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Propagation and oblique collision of electron-acoustic solitons in two-electron-populated quantum plasmas

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Abstract. Oblique interaction of small- but finite-amplitude KdV-type electron-acoustic solitary excitations is examined in an unmagnetized two-electron-populated degenerate quantum electron–ion plasma in the framework of quantum hydrodynamics model using the extended Poincaré–Lighthill–Kuo (PLK) perturbation method. Critical plasma parameter is found to distinguish the types of solitons and their interaction phase-shifts. It is shown that, depending on the critical quantum diffraction parameter $H_{\rm cr}$, both compressive and rarefactive solitary excitations may exist in this plasma and their collision phase-shifts can be either positive or negative for the whole range of collision angles $0 < \theta < \pi$.

Keywords. Electron-acoustic solitary excitations; quantum degeneracy; quantum hydrodynamics; oblique collision; phase-shift.

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1. Introduction

There has been early reports on the existence of ordinary plasmas, namely, two-temperature-electron (2Te) plasmas (plasmas populated with two different electron species) [1,2]. One of the distinct nonlinear features of 2Te plasmas is their high-frequency electron-acoustic (EA) spectrum compared to that of ion-acoustic waves (IAWs). Properties of EA and ion-acoustic (IA) wave propagation and their modulational instability in ordinary 2Te plasmas were extensively studied by many authors [3–8]. Electron-acoustic solitary waves (EASWs) have been recently studied in unmagnetized [9] and magnetized [10] quantum plasmas also. It has been found that, the characteristic features of the EASWs are significantly affected by the variation in the ratio of hot-to-cold (degenerate-to-nondegenerate) electron concentration [11] in planar as well as non-planar geometries. Recent investigation of electron-acoustic solitary propagations has shown that critical hot-to-cold electron densities may exist in two-electron species quantum plasmas [12]. It is also noted that the propagation of the electron-acoustic waves remain undamped in the range $0.25 < n_{\rm c0}/n_{\rm h0} < 4$

for ordinary 2Te plasmas [13], where n_{c0} and n_{h0} denote the equilibrium densities of cold and hot electrons respectively. However, in a quantum plasma the degeneracy of one electron species is characterized by higher relative number density of that species, that is, in quantum plasmas where $n_{c0}/n_{h0} \ll 1$.

Recently, quantum plasma has attracted great attention and has become an intense field of plasma research [14–18] due to its broad application in the manufacture of the micro- and nanostructured electronic devices [19]. Quantum plasma is characterized by high densities and low temperatures contrary to ordinary plasma. The quantum hydrodynamics (QHD) model has also been applied by Gardner and Ringhofer [20] to study the electron–hole dynamics in semiconductors. The quantum effects emerge when the de Broglie thermal wavelength $\lambda_{\mathrm{D}} = h/(2\pi m_e k_{\mathrm{B}} T)^{1/2}$ of electrons become comparable to the interparticle distances [21], the condition which is well satisfied for metallic and semiconductor compounds. The state of quantum plasma where the thermal energy of the electron is much less than its Fermi energy is governed by the Fermi statistics and this state is called degenerate state. In a normal metal, at room temperature the conduction electrons are mostly in degenerate state. Such conditions can also be found in the laboratory laser-produced plasmas [22–24]. The propagation of ion acoustic solitary waves (IASWs) in quantum 2Te plasma has been recently studied in refs [25,26]. However, to the best of author's current knowledge, solitary wave interactions in such plasmas have not been studied so far.

As was first discovered by Zabysky and Kruskal [27], a unique feature of the solitary interaction is the asymptotic preservation of solitary wave amplitude. However, the imprint of solitary collisions can be a phase-shift in the post-collision trajectory of each wave. Two particular cases of solitary collisions are known as head-on and overtaking collisions. The former case has been extensively investigated using extended Poincaré–Lighthill–Kuo (PLK) perturbation method in one-dimensional ordinary [28,29] and quantum [30] plasmas. On the other hand, the latter case, in which the solitary waves propagate in the same direction, was studied using the inverse scattering transformation method [31]. A more general case is a planar collision under an arbitrary angle which requires two- or three-dimensional [32] treatment. The two-dimensional interaction of KdV-type solitons has been carried out in electron-ion (e-i) and dusty plasmas in non-relativistic [33–35] as well as weakly relativistic [36] cases.

In the present work we aim at investigating the effect of relative electron degeneracy population on propagation and interaction of small-amplitude electron-acoustic solitary waves in a quantum two-electron-populated plasma. Although the focus is on two-electron-populated (partially degenerated) quantum plasmas, it is easily noticed that current approach can be extended to the two-ion-populated quantum plasmas. The organization of the article is as follows. Description of quantum hydrodynamics state equations is given in §2. Reductive perturbation method is applied and the KdV evolution equation as well as phases are obtained in §3. Section 4 presents the discussions based on numerical analysis and, finally, §5 presents the concluding remarks.

2. QHD description of plasma

Consider a three-component dense plasma in which two types of inertial-less electrons exist in the presence of background inertial positive heavy ions. Also consider that one type of electrons is degenerate while the other is non-degenerate. The classical pressure can be ignored for inertial heavy ions compared to much higher degeneracy pressure of electrons. The two-dimensional quantum continuum-pressure continuity equation closed set, which includes quantum tunnelling effect for electrons [20], can be written as

$$\frac{\partial n_{\rm d}}{\partial t} + \nabla \cdot n_{\rm d}\vec{V}_{\rm d} = 0,$$
$$\frac{\partial n_{\rm n}}{\partial t} + \nabla \cdot n_{\rm n}\vec{V}_{\rm n} = 0,$$
$$\frac{\partial u_{\rm d}}{\partial t} + \left(\vec{V}_{\rm d} \cdot \nabla\right)\vec{V}_{\rm d} = \frac{e}{m_e}\nabla\varphi - \frac{1}{m_e n_{\rm d}}\nabla P_{\rm d} + \frac{\hbar^2}{2{m_e}^2}\nabla\left[\frac{\nabla^2\sqrt{n_{\rm d}}}{\sqrt{n_{\rm d}}}\right],$$
$$\frac{\partial u_{\rm n}}{\partial t} + \left(\vec{V}_{\rm n} \cdot \nabla\right)\vec{V}_{\rm n} = \frac{e}{m_e}\nabla\varphi + \frac{\hbar^2}{2{m_e}^2}\nabla\left[\frac{\nabla^2\sqrt{n_{\rm n}}}{\sqrt{n_{\rm n}}}\right],$$
$$\nabla^2\varphi = \frac{e}{\varepsilon_0}\left(n_{\rm d} + n_{\rm n} - z_i n_i\right), \tag{1}$$

where n and d stand for non-degenerate and degenerate species, respectively. The parameters $\hbar$, N and Z indicate the scaled Plank constant, background ion density and atomic number. The quantities $\vec{V}_{\rm n}$ ($\vec{V}_{\rm d}$), $n_{\rm n}$ ($n_{\rm d}$) and φ refer to the velocity, number density of non-degenerate (degenerate) electrons and the electrostatic potential, respectively. The normalized parameter $H = \hbar\omega_{pd}/2k_{\rm B}T_{\rm Fd}$ is the quantum diffraction parameter, which is the ratio of (d)-electron plasmon energy to (d)-electron Fermi energy. It is noted that, in a dense and degenerate plasma environment and under the zero-temperature Fermi gas assumption, the degeneracy pressure of (d)-electrons obeys the Pauli exclusion principle and is related to the equilibrium (d)-electron number density through the following relations in two dimensions [37]:

$$P = \frac{m_e v_{\rm Fe}^2 n^{(0)}}{4}\left(\frac{n}{n^{(0)}}\right)^2, \quad v_{\rm Fe} = \sqrt{\frac{2E_{\rm Fe}}{m_e}}, \quad E_{\rm Fe} = k_{\rm B}T_{\rm Fe}, \tag{2}$$

where quantities $v_{\rm Fe}$, $E_{\rm Fe}$ and $T_{\rm Fe}$ are respectively the Fermi velocity, Fermi energy and Fermi temperature of degenerate electrons. The quantity $n^{(0)}$ denotes the equilibrium number density of the electrons. From the standard definitions, it is also known that in a two-dimensional degenerate Fermi gas the quantum equilibrium number density is related to the Fermi temperature of degenerate electrons, i.e. [38]

$$T_{\rm Fe} = \frac{\hbar^2}{2m_e}\left(\frac{2\pi n^{(0)}}{k_{\rm B}}\right). \tag{3}$$

From eq. (3), it is observed that even when $H \to 0$ the classical model is not retained. The normalized equations may be obtained using the following re-scalings from basic equations (eqs (1)):

$$x \to \frac{C_{sd}}{\omega_{pd}}x, \quad t \to \frac{t}{\omega_{pd}}, \quad n \to n n_{\rm d}^{(0)}, \quad \vec{V} \to \vec{V} v_{\rm Fe}, \quad \varphi \to \varphi\frac{2k_{\rm B}T_{\rm Fd}}{e}, \tag{4}$$

where $\omega_{pd} = \sqrt{e^2 n_{\rm d}^{(0)}/\varepsilon_0 m_e}$ and $v_{\rm Fe} = \sqrt{2k_{\rm B}T_{\rm Fd}/m_e}$ are the characteristic plasma frequency and Fermi speed, respectively. The normalized set of closed QHD equations, therefore, read as

$$\begin{aligned}
&\frac{\partial n_\alpha}{\partial t} + \nabla \cdot n_\alpha \vec{V}_\alpha = 0, \quad \vec{V}_\alpha = \hat{i} u_\alpha + \hat{j} v_\alpha, \\
&\frac{\partial \vec{V}_\alpha}{\partial t} + \left(\vec{V}_\alpha \cdot \nabla\right)\vec{V}_\alpha = \nabla\varphi - \frac{D_\alpha}{2}\nabla n_\alpha + \frac{H^2}{2}\nabla\left[\frac{\nabla^2\sqrt{n_\alpha}}{\sqrt{n_\alpha}}\right], \\
&\nabla^2\varphi = \sum_\alpha n_\alpha - z_i n_i.
\end{aligned} \tag{5}$$

Here, the label α is used to denote (d)/(n)-electrons, for simplicity. The new quantities, introduced here, are $D_\alpha = \{1, 0\}$ and $n_\alpha^{(0)} = \{1, \beta\}$ for $\alpha = \{d, n\}$, respectively. On the other hand, the quasi-neutrality condition at thermodynamics equilibrium is given by the Poisson's relation as

$$n_{\rm d}^{(0)} + n_{\rm n}^{(0)} - z_i n_i = 0, \tag{6}$$

or in a reduced form

$$\delta = 1 + \beta, \quad \beta = \frac{n_{\rm n}^{(0)}}{n_{\rm d}^{(0)}}, \quad \delta = \frac{z_i n_i}{n_{\rm d}^{(0)}}. \tag{7}$$

If two small perturbations which move at angle θ, in the x–y plane with different velocities approach each other, they can interact at some event and finally depart leaving a phase-shift on each other's trajectories. To evaluate the dynamics of this collision, we use asymptotic expansion of plasma variables around thermodynamics equilibrium state in an appropriate strained coordinate which contains the phase records of each wave. The technique is the so-called extended Poincaré–Lighthill–Kuo (PLK) method [39,40]. The reductive perturbation method in nonlinear wave propagation has been put forward by Taniuti and Wei [41], where it was shown that the stretching of $\xi = \varepsilon^a(x - ct)$, $\tau = \varepsilon^{a+1}$ and $a = (p-1)^{-1}$ with $p = 2$, $p = 3$ admit reductions to Burgers and KdV equations, respectively. Hence, normalized equation sets are introduced to the following strained coordinate:

$$\begin{aligned}
&\xi = \varepsilon\left(k_1 x + l_1 y - c_\xi t\right) + \varepsilon^2 P_0(\eta, \tau) + \varepsilon^3 P_1(\xi, \eta, \tau) + \cdots, \\
&\eta = \varepsilon\left(k_2 x + l_2 y - c_\eta t\right) + \varepsilon^2 Q_0(\xi, \tau) + \varepsilon^3 Q_1(\xi, \eta, \tau) + \cdots, \\
&\tau = \varepsilon^3 t, \\
&c_\xi = c_1, \quad c_\eta = c_2.
\end{aligned} \tag{8}$$

The functions P_j and Q_j ($j = 0, 1, 2, ...$) are the phase records of the interacting waves in space-time and are to be determined later along with the evolution equations in §3. Initial wave trajectories are given by vectors $\mathbf{r}_1 = (k_1, l_1)$ and $\mathbf{r}_2 = (k_2, l_2)$ and the collision angle θ is defined by

$$\cos\theta = \frac{\mu}{\lambda_1\lambda_2}, \tag{9}$$

where

$$\begin{aligned}
&\mu = \left(k_1 k_2 + l_1 l_2\right), \\
&\lambda_1 = \left(k_1^2 + l_1^2\right)^{1/2}, \\
&\lambda_2 = \left(k_2^2 + l_2^2\right)^{1/2}.
\end{aligned}$$

λ_1 and λ_2 are the normalized wave numbers. The asymptotic expansion of the dependent plasma variables in powers of ε in the states away from thermodynamics equilibrium is carried out using the following orderings:

$$\begin{pmatrix} n_\alpha \\ u_\alpha \\ v_\alpha \\ \varphi \end{pmatrix} = \begin{pmatrix} n_\alpha^{(0)} \\ 0 \\ 0 \\ 0 \end{pmatrix} + \varepsilon^2 \begin{pmatrix} n_\alpha^{(1)} \\ u_\alpha^{(1)} \\ v_\alpha^{(1)} \\ \varphi^{(1)} \end{pmatrix} + \varepsilon^4 \begin{pmatrix} n_\alpha^{(2)} \\ u_\alpha^{(2)} \\ v_\alpha^{(2)} \\ \varphi^{(2)} \end{pmatrix} + \cdots, \tag{10}$$

where the parameter ε is a very small, positive and real value proportional to the perturbation amplitude. Note also that, we use the stretching employed in ref. [36], which is completely different from two- and three-dimensional magnetized [42,43] and unmagnetized one-dimensional collisions [28]. The reduced set of plasma equations in strained coordinates are given in Appendix A, for simplicity. From the leading-order in ε in eqs (A.1)–(A.4), we isolate the following relations:

$$\left(c_1 \frac{\partial}{\partial \xi} + c_2 \frac{\partial}{\partial \eta}\right) n_\alpha^{(1)} = n_\alpha^{(0)} \left(k_1 \frac{\partial}{\partial \xi} + k_2 \frac{\partial}{\partial \eta}\right) u_\alpha^{(1)} + n_\alpha^{(0)} \left(l_1 \frac{\partial}{\partial \xi} + l_2 \frac{\partial}{\partial \eta}\right) v_\alpha^{(1)}, \tag{11}$$

$$\left(c_1 \frac{\partial}{\partial \xi} + c_2 \frac{\partial}{\partial \eta}\right) u_\alpha^{(1)} = \frac{D_\alpha}{2} \left(k_1 \frac{\partial}{\partial \xi} + k_2 \frac{\partial}{\partial \eta}\right) n_\alpha^{(1)} - \left(k_1 \frac{\partial}{\partial \xi} + k_2 \frac{\partial}{\partial \eta}\right) \varphi^{(1)}, \tag{12}$$

$$\left(c_1 \frac{\partial}{\partial \xi} + c_2 \frac{\partial}{\partial \eta}\right) v_\alpha^{(1)} = \frac{D_\alpha}{2} \left(l_1 \frac{\partial}{\partial \xi} + l_2 \frac{\partial}{\partial \eta}\right) n_\alpha^{(1)} - \left(l_1 \frac{\partial}{\partial \xi} + l_2 \frac{\partial}{\partial \eta}\right) \varphi^{(1)}, \tag{13}$$

$$\sum_\alpha n_\alpha^{(1)} = 0. \tag{14}$$

Consequently, we obtain the following first-order perturbed plasma components by solving the coupled equations (eqs (11)–(14))

$$\begin{pmatrix} n_\alpha^{(1)} \\ u_\alpha^{(1)} \\ v_\alpha^{(1)} \\ \varphi^{(1)} \end{pmatrix} = \begin{pmatrix} A_{1\alpha} \\ A_{2\alpha} \\ A_{3\alpha} \\ 1 \end{pmatrix} \varphi^{(1)}(\xi, \tau) + \begin{pmatrix} B_{1\alpha} \\ B_{2\alpha} \\ B_{3\alpha} \\ 1 \end{pmatrix} \varphi^{(1)}(\eta, \tau). \tag{15}$$

The quantities $\varphi^{(1)}(\xi, \tau)$ and $\varphi^{(1)}(\eta, \tau)$ describe the first-order amplitude evolution of two distinct solitary waves in oblique directions $\eta_\perp$ and $\xi_\perp$, respectively, as we shall see in the next section. In the proceeding calculations we shall use the notations $\varphi_\xi^{(1)}$ and $\varphi_\eta^{(1)}$ instead of $\varphi^{(1)}(\xi,\tau)$ and $\varphi^{(1)}(\eta,\tau)$, for brevity. The coefficients of first-order components of plasma variable approximations for (d)- and (n)-electrons (labelled by α), are given as

$$\begin{aligned}
A_{1\alpha} &= \frac{2n_\alpha^{(0)}\lambda_1^2}{n_\alpha^{(0)}D_\alpha\lambda_1^2 - 2c_1^2}, \quad B_{1\alpha} = \frac{2n_\alpha^{(0)}\lambda_2^2}{n_\alpha^{(0)}D_\alpha\lambda_2^2 - 2c_2^2},\\
A_{2\alpha} &= \frac{2c_1k_1}{n_\alpha^{(0)}D_\alpha\lambda_1^2 - 2c_1^2}, \quad B_{2\alpha} = \frac{2c_2k_2}{n_\alpha^{(0)}D_\alpha\lambda_2^2 - 2c_2^2},\\
A_{3\alpha} &= \frac{2c_1l_1}{n_\alpha^{(0)}D_\alpha\lambda_1^2 - 2c_1^2}, \quad B_{3\alpha} = \frac{2c_2l_2}{n_\alpha^{(0)}D_\alpha\lambda_2^2 - 2c_2^2}.
\end{aligned} \tag{16}$$

The nonlinear dispersion relation, which is deduced from the first-order approximations, can be written in the following compact form:

$$\sum_\alpha \frac{n_\alpha^{(0)}}{2c_1^2 - D_\alpha n_\alpha^{(0)}\lambda_1^2} = \sum_\alpha \frac{n_\alpha^{(0)}}{2c_2^2 - D_\alpha n_\alpha^{(0)}\lambda_2^2} = 0. \tag{17}$$

Also, the normalized phase speeds c_1 and c_2 of waves are given as

$$c_1 = \lambda_1\sqrt{\frac{\beta}{2(1+\beta)}}, \quad c_2 = \lambda_2\sqrt{\frac{\beta}{2(1+\beta)}}. \tag{18}$$

3. Solitary collision dynamics

In this section from the second-order, we derive the nonlinear wave evolution equations and the related phase functions introduced in eqs (8). We use eqs (A.1)–(A.4) to deduce relations among higher-order (second-order in ε) plasma variables, and consequently, by solving the coupled differential equations in this approximation level and by using the dispersion relations (eq. (17)), the second-order d/n-electrons number-density perturbation components are obtained in the following form:

$$\begin{aligned}
n_\mathrm{d}^{(2)} = {} & K_1N_1\left[\frac{\partial\varphi_\xi^{(1)}}{\partial\tau} + A_1\varphi_\xi^{(1)}\frac{\partial\varphi_\xi^{(1)}}{\partial\xi} + B_1\frac{\partial^3\varphi_\xi^{(1)}}{\partial\xi^3}\right]\eta\\
&+K_2N_2\left[\frac{\partial\varphi_\eta^{(1)}}{\partial\tau} + A_2\varphi_\eta^{(1)}\frac{\partial\varphi_\eta^{(1)}}{\partial\eta} + B_2\frac{\partial^3\varphi_\eta^{(1)}}{\partial\eta^3}\right]\xi\\
&+K_1E_2\left[P_0(\eta,\tau) - \frac{E_1}{E_2}\int\varphi_\eta^{(1)}\mathrm{d}\eta\right]\frac{\partial\varphi_\xi^{(1)}}{\partial\xi}\\
&+K_2E_2'\left[Q_0(\xi,\tau) - \frac{E_1'}{E_2'}\int\varphi_\xi^{(1)}\mathrm{d}\xi\right]\frac{\partial\varphi_\eta^{(1)}}{\partial\eta}\\
&+\left[\frac{K_1C_1}{2}\left(\varphi_\xi^{(1)}\right)^2 + (K_1D_1+K_2D_2)\varphi_\xi^{(1)}\varphi_\eta^{(1)} + \frac{K_2C_2}{2}\left(\varphi_\eta^{(1)}\right)^2\right]
\end{aligned}$$

$$+\left[K_1 N_1 \int \frac{\partial \varphi_\xi^{(1)}}{\partial \tau} \mathrm{d}\xi + K_2 N_2 \int \frac{\partial \varphi_\eta^{(1)}}{\partial \tau} \mathrm{d}\eta\right]$$

$$-\left[(L\lambda_1^2 + K_1 R_1)\frac{\partial^2 \varphi_\xi^{(1)}}{\partial \xi^2} + (L\lambda_2^2 + K_2 R_2)\frac{\partial^2 \varphi_\eta^{(1)}}{\partial \eta^2}\right]$$

$$+F(\xi, \tau) + G(\eta, \tau),$$

$$n_n^{(2)} = \left[\lambda_1^2 \frac{\partial^2 \varphi_\xi^{(1)}}{\partial \xi^2} + \lambda_2^2 \frac{\partial^2 \varphi_\eta^{(1)}}{\partial \eta^2}\right] - n_{\mathrm{d}}^{(2)}, \tag{19}$$

where $F(\xi, \tau)$ and $G(\eta, \tau)$ represent the homogeneous solutions of integrals involved in $n_\alpha^{(2)}$ differential equations. The unknown coefficients appearing in eq. (19) are given below:

$$\frac{A_1}{\lambda_1} = \frac{A_2}{\lambda_2} = -\frac{(1-\beta)(3+2\beta)}{1+3\beta}\sqrt{\frac{2(1+\beta)}{\beta}}, \tag{20}$$

$$\frac{B_1}{\lambda_1^3} = \frac{B_2}{\lambda_2^3} = \frac{H^2(1+\beta)^3 - \beta}{2\sqrt{2}\sqrt{\beta}(1+\beta)^{3/2}(1+3\beta)}, \tag{21}$$

$$E_1 = E_1' = \frac{4(1+\beta)^2(3+\beta(8+\beta) - (1-\beta)\beta\cos\theta)}{\beta}, \tag{22}$$

$$\frac{E_2\lambda_1}{\lambda_2} = \frac{E_2'\lambda_2}{\lambda_1} = 2(1+\beta)(1+3\beta)(1-\cos\theta), \tag{23}$$

$$\frac{C_1\lambda_2}{\lambda_1} = \frac{C_2\lambda_1}{\lambda_2} = \frac{4(1+\beta)^2(2\beta(\beta-1) - (1+\beta^2)\cos\theta)}{\beta}, \tag{24}$$

$$\frac{D_1\lambda_2}{\lambda_1} = \frac{D_2\lambda_1}{\lambda_2} = -\frac{4(1+\beta)^2(1-\beta)(\beta+\cos\theta)}{\beta}, \tag{25}$$

$$\frac{K_1\lambda_2}{\lambda_1} = \frac{K_2\lambda_1}{\lambda_2} = \frac{L}{2} = \frac{1}{2(1+\beta)(1-\cos\theta)}, \tag{26}$$

$$N_1\lambda_1 = N_2\lambda_2 = \frac{2\sqrt{2}(1+3\beta)(1+\beta)^{3/2}}{\sqrt{\beta}}, \tag{27}$$

$$\frac{R_1}{\lambda_1^2} = \frac{R_2}{\lambda_2^2} = -\frac{H^2(1+\beta)^3}{\beta}, \tag{28}$$

In eq. (19), it is noted that, the first and the second terms diverge as $\xi \to \pm\infty$ and $\eta \to \pm\infty$. Thus, their coefficients must vanish to eliminate the secularities. This leads to a pair of distinct KdV-type evolution equations which describe the propagations of two solitary structures moving in $\xi_\perp$ and $\eta_\perp$ directions. The next two terms in eq. (19) although are not secular at this order will become secular in the next order. It has been shown [44] that, the method of multiple scales combined with the reductive perturbation technique, not only eliminates the secularities arising in the second-order correction but also modifies the phase factor of the lowest KdV soliton proportional to its amplitude. A fuller discussion of the elimination of secularities in higher-order amplitude approximation is given in [40]. From the latter requirement the phase-shifts introduced in eqs (8) are derived. Therefore, by considering the mentioned requirements, the low-amplitude dynamics (propagation and collision) of both solitary waves are defined by the following two-coupled set of equations:

$$\frac{\partial \varphi_\xi^{(1)}}{\partial \tau} + A_1 \varphi_\xi^{(1)} \frac{\partial \varphi_\xi^{(1)}}{\partial \xi} + B_1 \frac{\partial^3 \varphi_\xi^{(1)}}{\partial \xi^3} = 0, \tag{29}$$

$$P_0(\eta, \tau) = \frac{E_1}{E_2} \int \varphi_\eta^{(1)} \mathrm{d}\eta, \tag{30}$$

$$\frac{\partial \varphi_\eta^{(1)}}{\partial \tau} + A_2 \varphi_\eta^{(1)} \frac{\partial \varphi_\eta^{(1)}}{\partial \eta} + B_2 \frac{\partial^3 \varphi_\eta^{(1)}}{\partial \eta^3} = 0, \tag{31}$$

$$Q_0(\xi, \tau) = \frac{E_1'}{E_2'} \int \varphi_\xi^{(1)} \mathrm{d}\xi. \tag{32}$$

Therefore, after eliminating the secularities, the second-order d-electrons number density read as

$$\begin{aligned} n_{\mathrm{d}}^{(2)} = {} & \left[K_1 \left(\frac{C_1}{2} - A_1 N_1 \right) \varphi_\xi^{(1)^2} + K_2 \left(\frac{C_2}{2} - A_2 N_2 \right) \varphi_\eta^{(1)^2} \right] \\ & - \left(K_1 N_1 B_1 + L\lambda_1^2 + K_1 R_1 \right) \frac{\partial^2 \varphi_\xi^{(1)}}{\partial \xi^2} - \left(K_2 N_2 B_2 + L\lambda_2^2 + K_2 R_2 \right) \frac{\partial^2 \varphi_\eta^{(1)}}{\partial \eta^2} \\ & + \left(K_1 D_1 + K_2 D_2 \right) \varphi_\xi^{(1)} \varphi_\eta^{(1)} + F(\xi, \tau) + G(\eta, \tau). \end{aligned} \tag{33}$$

There are multi-soliton solutions for eqs (29) and (31). However, unique stationary single-soliton solutions are obtained by applying appropriate boundary conditions, which require the perturbed potential components and their derivatives to vanish at infinities, i.e.

$$\lim_{\zeta \to \pm\infty} \left\{ \varphi_\zeta^{(1)}, \frac{\partial \varphi_\zeta^{(1)}}{\partial \zeta}, \frac{\partial^2 \varphi_\zeta^{(1)}}{\partial \zeta^2} \right\} = 0, \quad \zeta = \xi, \eta. \tag{34}$$

These suitable solutions are given as

$$\begin{aligned} \varphi_\xi^{(1)} &= \frac{\varphi_{\xi 0}}{\cosh^2\left(\frac{\xi - u_{\xi 0}\tau}{\Delta_\xi} \right)}, \\ \varphi_{\xi 0} &= \frac{3u_{\xi 0}}{A_1}, \quad \Delta_\xi = \left(\frac{4B_1}{u_{\xi 0}} \right)^{1/2}, \end{aligned} \tag{35}$$

$$\begin{aligned} \varphi_\eta^{(1)} &= \frac{\varphi_{\eta 0}}{\cosh^2\left(\frac{\eta - u_{\eta 0}\tau}{\Delta_\eta} \right)}, \\ \varphi_{\eta 0} &= \frac{3u_{\eta 0}}{A_2}, \quad \Delta_\eta = \left(\frac{4B_2}{u_{\eta 0}} \right)^{1/2}. \end{aligned} \tag{36}$$

where $\varphi_{\xi 0}$ ($\varphi_{\eta 0}$) and Δ_ξ (Δ_η) represent the soliton amplitude and width, respectively, and $u_{\xi 0}$ ($u_{\eta 0}$) is an arbitrary value for relative soliton speed.

The first-order approximations for the collisional phase-shifts due to elastic collision of solitary excitations are obtained from eqs (30) and (32) using KdV solutions (eqs (35) and (36))

$$P_0(\eta,\tau) = \frac{E_1}{E_2}\varphi_{\eta 0}\Delta_\eta \tanh\left(\frac{\eta - u_{\eta 0}\tau}{\Delta_\eta}\right), \tag{37}$$

$$Q_0(\xi,\tau) = \frac{E_1'}{E_2'}\varphi_{\xi 0}\Delta_\xi \tanh\left(\frac{\xi - u_{\xi 0}\tau}{\Delta_\xi}\right). \tag{38}$$

Therefore, the trajectories and the phase variations of the two collided solitary excitations up to order $O(\varepsilon^2)$ are fully determined by the following oblique coordinate:

$$\begin{aligned}\xi &= \varepsilon(k_1x + l_1y - c_1t) - \varepsilon^2\frac{E_1}{E_2}\varphi_{\eta 0}\Delta_\eta \tanh\left(\frac{\eta - u_{\eta 0}\tau}{\Delta_\eta}\right) + O(\varepsilon^3),\\ \eta &= \varepsilon(k_2x + l_2y - c_2t)] - \varepsilon^2\frac{E_1'}{E_2'}\varphi_{\xi 0}\Delta_\xi \tanh\left(\frac{\xi - u_{\xi 0}\tau}{\Delta_\xi}\right) + O(\varepsilon^3).\end{aligned} \tag{39}$$

The overall phase-shifts, however, are obtained by comparing the phases of each wave long before and after the collision. In other words, the past-collision state of soliton labelled ‘2’ moving in $\xi_\perp$ direction, described from the rest frame $\eta = 0$ of soliton labelled ‘1’, is ($\xi = -\infty, \eta = 0$) at a fixed time t = const. and the post-collision state is ($\xi = +\infty, \eta = 0$) at a fixed time (t = const.). On the other hand, the past-collision state of soliton labelled ‘1’ moving in $\eta_\perp$ direction, described from the rest frame $\xi = 0$ of soliton labelled ‘2’, is ($\xi = 0, \eta = -\infty$) at a fixed time (t = const.) and the post-collision state is ($\xi = 0, \eta = +\infty$) at a fixed time (t = const.). Thus, we can deduce the overall phase-shifts in the trajectories of each soliton in the following mathematical manner:

$$\begin{aligned}\Delta P_0 &= P_{\text{post-collision}} - P_{\text{past-collision}}\\ &= \lim_{\xi=0,\eta\to+\infty}[\varepsilon(k_1x + l_1y - c_1t)] - \lim_{\xi=0,\eta\to-\infty}[\varepsilon(k_1x + l_1y - c_1t)],\\ \Delta Q_0 &= Q_{\text{post-collision}} - Q_{\text{past-collision}}\\ &= \lim_{\eta=0,\xi\to+\infty}[\varepsilon(k_2x + l_2y - c_2t)] - \lim_{\eta=0,\xi\to-\infty}[\varepsilon(k_2x + l_2y - c_2t)],\end{aligned} \tag{40}$$

where ΔP_0 and ΔQ_0 denote the overall phase-shifts of waves ‘1’ and ‘2’ in oblique collision. By using eqs (37) and (38), we readily obtain

$$\begin{aligned}\Delta P_0 &= 2\varepsilon^2\frac{E_1}{E_2}\varphi_{\eta 0}\Delta_\eta,\\ \Delta Q_0 &= 2\varepsilon^2\frac{E_1'}{E_2'}\varphi_{\xi 0}\Delta_\xi.\end{aligned} \tag{41}$$

Similar representations are given in ref. [36].

4. Numerical analysis and discussion

Before the discussion a few comments concerning the possibility of solitary excitations in this plasma are in order. As it is evident from the KdV equation coefficients, the coefficient A vanishes at the critical fractional degenerate-electron number density value, $\beta = 1$, where it is negative for $\beta < 1$ (note that in experiments such as laser–solid interactions, the population of non-degenerate electrons are relatively less than that of degenerate electrons, hence, $\beta \ll 1$) and is positive otherwise. Other such critical β-values have been reported for quantum two-temperature-electron plasmas [9]. Also, the coefficient B vanishes at a critical quantum diffraction parameter value H_{cr} defined as

$$H_{\mathrm{cr}} = \frac{1}{1+\beta}\sqrt{\frac{\beta}{1+\beta}}, \tag{42}$$

with a maximum value of $H_m \simeq 0.39$ at $\beta = 0.5$. This feature is very similar to the ones previously reported for ion-acoustic [14] as well as electrostatic [47] solitary excitations in quantum plasmas. Another similar behaviour to be mentioned is the independence of coefficient A (i.e. first-order soliton amplitude) from the quantum diffraction parameter, H, which is also reported in refs [14,47].

Figure 1a indicates two distinct (dark–bright) regions in H–β plane, separated by H_{cr} curves (eq. (42)), where bright or dark solitary excitations may occur. Also, according to figure 1b, increase in relative non-degenerate electron density, β, for other fixed plasma parameters leads to increase in the soliton amplitude for both rarefactive and compressive solitary excitations. On the other hand, figure 1c shows that for $H = 0.25$ with increase in the value of β, the soliton width (coefficient B) vanishes at some β-values (defined through eq. (42)) but $H = 0.5$ it does not. Therefore, for a given fixed value of $H < H_m$, by increasing the value of β, one critical β-value is always encountered (figures 1a and 1c) but for $H > H_m$ critical β-value does not exist. It is important to note that vanishing one of the coefficients A or B (such as in critical values) destroys the KdV evolution equation and no possible KdV-type solitary excitations occur for these special cases, and hence, they are excluded from this work. Surprisingly, the shape of solitary excitation is defined by H_{cr}, although the soliton amplitude is completely independent of the value of quantum diffraction parameter, H. For instance, for a fixed β-value, increase in the value of quantum diffraction parameter, H, results in different compressive (shown by solid lines in figure 1d) and rarefactive (shown by dashed lines in figure 1d) branches connected at critical H-values. This is much similar to the findings in ref. [47].

Figures 1e and 1f show the dependence of the collision phase-shift on the relative non-degenerate electron population, β, and the collision angle for other fixed plasma parameters. It is revealed from figure 1e that, for the compressive/rarefactive wave branches (solid curves/dashed curves) the collision phase-shift is negative/positive. Therefore, the sign of the collision phase-shift also can be determined by figure 1a (eq. (42)). Consequently, the increase in β-value leads to decrease/increase in the value of collision phase-shift for compressive/rarefactive solitary collisions. It should be noted that, a negative phase-shift in a collision indicates that the collided parts of solitons lag behind the initial wave trajectory, while, a positive phase-shift means that, the collided parts of solitons move ahead of the initial wave trajectory [34]. Furthermore, it is observed from figure 1f that for

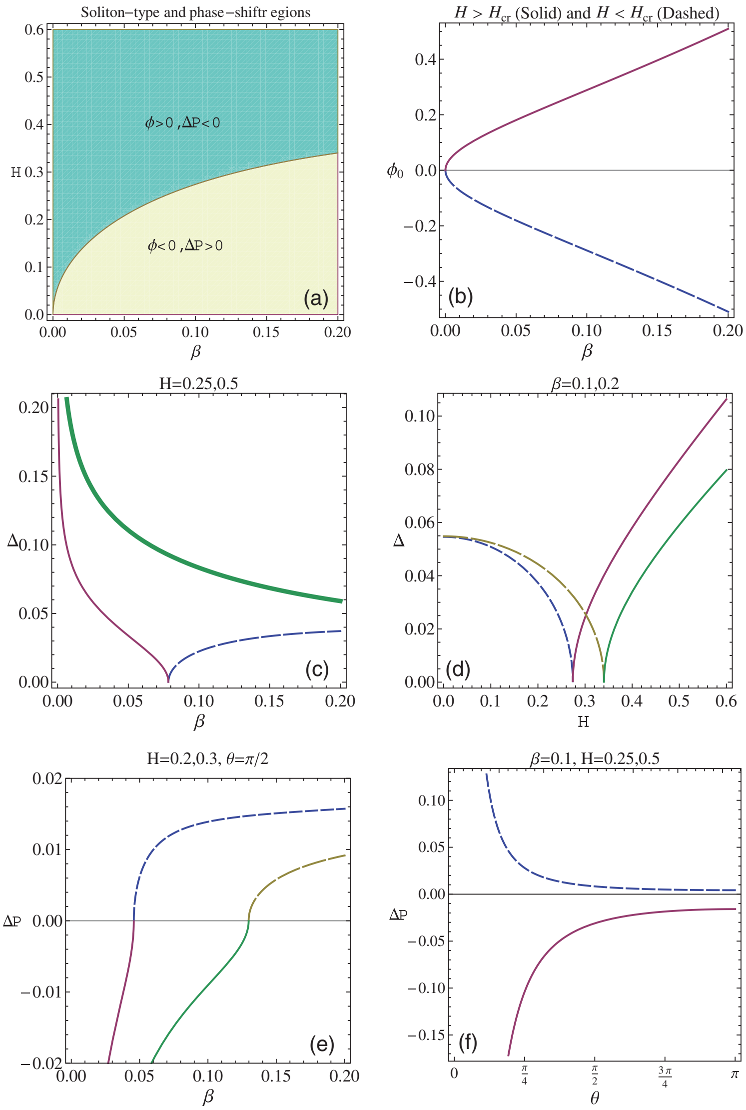

Figure 1. (**a**) shows the dark–bright regions in β–H plane for which the solitary excitations are compressive (grey regions) or rarefactive (bright regions) and accordingly where the collision phase-shifts are positive (bright regions) or negative (grey regions). (**b**), (**c**) and (**d**) show the variations of soliton amplitude and width with respect to the fractional non-degenerate electron population β and quantum diffraction parameter H for different values of plasma parameters (critical β- and H-values are shown in figures 1c and 1d). The thick curve in c corresponds to $H = 0.5$. Figures 1e and 1f depict the variation of collision phase-shift with respect to fractional non-degenerate electron population β and the collision angle θ for compressive (solid curves) and rarefactive (dashed curve) in oblique solitary interactions. The values of $\varepsilon = 0.1$ and $\lambda_1 = \lambda_2 = 0.1$ are used in all plots. Different dash sizes in d and e are used to appropriately present different values of varied parameters in each plot.

both compressive (solid curve) and rarefactive (dashed curve) collisions the absolute value of phase-shift decreases with increase of the collision angle, θ, but never changes the sign as it has been reported for some plasmas [34].

5. Concluding remarks

The extended Poincaré–Lighthill–Kuo (PLK) reductive perturbation method was used to investigate a two-dimensional collision of small-amplitude electron-acoustic solitary waves in an unmagnetized two-electron-populated quantum plasma. It is remarked that the propagation and collision properties of these waves are determined by a critical plasma value which significantly depend on the degeneracy population. It is also observed that the collision phase-shift is affected by both relative non-degenerate electron population and the quantum diffraction parameter in such plasmas.

Appendix A. Normalized plasma equations in strained coordinate

$$\begin{aligned}
\varepsilon^2\frac{\partial n_\alpha}{\partial \tau} &- c_1\frac{\partial n_\alpha}{\partial \xi} - \varepsilon^2 c_1\frac{\partial Q_0}{\partial \xi}\frac{\partial n_\alpha}{\partial \eta} - c_2\frac{\partial n_\alpha}{\partial \eta} - \varepsilon^2 c_2\frac{\partial P_0}{\partial \eta}\frac{\partial n_\alpha}{\partial \xi} + k_1\frac{\partial n_\alpha u_\alpha}{\partial \xi} \\
&+ \varepsilon^2 k_1\frac{\partial Q_0}{\partial \xi}\frac{\partial n_\alpha u_\alpha}{\partial \eta} + k_2\frac{\partial n_\alpha u_\alpha}{\partial \eta} + \varepsilon^2 k_2\frac{\partial P_0}{\partial \eta}\frac{\partial n_\alpha u_\alpha}{\partial \xi} + l_1\frac{\partial n_\alpha v_\alpha}{\partial \xi} \\
&+ \varepsilon^2 l_1\frac{\partial Q_0}{\partial \xi}\frac{\partial n_\alpha v_\alpha}{\partial \eta} + l_2\frac{\partial n_\alpha v_\alpha}{\partial \eta} + \varepsilon^2 l_2\frac{\partial P_0}{\partial \eta}\frac{\partial n_\alpha v_\alpha}{\partial \xi} + \cdots = 0, \qquad \text{(A.1)}
\end{aligned}$$

$$\begin{aligned}
\varepsilon^2 n_\alpha\frac{\partial u_\alpha}{\partial \tau} &- c_1 n_\alpha\frac{\partial u_\alpha}{\partial \xi} - \varepsilon^2 c_1 n_\alpha\frac{\partial Q_0}{\partial \xi}\frac{\partial u_\alpha}{\partial \eta} - c_2 n_\alpha\frac{\partial u_\alpha}{\partial \eta} - \varepsilon^2 c_2 n_\alpha\frac{\partial P_0}{\partial \eta}\frac{\partial u_\alpha}{\partial \xi} \\
&+ k_1 n_\alpha u_\alpha\frac{\partial u_\alpha}{\partial \xi} + \varepsilon^2 k_1 n_\alpha u_\alpha\frac{\partial Q_0}{\partial \xi}\frac{\partial u_\alpha}{\partial \eta} + k_2 n_\alpha u_\alpha\frac{\partial u_\alpha}{\partial \eta} \\
&+ \varepsilon^2 k_2 n_\alpha u_\alpha\frac{\partial P_0}{\partial \eta}\frac{\partial u_\alpha}{\partial \xi} + l_1 n_\alpha v_\alpha\frac{\partial u_\alpha}{\partial \xi} + \varepsilon^2 l_1 n_\alpha v_\alpha\frac{\partial Q_0}{\partial \xi}\frac{\partial u_\alpha}{\partial \eta} \\
&+ l_2 n_\alpha v_\alpha\frac{\partial u_\alpha}{\partial \eta} + \varepsilon^2 l_2 n_\alpha v_\alpha\frac{\partial P_0}{\partial \eta}\frac{\partial u_\alpha}{\partial \xi} - k_1 n_\alpha\frac{\partial \varphi}{\partial \xi} \\
&- \varepsilon^2 k_1 n_\alpha\frac{\partial Q_0}{\partial \xi}\frac{\partial \varphi}{\partial \eta} - k_2 n_\alpha\frac{\partial \varphi}{\partial \eta} - \varepsilon^2 k_2 n_\alpha\frac{\partial P_0}{\partial \eta}\frac{\partial \varphi}{\partial \xi} \\
&+ \frac{D_\alpha}{2} n_\alpha k_1\frac{\partial n_\alpha}{\partial \xi} + \frac{D_\alpha}{2}\varepsilon^2 n_\alpha k_1\frac{\partial Q_0}{\partial \xi}\frac{\partial n_\alpha}{\partial \eta} + \frac{D_\alpha}{2} n_\alpha k_2\frac{\partial n_\alpha}{\partial \eta} \\
&+ \frac{D_\alpha}{2}\varepsilon^2 n_\alpha k_2\frac{\partial P_0}{\partial \eta}\frac{\partial n_\alpha}{\partial \xi} - \varepsilon^2 k_1\frac{H^2}{4}\frac{\partial^3 n_\alpha}{\partial \xi^3} - \varepsilon^2 k_2\frac{H^2}{4}\frac{\partial^3 n_\alpha}{\partial \eta^3} \\
&+ \cdots = 0, \qquad \text{(A.2)}
\end{aligned}$$

$$\begin{aligned}\varepsilon^2 n_\alpha \frac{\partial v_\alpha}{\partial \tau} &- c_1 n_\alpha \frac{\partial v_\alpha}{\partial \xi} - \varepsilon^2 c_1 n_\alpha \frac{\partial Q_0}{\partial \xi}\frac{\partial v_\alpha}{\partial \eta} - c_2 n_\alpha \frac{\partial v_\alpha}{\partial \eta} - \varepsilon^2 c_2 n_\alpha \frac{\partial P_0}{\partial \eta}\frac{\partial v_\alpha}{\partial \xi}\\ &+ k_1 n_\alpha u_\alpha \frac{\partial v_\alpha}{\partial \xi} + \varepsilon^2 k_1 n_\alpha u_\alpha \frac{\partial Q_0}{\partial \xi}\frac{\partial v_\alpha}{\partial \eta} + k_2 n_\alpha u_\alpha \frac{\partial v_\alpha}{\partial \eta}\\ &+ \varepsilon^2 k_2 n_\alpha u_\alpha \frac{\partial P_0}{\partial \eta}\frac{\partial v_\alpha}{\partial \xi} + l_1 n_\alpha v_\alpha \frac{\partial v_\alpha}{\partial \xi} + \varepsilon^2 l_1 n_\alpha v_\alpha \frac{\partial Q_0}{\partial \xi}\frac{\partial v_\alpha}{\partial \eta}\\ &+ l_2 n_\alpha v_\alpha \frac{\partial v_\alpha}{\partial \eta} + \varepsilon^2 l_2 n_\alpha v_\alpha \frac{\partial P_0}{\partial \eta}\frac{\partial v_\alpha}{\partial \xi} - l_1 n_\alpha \frac{\partial \varphi}{\partial \xi} - \varepsilon^2 l_1 n_\alpha \frac{\partial Q_0}{\partial \xi}\frac{\partial \varphi}{\partial \eta}\\ &- l_2 n_\alpha \frac{\partial \varphi}{\partial \eta} - \varepsilon^2 l_2 n_\alpha \frac{\partial P_0}{\partial \eta}\frac{\partial \varphi}{\partial \xi} + \frac{D_\alpha}{2} n_\alpha l_1 \frac{\partial n_\alpha}{\partial \xi}\\ &+ \frac{D_\alpha}{2}\varepsilon^2 n_\alpha l_1 \frac{\partial Q_0}{\partial \xi}\frac{\partial n_\alpha}{\partial \eta} + \frac{D_\alpha}{2} n_\alpha l_2 \frac{\partial n_\alpha}{\partial \eta} + \frac{D_\alpha}{2}\varepsilon^2 n_\alpha l_2 \frac{\partial P_0}{\partial \eta}\frac{\partial n_\alpha}{\partial \xi}\\ &- \varepsilon^2 l_1 \frac{H^2}{4}\frac{\partial^3 n_\alpha}{\partial \xi^3} - \varepsilon^2 l_2 \frac{H^2}{4}\frac{\partial^3 n_\alpha}{\partial \eta^3} + \cdots = 0, \end{aligned} \tag{A.3}$$

$$\varepsilon^2 \lambda_1^2 \frac{\partial^2 \varphi}{\partial \xi^2} + \varepsilon^2 \lambda_2^2 \frac{\partial^2 \varphi}{\partial \eta^2} + 2\varepsilon^2 \mu \frac{\partial^2 \varphi}{\partial \xi \partial \eta} - \sum_\alpha n_\alpha + z_i n_i + \cdots = 0. \tag{A.4}$$

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