the kHz QPOs can be related in any way to the spin frequency of the star. Recently Méndez and Belloni [20] re-examined all reliable available data of sources for which there exist values of two simultaneous kHz QPOs and spin frequencies and put forward the possibility that the difference between the upper and the lower QPOs, $\delta\nu$, are not related to each other (anticorrelated). We have used their data for these ten sources and we found a correlation between the two simultaneous kHz QPOs raised to the power $2/3$ (figure 1). Since QPOs are represented by peaks in the power spectrum, they are related to the relatively stable orbits of accreting matter from which particles fall onto the compact object – in our case a strange star – and these orbits are the maximally stable orbit given by an average radius of $R_{\rm ms}$ for $\nu_{\rm up}$ and R_0 for $\nu_{\rm low}$. This assumption leads to the possible determination of the mass and radius of the compact strange star.

Most compact stars showing a pair of QPOs in the X-ray power spectrum, have variable $\delta\nu$. The first star observed to show a constant X-ray pulse period accurately was the X-ray pulsar SAX J1808.4$-$3658 [21]. This source was found to show kHz QPOs with a frequency difference of 195 kHz. This is roughly half the spin frequency $\nu_{\rm spin}$ of the star which is measured very accurately to be 401 Hz. So a modified beat-frequency model was tried with $\nu_{\rm peak} \sim 0.5\nu_{\rm spin}$. But for many other stars the difference of kHz QPOs has no apparent correlation with the spin frequency of the star as suggested by the beat-frequency model.

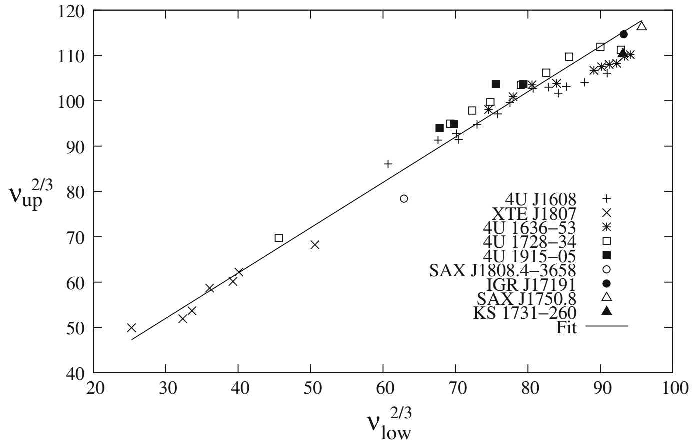

Figure 1. Lower and upper QPO frequencies raised to the 2/3rd power, for different LMXBs. They show an almost linear correlation.

There are now at least five other detected X-ray pulsars whose spins are steady and accurately measured and only for one of these, XTE J1807−294, kHz QPOs have been seen. We find, by applying the Titarchuck and Osherovich [7] model, that the masses (M) and radii (R) of these two stars (XTE J1807–294 and SAX J1808.4–3658) are very small and may fit those of strange stars.

3. Mass and radius limit of the compact star, using QPO frequencies

We assume the possibility that the lower QPO frequency is due to particles thickly clustered at the inner edge of the accretion disc (at a radial distance R_0) and the upper QPO frequency is due to particles thickly clustered at the marginally stable orbit (at a radial distance $R_{\rm ms}$). Bardeen *et al* [22] have given the expression for the marginally stable radius as:

$$R_{\rm ms} = R_{\rm g}\left\{3 + Z_2 - [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}\right\}, \tag{1}$$

where

$$Z_1 = 1 + \left[1 - (a/R_{\rm g})^2\right]^{1/3} \left[(1 + a/R_{\rm g})^{1/3} + (1 - a/R_{\rm g})^{1/3}\right], \tag{2}$$

$$Z_2 = \left[3(a/R_{\rm g})^2 + Z_1^2\right]^{1/3}, \tag{3}$$

where $R_{\rm g} = GM/c^2$ and $a = 2\pi\nu_{\rm spin}I/Mc$, I is the moment of inertia, c is the velocity of light and G is the gravitational constant. A lower limit for R_0 according to the Titarchuck and Osherovich model [7] and adapted by Li *et al* [8] is as follows:

$$R_0 = 3R_{\rm s} \times \left(\frac{M}{M_\odot}\right)^{1/3}, \tag{4}$$

where $R_{\rm s} = 2GM/c^2$. Hence it is clear that R_0 depends only on M whereas $R_{\rm ms}$ depends on both M and $\nu_{\rm spin}$. So for a particular star (fixed M and $\nu_{\rm spin}$), $R_{\rm ms}$ should be constant.

From figure 1 we see that $\nu_{\rm up}^{2/3} = 22 + \nu_{\rm low}^{2/3}$ or $({\rm d}\nu_{\rm up}^{2/3}/{\rm d}\nu_{\rm low}^{2/3}) \sim 1.0$ for most of the stars and one can calculate the radius r_0 for the most probable outermost orbit and consider its relation to R_0 given above. We tabulate $\nu_{\rm low}$, $\nu_{\rm up}$ and $\nu_{\rm spin}$ for 10 sources in table 1 for which figure 1 shows that $\nu_{\rm up}^{2/3}/\nu_{\rm low}^{2/3}$ is nearly constant.

From eqs (1) to (4) we obtain the difference

$$\nu_{\rm up} - \nu_{\rm low} = [(R_0/R_{\rm ms})^{3/2} - 1]\nu_{\rm low}, \tag{5}$$

Table 1. kHz QPO frequencies and spin frequencies for different stars.

Sources	$\nu_{\rm low}$	$\nu_{\rm up}$	$\nu_{\rm spin}$ (Hz)
	X-Ray pulsars		
SAX J1808.4−3658	499	694	401
XTE J1807−294	127.1	352.8	191.0
	184	374	
	195	393.4	
	216.5	449.6	
	246.3	466.3	
	253.9	490.6	
	359.9	563.6	
	Other sources		
KS 1731−260	898.3	1158.6	524
IGR J17191−2821	900	1227	294
SAX J1750.8−2900	936	1253	600.75
4U 1728−34	576.6	925.0	363
	614.9	967.6	
	647.1	995.2	
	702.3	1053	
	749.7	1094.2	
	793.0	1148.9	
	853.8	1183.2	
	894.2	1172.9	
	308	582	
4U 1608−52	472.8	798.3	619.0
	555.6	872.5	
	561.3	874.5	
	587.3	893.0	
	623.2	922.7	619.0
	660.0	956.5	
4U 1608−52	682.5	993.7	
	725.0	1040.9	
	754.0	1044.9	
	772.3	1024.9	
	787.7	1047.0	
	822.7	1061.5	
	866.9	1092.2	

Table 1. *Continued.*

Sources	$\nu_{\rm low}$	$\nu_{\rm up}$	$\nu_{\rm spin}$ (Hz)
4U 1636−53	644	971	582
	688	1013	
	723	1053	
	769	1058	
	841	1102	
	856	1114	
	871	1122	
	886	1126	
	901	1150	
	913	1156	
4U 1702−43	657.1	1000.1	330
	707.7	1037.7	
	770.0	1084.8	
4U 1915−05	706.9	1055.3	272
	656.9	906.4	
	558.3	911.2	
	583.2	923.7	

so that we can seek the stellar mass, the corresponding stellar radius and the moment of inertia to fit this difference. If the difference remains constant as in the case of SAX J1808.4−3658 (at 195 Hz) we can predict the mass to be $0.904M_\odot$. For this mass and spin $R_{\rm ms} = 7.409$ km. For strange stars, the corresponding radius is known from our EOS to be 6.89 km which falls nicely inside $R_{\rm ms}$. The derived value of $r_0 = 8.607$ km and the approximate relation $R_0 = 3 \times 2.9532M^{1/3}$ give a result $R_0 = 8.566$ km which is very close to r_0 so that our model leads to a consistent result. A recent estimate of the mass of SAX J1808.4−3658 by Elebert *et al* [23] from its optical spectroscopy and photometry showed that its mass is $0.9 \pm 0.4M_\odot$. This is consistent with the mass derived according to the strange star model.

Tolman–Oppenheimer–Volkoff equations in the framework of Einstein's general relativity give the structure of spherically symmetric compact objects like strange stars. The relation between mass and radius and the EOSs are of great importance. We have made a polynomial fitting of the mass $M/M_\odot$ of the strange star model of Dey *et al* [24] along with the corresponding moment of inertia I against the radius R, for the range where the radius increases monotonically with mass as follows:

$$M/M_\odot(R) = aR^c + bR^d, \tag{6}$$

$$I(R) = pR^r + qR^s. \tag{7}$$

The parameters are given in table 2. This is for the EOS A which gives the right masses and magnetic moments of baryons in a finite system calculation with our chosen interquark force as given in Bagchi *et al* [25]. We would like to stress that this SS model has the Richardson potential modified with two appropriate but different scales for confinement

Table 2. Parameters for fitting mass and moment of inertia for EOS A.

Parameters	$M/M_{\odot}$	Parameters	I
a	4.6×10^{-2}	p	8.9×10^{-4}
b	3.0×10^{-6}	g	1.7×10^{-7}
c	3.0	r	5.0
d	7.2	s	9.0

and asymptotic freedom for the interquark force. The parameters of this force are all fixed from elaborate baryon mass and magnetic moment calculations and satisfy the stringent binding energy requirement that

$$E/A(uds) \ \leq \ E/A(\mathrm{Fe}^{56}) \ \leq \ E/A(ud). \tag{8}$$

4. Other low-mass X-ray binaries

We have extended our idea to test a few other LMXBs. For KS 1731–260, SAX J1750.8−2900 and IGR J17191–2821, the masses and radii are estimated to be 0.878, 0.895, $0.843M_{\odot}$ and 6.84, 6.88, 6.762 km, R_{ms} = 7.00, 7.031, 7.051 km respectively. Clearly the value of R_{ms} is quite close to the stellar radius and this was what had made the estimate for 4U 1728−34 reliable in the paper by Li *et al* [8].

For the other millisecond pulsar XTE J1807−294 if we estimate the mass to be $M = 0.782M_{\odot}$ and radius to be $R = 6.62$ km with R_{ms} very close ∼6.672 km and R_0 to be 8.162 km, then for the seven sets of QPOs given by Méndez and Belloni [20], we find r_0 to be 13.177, 10.705, 10.652, 10.859, 10.210, 10.350 and 8.997 km [26]. Since R_0 is just a limit, our results could mean that the outer orbit varies for this and the other stars, for example, due to variations in the accretion rate.

5. Discussions

Owing to lack of direct evidence for the existence of strange stars, people are still sceptical of the existence of strange quark matter (SQM) and strange stars. But indirect evidence is now available because (a) there are some observed features of certain compact objects which challenge neutron star models and (b) there are some features which are easily explained if strange stars exist. The problem (a) has been discussed, among others, by Klahn *et al* [27]. The two stars PSR J0751+1807 [28] and EXO 0748−676 [29] have masses $\sim 2.1M_{\odot}$. These masses can originate only from the rather stiff neutron matter EOS. Such stiff EOSs cannot produce low mass stars like the recently found binary pulsars J0737−3039A and B having masses 1.338 and $1.249M_{\odot}$ respectively. The moment of inertia of J0737−3039B is likely to be determined very soon from radio observations.

The mystery surrounding the kHz QPOs still continues. For the last decade and half or so, scientists were searching for a relationship between QPO frequencies and spin frequency of the star. In recent years, Belloni *et al* [30,31] have shown that ν_{low} and ν_{up} are strongly correlated along a curve passing through the origin. Again, Yin *et al* [32] showed that, for six atoll sources, the relation between $\delta\nu$ and $\nu_{\rm spin}$ can be fitted by a linear relation

$$\langle\delta\nu\rangle = -(0.19 \pm 0.05)\nu_{\rm spin} + (389.40 \pm 21.67)\ \text{Hz}. \tag{9}$$

However, according to these authors, this anticorrelation is just a conjecture since it is based on data of only six sources. As we have stated before, examining all available data of sources for which there exist measurements of two simultaneous kHz QPOs and $\nu_{\rm spin}$, Méndez and Belloni [20] have advanced the possibility that $\delta\nu$ and $\nu_{\rm spin}$ are not related to each other. On the other hand, we find that the correlation between $\nu_{\rm up}^{2/3}$ and $\nu_{\rm low}^{2/3}$ which is expected from the Titarchuck and Osherovich model, is in agreement with the findings of Belloni *et al* [31]. Though presently we have data for ten sources only, we hope that in future with advanced X-ray satellites, like the ASTROSAT to be launched by India in 2011 and the satellite to be launched by China in 2013, we shall be able to get more data on kHz QPOs which will help us to re-examine our findings and hopefully give a better answer to the quest.

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