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Gravitational waves from binary black holes

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Abstract. It is almost a century since Einstein predicted the existence of gravitational waves as one of the consequences of his general theory of relativity. A brief historical overview including Chandrasekhar's contribution to the subject is first presented. The current status of the experimental search for gravitational waves and the attendant theoretical insights into the two-body problem in general relativity arising from computations of gravitational waves from binary black holes are then broadly reviewed.

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1. Einstein's equations and gravitational waves

The effect of gravity cannot be transmitted faster than light, if a theory of gravitation is consistent with principle of special relativity (SR). Thus, if the gravitational field of an object changes, the changes propagate through space and take a finite time to reach other objects. These propagating oscillations are called gravitational radiation or gravitational waves (GW). Einstein's theory of gravitation, general relativity (GR), is consistent with SR and predicts that just like electromagnetic waves, in GR too, GW travel with the speed of light, are transverse and have two states of polarization. Conservation of energy rules out monopole gravitational radiation. Conservation of linear momentum and angular momentum forbid gravitational dipole radiation. GW are intrinsically non-linear since wave energy density itself 'gravitates'. GW propagate essentially unperturbed through space as they interact weakly with matter. Thus the theorist's paradise of GW providing an uncorrupted view of the Universe goes hand in hand with the experimenters hell of GW being almost impossible to detect.

In the 1916 paper exploring the physical implications of GR, Einstein proposed the existence of GW as one of its important consequences. In 1918, he calculated the flux of energy far from source, the famous quadrupole formula leading to gravitational radiation reaction, or gravitational radiation damping. Einstein distinguished between energy carrying physical waves and non-energy carrying wave-like coordinate (gauge) artefacts. In 1922, Eddington corrected a factor of 2 in Einstein's work, and pointed out the inapplicability of Einstein's derivation for self-gravitating systems. In his characterstic style, he described the situation regarding gauge effects by the aphorism, GW propagate at speed of thought! It appears that Einstein wished to forget he had predicted GW. With his experience in the patent office, probably, he had a reasonable judgement about the slim chance that GW might be detected. For more historical details on this and related aspects, see the interesting book by Kennefick [1].

2. Improved theoretical understanding of GW

Conservative PN dynamics is an expansion in v^2/c^2. With this choice of expansion parameter, nPN means corrections of order $(v^2/c^2)^n$ relative to the leading order. Thus, 2.5PN equations of motion (EOM) means EOM corrrect up to v^5/c^5. Landau–Lifshiftz (1941) and Fock (1955) extended Einstein's quadrupole formula to weakly self-gravitating systems [2]. These constitute two different approaches – DIRE [3,4] and MPM [5], which we discuss later – to GW generation today. Where does the complication come from? For self-gravitating systems, orders in velocity are related to orders in non-linearity [6]. Virial theorem implies that $\Phi = GM/R$ is of the same order as v^2. Thus reaction terms of order $(v/c)^5$ from linear theory will be accompanied by terms $(v/c)^3\, \Phi/c^2$, $(v/c)^1\, (\Phi/c^2)^2$. Thus, higher-order PN calculations require dealing with higher-order non-linearities. In the 1950s, in the work of Goldberg, Havas, Pirani, Bondi, Metzner, Sachs involving a mathematically precise discussion of asymptotics in GR, rigorous work showed that GW transfer energy! More physically, Feynman and Bondi argued that GW could in principle heat a suitably contrived mechanical system [1]!

3. Chandrasekhar and GW

There are three important topics in relativistic astrophysics to which Chandrasekhar made seminal contributions: the influence of GR on pulsation and stability of stars; PN approximations to GR and its astrophysical applications and finally the back reaction of GW on their sources. Chandrasekhar provided the first direct calculation *à la* Lorentz, of radiation reaction force in agreement with quadrupole radiation losses. The PN approximation has become standard working tool of physics and astrophysics. It is used in studies of stars, star clusters, gravitational wave generation, motion of planets and moon, and the development of parametrized post-Newtonian (PPN) formalism to compare GR with other theories of gravity and experiment. Regarding Chandra's contributions to this field Kip Thorne said in his Foreword to [7]: "If Chandra had left us no other relativistic legacy beyond the PN and PPN formalisms, he would still deserve a place among the great contributers to our subject."

In the 1960s Chandrasekhar addressed the radiation reaction problem [6]. How does the emission of GW affect the emitting system, when it is self-gravitating? Chandra was the first to show conceptually that the radiation reaction problem could be solved for continuous systems. At that time the situation was indeed a lot confusing since damping, null and even antidamping results existed. This confused situation led some to even doubt the reality of gravitational radiation and the possibility of associating some kind of conserved energy with GW. It was unsatisfactory both from the physical and astrophysical points of view. Chandra saw the need for a careful step-by-step approach starting from the Newtonian limit and proceeding PN order-by-order. No one had attempted the PN approximation for continuous bodies in an exhaustive way. He realized that previous works used simplifications that could have led to fallacies and decided to avoid any tricks and do a complete and honest calculation. He studied previous works until he knew what to emulate and what to discard. Technical issues in these different approaches relate to the use of point particles and the consequent infinite self-field energy problems in a non-linear theory, imposition of no-incoming boundary condition in a PN scheme, dealing separately with conservative (even in v/c) and dissipative (odd in v/c) effects, validity of the use of matched asymptotic expansions to isolate terms in EOM coupling to radiation far away. He assembled together the essential ingredients: Landau–Lifshiftz pseudotensor to include the non-linearities of the gravitational field, retarded potentials and near-zone expansion to implement outgoing boundary conditions, following Trautman. His calculation gave astrophysicists the confidence that GR was physically reasonable and well behaved. Energy and angular momentum radiated as GW was correctly balanced by the loss of mechanical energy and angular momentum. Important applications immediately followed. (i) The GW induced non-axisymmetric instability of rotating stars by Chandrasekhar himself and (ii) an understanding of cataclysmic binary systems as a competition between GW radiation reaction and mass transfer by Faulkner.

However there were problems: (i) (in the gauge he chose to work in) some terms at 2PN were divergent and thus the expressions were only formal; (ii) the appearance of terms diverging at infinity for continuous sources and their reconciliation only by accepting for the metric a solution only in the distributional sense. The divergences cast doubt on the validity of Chandra's treatment for more mathematically demanding relativists and proved to be a barrier for extension of the treatment to higher PN orders. Chandra (and Thorne) did not find the infinities worrying because they felt they had a physicist's intuition for the correctness of the method used and results obtained. By brute force, insight and attention to detail, Chandra first achieved what many relativists had tried for decades. Chandra was always unhappy about the criticism regarding the divergent terms since it prevented him from being given adequate credit for the significance of his PN work. It is the only body of work not immortalized by a book unlike all his other research endeavours!

4. A century of waiting for direct detection but GW exist!

It is almost a century since GW were theoretically predicted but still there is no experimental confirmation *à la* Hertz. The reason is connected to two fundamental differences between electromagnetism and gravitation: The weakness of the gravitational interaction relative to EM (10^{-39}) and the spin-2 nature of gravitation compared to the spin-1 nature of EM that forbids dipole radiation in GR. This implies a low efficiency for conversion of mechanical energy to gravitational radiation and feeble effects of GW on any potential detector. A GW Hertz experiment is ruled out and it is only signals produced by astrophysical systems where there are potentially huge masses accelerating very strongly that are the likely sources.

However, there is high-quality data that is proof that GW exist [8]. In 1974, Hulse and Taylor discovered the binary pulsar 1913+16. The system has now been monitored for more than thirty years. If general relativity is right (and Newtonian gravity is incorrect), the system must emit GW and its orbit shrinks by a tiny 3 mm per orbit. Indeed, observations show that the orbital period is slowly decreasing at just the rate predicted by GR for the emission of GW! Hulse and Taylor received the Nobel Prize in 1993 'for their discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation.'

5. Quadrupole formula controversy: Genesis of mark II approximation methods

The prospects of testing PN theory against the Hulse–Taylor system once again revived more critical questions regarding the existing treatment of GW: (i) Does it apply to orbital motion of the two neutron stars even though it does not apply to their internal structure with strong gravitational fields? (ii) Even for weak fields, is it a valid approximation of GR due to the divergent terms in the PN equations? The first issue needs methods to treat weak orbital fields without an assumption on internal fields. One can show that orbits and interactions of stars are independent of compactness modulo tidal distortions. Further, if radiation is present, it is inconvenient to iterate the Einstein equations using Newtonian-like Poisson equations. Retardation effects cannot be neglected. Successful formalisms are all formulated in terms of retarded integrals rather than Poisson-like Green functions.

The high-quality binary pulsar data forced a revisit to approximation methods in GR to remedy the mathematical shortcomings in the existing approaches. Insights of a newer generation more comfortable with techniques in field theory to deal with divergences helped. For example, Damour (whose thesis was on regularization in classical field theories) critically looked at the problem and realized the need to carefully deal with the ultra-violet (UV) divergences arising from the use of delta functions to model point particles in a non-linear theory. He proposed iteration algorithms including Riesz regularization to deal with such divergences and iterated Einstein's equations using the Einstein–Infeld–Hoffmann approach to sufficient order of non-linearity to obtain the EOM of compact binaries including v^5/c^5 terms [9].

6. Laser interferometric gravitational-wave detectors and the GW spectrum

Binary pulsars establish the reality of gravitational radiation and the validity of GR in strong fields. There is excellent evidence for GW but the evidence is indirect. Can detectors be built to attempt a direct detection of these GW? GW are transverse and can tidally distort a system in directions perpendicular to the propagation direction of the waves. The effect is quantified by the dimensionless strain $h = 2(\Delta L)/L$ it produces. For a typical neutron star binary of mass M and radius R at distance D, the strain is given by

$$h \sim \frac{4G}{c^4 D} K_{\text{nonsph}} \sim 2\,\frac{GM}{Rc^2}\,\frac{GM}{Dc^2}.$$

Thus, for a neutron star binary in the Virgo cluster ($D = 18$ Mpc; 5.6×10^{20} km) the strain is $\sim$1.5 $\times 10^{-21}$. The miniscule strain and associated tiny displacement must be measured to detect the GW. As a GW passes, the arm lengths of km scale interferometers change (by 10^{-18} m) tidally causing the interference pattern to change at the photodiode. The direct detection of GW is the first mandate of laser interferometric GW detectors. The promised and real excitement is because they open a new observational window with a potential to become a tool for astrophysics and subsequently an experimental probe for basic physics. Contrary to Weber's bar detectors which were narrow-band detectors, the laser interferometric detectors are broadband detectors. For the fascinating story related to the odyssey of GW detection, see the book by Collins [10].

The field of GW detection has reached a milestone with decades-old plans to build and operate kilometer scale interferometric GW detectors now realized at several locations worldwide [11]. The S5 observation in LIGO lasted from November 2005 to September 2007 including some coincidence runs with Virgo. Though no detection has been reported, the unprecedented sensitivity allowed the LIGO Science Collaboration and the Virgo Collaboration to place upper limits on GW from a variety of astrophysical sources [12,13].

GW detectors are sensitive to the amplitude of the radiation which falls of as inverse of the source distance. Thus, a factor N increase in sensitivity leads to factor N^3 increase in probed volume and hence event rates. From inception, LIGO and Virgo are envisaged as an ongoing observatory with infrastructure to go to higher sensitivity. Enhanced detectors (2009–2011) achieved using 35 W laser power and more efficient readout for GW channel are expected to have twice the sensitivity, leading to eight times larger event rate. LIGO and Virgo will be upgraded to advanced detectors around 2015 with 10 times increase in sensitivity leading to 1000 times larger event rates. This is possible due to signal recycling, a 200 W laser, a test mass of 40 kg that would reduce radiation pressure noise, larger beams, better dielectric mirror coatings, four cascaded stages of passive isolation, fused silica fibres with low mechanical loss reducing the suspension noise 100 times and two-stage active isolation to go down to 10 Hz. The Einstein telescope (2027 –) is the proposed third-generation (3G) detector under conceptual design study with 100 times increase in sensitivity leading to over million times increase in event rate. In addition to the ground-based detectors, there are planned detectors in space like Laser Interferometer Space Antenna (LISA) with 5 million km baseline that are needed to search for low-frequency GW from supermassive black hole binaries. LISA is a planned ESA–NASA mission and hopes to fly by 2025.

EMW are studied over 20 orders of magnitude from ultra-low-frequency radio (10^3 Hz; 10^6 m) to high-energy γ-rays (10^{22} Hz; 10^{-13} m). With LISA and LIGO similarly, one covers 10 orders of magnitude in frequency. In addition, there are GW windows at very low frequency (10^{-7}–10^{-9} Hz) and extreme low frequency (10^{-15}–10^{-18} Hz).

6.1 *Expected annual coalescence rates for ICB*

In a 95% confidence interval, event rates for ICB are uncertain by three orders of magnitude and expected to be the following for our three prototypical systems: NS-NS (0.4−400) yr^{-1};

NS-BH (0.2−300) yr^{-1}; BH-BH (2−4000) yr^{-1}. The rates are based on extrapolations from observed binary pulsars, stellar birth rate estimates and population synthesis models. The mean rates for NS-NS binaries are 0.02, 0.1 and 40 for initial, enhanced and advanced LIGO respectively. The corresponding numbers for NS-BH binaries are 0.006, 0.04 and 10 while for BH-BH binaries they are 0.0009, 0.07 and 20 respectively. For more details, see [14,15].

7. The next decade: Black hole binaries, GW astronomy and LIGO-Australia

There is evidence for BH in the Universe on a range of mass scales: stellar mass BH (3–30)$M_\odot$; IMBH (10^2–10^4)$M_\odot$ and MBH (10^4–10^9)$M_\odot$. BH mergers are monumental astrophysical events releasing tremendous amounts of energy in the form of GW. Each merger releases more power than the combined light from all stars in the visible Universe. They are key sources both for ground-based and space-based GW detectors. BH and GW, two exotic and amazing predictions of GR, come together in binary BH that are expected to be the strongest emitters of GW. Successful direct detection and opening of GW astronomy would rely heavily on a global network of GW observatories set up at optimal locations. New observatories are required for improved duty cycle, improved sky coverage, polarization coverage, enhanced signal-to-noise ratio and most importantly improved angular resolution. The recent report of the Gravitational Wave International Committee (GWIC) recognizes the need for a detector in Asia–Pacific region – Japan, Australia, and in the future possibly India and China.

An exciting opportunity called LIGO-Australia [17] has come about last year. The LIGO Lab has agreed to transfer to Australia an advanced GW detector provided Australia funds the construction of a national facility to house it and commits to running costs for 10 years. NSF has approved LIGO-Australia provided ACIGA [18] makes a commitment before October 2011. Indian Initiative in Gravitational-wave Observations (IndIGO) Consortium [19] that has been exploring a road-map for Indo–Australian collaboration in GW astronomy since 2009 has plans to seek funds to collaborate with ACIGA and participate in LIGO-Australia (figure 1).

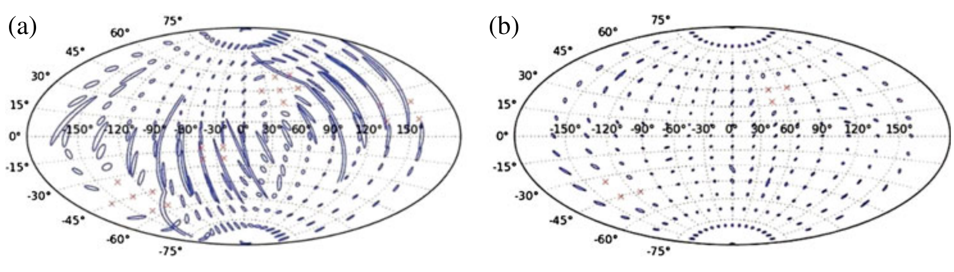

Figure 1. Improvement of source localization with only LIGO Hanford, Livingston and Virgo (**a**) compared with addition of LIGO-Australia (**b**) [16].

8. The last three minutes: Can chirping compact binaries be detected?

Can we detect the GW from 1913+16 today? The answer is in the negative because an orbital period of 8 h corresponds to a GW frequency $f_{\rm GW} \sim 10^{-4}$ Hz and interferometers on the Earth cannot be sensitive enough at this frequency due to seismic noise. However, due to gravitational radiation reaction, the binary spirals in with increasing velocity and frequency (and also increasing amplitude due to the stronger gravitational field). In 300 million years the GW frequency rises to $f_{\rm GW} \sim 10$ Hz and fifteen minutes later to $f_{\rm GW} \sim 1000$ Hz leading to 16,000 cycles in the last three minutes before coalescence. All this brings the system in the sensitivity bandwidths of Earth-bound detectors. The orbital eccentricity would reduce from $e = 0.617$ to $e \to 0$. In 2003, a new binary pulsar J0737-3039, the famous double pulsar, with an orbital period of 2.5 h ($e = 0.0877$) was discovered which will coalesce in 86 Myrs. The infall due to gravitational radiation damping is 7 mm/day! This is an even more unique laboratory for relativistic gravitational physics in the strong field regime. The late inspiral and merger epochs of compact binaries of neutron stars or black holes provide us possible strong sources of GW for LIGO and Virgo in the 'high' frequency range 10 Hz–10 kHz. We have guaranteed sources for the GW detectors if there are enough of them. The waveform is a chirp, i.e. a signal whose amplitude and frequency increase with time. Even GW from strongest astrophysical sources are weak signals buried in the strong noise of the detector. They require matched filtering both for their detection or extraction and parameter estimation or characterization. The success of matched filtering requires accurate model of the signal using general relativity and thus favours sources like ICB (NS-NS, BH-BH, NS-BH) over unmodelled sources like supernovae or GRB. This dire experimental need has thus led to progress in the improved understanding of the theoretical aspects of the two-body problem in GR.

When LIGO was funded in the early nineties, and efforts to construct accurate ICB waveforms started, it was soon realized that far more accurate higher-order PN waveforms would be needed to describe GW in the final stages of inspiral and merger than in binary pulsar timing analysis. Numerical relativity was far from mature and a grand challenge program was started towards this goal. Physical insights were essential to simplify the treatment and achieve the required waveforms. These include, realizing that (i) garden variety ICB would have radiated away their eccentricity and be moving in quasicircular orbits during the late inspiral, (ii) since matched filtering is sensitive to the phase it is more important to first control higher-order phasing than higher-order amplitudes – Newtonian amplitude and best available phasing or the restricted waveform, (iii) the inspiral can be treated in the adiabatic approximation as a sequence of circular orbits. This allows one to treat separately the radiation reaction effects and the conservative effects. One can go to higher PN orders in the inspiral without getting technically bogged down in controlling the much more difficult higher-order conservative PN terms, (iv) for compact objects the effects of finite size and quadrupole distortion induced by tidal interactions are of order 5PN. Hence, neutron stars and black holes can be modelled as point particles represented by Dirac δ-functions. Thus modelling ICB waveforms involves three modules: (a) Motion: given a binary system, iterate Einstein's equations (EE) to discuss conservative motion of the system. Compute the centre of mass energy E. (b) Generation: given the motion of the binary system on a fixed orbit, iterate EE to compute multipoles of the gravitational field and hence the far-zone flux of energy carried by GW. Compute $\mathcal{L}$. (c) Radiation reaction: given the conserved energy and radiated flux of energy assume the balance equations to compute the effect of radiation on the orbit. Compute $F(t)$ and $\phi(t)$. The (GW) phasing of binary is the analog of the (EMW) timing of pulsars. In the adiabatic approximation and for restricted WF (GW phase twice orbital phase) the phasing specified by a pair of differential equations (using units where $G = c = 1$)

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} - \frac{v^3}{M} = 0, \quad \frac{\mathrm{d}v}{\mathrm{d}t} + \frac{\mathcal{F}(v)}{ME'(v)} = 0,$$

where M is the total mass of the binary, F the GW frequency, $v = (\pi MF)^{1/3}$, $\mathcal{F}(v)$ is the GW energy flux, $E(v)$ is the binding energy and prime denotes the derivative with respect to v. The more general elliptical orbit case can be dealt with by including the conserved angular momentum and the associated angular momentum flux. For more details on template families that can be constructed from the PN inputs, see [20].

9. Major technical tools and approximation methods

The above problem is complex and needed to employ most of the useful technical tools of mathematical physics for its resolution [5]. These include: (a) Multipole (M) expansions or expansion in irreducible representations of the rotation group using STF tensors or tensor spherical harmonics, (b) post-Minkowskian (PM) approximation or the non-linearity expansion or expansion in G. This is valid in all the weak-field regions including infinity. (c) post-Newtonian (PN) approximation or the slow motion expansion or expansion in v/c. This is valid only in the near zone. Successful wave-generation formalisms are a subtle cocktail of different approximations. The multipolar post-Minkowskian (MPM) formalism and the PN approach are employed to deal with arbitrary mass ratio inspiral in the slow-motion weak field regime. Perturbations about BH space-time [21,22] is employed in the test particle limit to deal with extreme mass ratio inspiral (EMRI) and quasinormal mode (QNM) ringing. The self-force approach [23] is used to compute μ/M corrections to the above perturbation results for extreme mass ratio inspiral even in strong field regime. To go beyond PN approximants, one uses a Padé resummation [24]. The effective one-body approach [25] is a more complete resummation that can deal with inspiral, late inspiral, merger and ringdown. Finally, numerical relativity is the only route to deal with the final coalescence, merger and ringdown.

9.1 *Multipolar post-Minkowskian (MPM) generation formalism*

The MPM formalism [5] developed for binary pulsar work is a good example of the advantage that a complete and mathematically rigorous treatment of a problem can eventually bring in the future for more demanding applications that could be around the corner. MPM is currently the most successful since it can deal with all aspects: the conservative EOM, radiation field at infinity, non-linear effects related to tails etc. It has evolved over the last two decades into a consistent algorithmic approach to analytical GW computations [5,26] leading to 3.5PN results for non-spinning ICB on quasicircular orbits, 3PN results for non-spinning ICB on quasielliptical orbits and 2.5PN results for spinning binaries. EE are integrated using flat-space retarded integrals. To perform PN expansion, one expands retarded integrals in powers of $1/c$. Divergent integrals appear due to the near-zone nature of PN expansion. The MPM generation formalism of Blanchet, Damour and Iyer following Fock essentially involves a MPM expansion of the gravitational field in the region outside the source including infinity. This is followed by a PN expansion in the near zone and their matching in an intermediate zone. Infra-red (IR) divergences are handled by complex analytic continuation. This specifies the field exterior to the source completely. In the DIRE approach of Will and Wiseman [3] retarded integrals are evaluated beyond the near zone without expansion using null coordinates.

There are two independent aspects of the MPM formalism addressing two different problems: (i) The general method (MPM expansion) applicable to extended or fluid sources with compact support, based on the mixed PM and multipole expansion matched to some PN (slowly moving, weakly gravitating, small-retardation) source. IR divergences arising from the retardation expansion dealt by analytic continuation, (ii) the particular application to describe inspiralling compact binaries (ICB) using point particle models. Self-field regularization to deal with ultra-violet (UV) divergences arising from the use of delta functions to model point particles like Riesz, Hadamard partie finie, etc. work well up to 2PN and are consistent with each other. However, they lead to ambiguities at 3PN both in the EOM and flux via the mass quadrupole multipole moment and thus needed more sophisticated gauge invariant dimensional regularization for resolution of the inconsistencies and completion of the problem.

The MPM generation formalism enables one to compute the radiative moments as nonlinear functionals of the source moments via canonical moments. Both radiative and canonical moments are of two types: mass type and current type. The whole idea is to connect the two radiative moments (U and V) which the detector sees, via the two canonical source moments (M and S) to the six general 'source' moments (I, J, W, X, Y, Z) where the last four are gauge moments. Computation of the source moments is so far done for slow moving, weakly stressed (PN) sources. The relationship between the radiative and source moments involve many nonlinear multipole interactions,

$$\begin{aligned}
U_{ij}(U) = {} & I^{(2)}_{ij}(U) + \frac{2GM}{c^3}\int_0^{+\infty} \mathrm{d}\tau \left[\ln\left(\frac{c\tau}{2r_0}\right) + \frac{11}{2}\right] I^{(4)}_{ij}(U-\tau) \\
& + \frac{G}{c^5}\left\{ -\frac{2}{7}\int_0^{+\infty} \mathrm{d}\tau\ I^{(3)}_{a<i}(U-\tau) I^{(3)}_{j>a}(U-\tau) \right. \\
& \quad + \frac{1}{7} I^{(5)}_{a<i} I_{j>a} - \frac{5}{7} I^{(4)}_{a<i} I^{(1)}_{j>a} - \frac{2}{7} I^{(3)}_{a<i} I^{(2)}_{j>a} + \frac{1}{3}\varepsilon_{ab<i} I^{(4)}_{j>a} J_b \\
& \quad \left. + 4\left[W^{(2)} I_{ij} - W^{(1)} I^{(1)}_{ij}\right]\right\} \\
& + 2\left(\frac{GM}{c^3}\right)^2 \int_0^{+\infty} \mathrm{d}\tau\, I^{(5)}_{ij}(U-\tau) \\
& \times \left[\ln^2\left(\frac{c\tau}{2r_0}\right) + \frac{57}{70}\ln\left(\frac{c\tau}{2r_0}\right) + \frac{124627}{44100}\right] \\
& + \mathcal{O}(7),
\end{aligned}$$

causing different types of contributions to the waveform and fluxes. For instance, the energy flux to 3PN order is given by

$$\left(\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t}\right)_{\text{far-zone}} = \frac{G}{c^5}\left\{\frac{1}{5}U^{(1)}_{ij}U^{(1)}_{ij}\right.$$

$$+\frac{1}{c^2}\left[\frac{1}{189}U^{(1)}_{ijk}U^{(1)}_{ijk}+\frac{16}{45}V^{(1)}_{ij}V^{(1)}_{ij}\right]+\frac{1}{c^4}\left[\frac{1}{9072}U^{(1)}_{ijkm}U^{(1)}_{ijkm}+\frac{1}{84}V^{(1)}_{ijk}V^{(1)}_{ijk}\right]$$

$$\left.+\frac{1}{c^6}\left[\frac{1}{594000}U^{(1)}_{ijkmn}U^{(1)}_{ijkmn}+\frac{4}{14175}V^{(1)}_{ijkm}V^{(1)}_{ijkm}\right]+\mathcal{O}(8)\right\}.$$

For a given PN order only a finite number of multipoles contribute. At a given PN order the mass l-multipole is accompanied by the current $l-1$-multipole as is familiar from EM. To go to a higher PN order flux requires new higher-order l-multipoles and more importantly higher PN accuracy in the known lower-order multipoles. 3PN energy flux requires 3PN accurate mass quadrupole, 2PN accurate mass octupole, 2PN accurate current quadrupole, and N Mass 2^5-pole, current 2^4-pole.

Introducing, $x \equiv (\pi GMF/c^3)^{2/3} \sim v^2$, the 3.5PN energy flux is given by [27,28],

$$\mathcal{L} = \frac{32c^5}{5G}x^5\nu^2\left\{1+\left(-\frac{1247}{336}-\frac{35}{12}\nu\right)x+4\pi x^{3/2}\right.$$

$$+\left(-\frac{44711}{9072}+\frac{9271}{504}\nu+\frac{65}{18}\nu^2\right)x^2+\left(-\frac{8191}{672}-\frac{535}{24}\nu\right)\pi x^{5/2}$$

$$+\left(\frac{6643739519}{69854400}+\frac{16\pi^2}{3}-\frac{1712}{105}C-\frac{856}{105}\ln(16x)\right.$$

$$\left.+\left[\frac{41\pi^2}{48}-\frac{134543}{7776}\right]\nu-\frac{94403}{3024}\nu^2-\frac{775}{324}\nu^3\right)x^3$$

$$\left.+\left(-\frac{16285}{504}+\frac{176419}{1512}\nu+\frac{19897}{378}\nu^2\right)\pi x^{7/2}+\mathcal{O}(x^4)\right\}.$$

It should be noted that the 3.5PN flux includes important hereditary contributions arising from tails, tails of tails and tail-squared terms. The corresponding waveforms for the non-spinning case are available in [31]. The other input required for computing the phasing is the 3PN energy flux. It is given by [29,30]

$$E_3(x) = -\frac{1}{2}\nu x\left[1-\left(\frac{3}{4}+\frac{1}{12}\nu\right)x-\left(\frac{27}{8}-\frac{19}{8}\nu+\frac{1}{24}\nu^2\right)x^2\right.$$

$$\left.-\left\{\frac{675}{64}-\left(\frac{34445}{576}-\frac{205}{96}\pi^2\right)\nu+\frac{155}{96}\nu^2+\frac{35}{5184}\nu^3\right\}x^3\right].$$

By assessing the number of cycles that each PN order contributes for different types of binaries one can see that from the inspiral phase the contributions are less than a cycle by this PN order for even the advanced detectors [27,28]. For results in the eccentric case see [32] and in the spinning case see [33] and references therein.

9.2 *Implications of the high accuracy waveforms*

High accuracy waveforms underlie the construction of GW templates used in GWDA pipelines of LIGO, Virgo and LISA. They allow one to explore higher mass ranges than with the RWF based on the dominant harmonic [34]. They also lead to more accurate parameter extraction, crucial to extract astrophysical information from LIGO and Virgo and cosmological information (like the EOS of dark energy) from GW observations in LISA [35] so that it could have implications for Science case of LISA. They also have implications for test of GR and the extent to which GW observations of stellar mass BHB and IMBHB by advanced LIGO and ET and SMBHB by LISA can test PN theory [36,37].

9.3 *Effective-one-body (EOB)*

The effective-one-body (EOB) approach [38] is a new resummation, to extend the validity of suitably resummed PN results beyond the LSO, and up to the merger. At Newtonian approximation, the Hamiltonian $H_0(\mathbf{q},\mathbf{p})$ can be thought of as describing a 'test particle' of mass μ orbiting around an 'external mass' GM ($M \equiv m_1 + m_2$ and $\mu = m_1 m_2/M$). EOB approach is the general relativistic generalization of this. It consists of looking for an 'external spacetime geometry' $g^{\text{ext}}_{\mu\nu}(x^\lambda; \text{GM})$ such that 'geodesic' dynamics of 'test particle' of mass μ within $g^{\text{ext}}_{\mu\nu}(x^\lambda, \text{GM})$ is equivalent (when expanded in powers of $1/c^2$) to the original, relative PN-expanded dynamics. The EOB estimated the complete GW signal emitted by inspiralling, plunging, merging and ringing binary black holes.

The four essential elements of the EOB approach are: (i) Hamiltonian H_{real} describing the conservative part of relative dynamics of 2 BH; (ii) radiation-reaction force $\mathcal{F}_\varphi$ describing the loss of (mechanical) angular momentum, and energy, of the binary system; (iii) definition of various multipolar components of 'inspiral-plus-plunge' (metric) waveform $h^{\text{insplunge}}_{\ell m}$; (iv) attachment of subsequent 'ringdown waveform' $h^{\text{ringdown}}_{\ell m}$ around certain (EOB-determined)'merger time' t_{m} inspired by classic 'plunging test-mass' result. The EOB predicted a blurred transition from inspiral to plunge that is a smooth continuation of inspiral and sharp transition around the merger of continued inspiral and ringdown signal.

10. Pretorius and the numerical relativity breakthrough

Recent advances in numerical relativity (NR) allow stable, robust BH merger simulations. Some key milestones in these advances include, initial data for BBH, methods like punctures for representing BH on computational grids, importance of hyperbolicity in formulating EE for NR, improved formulation of EE, 3D evolution codes and their use in distorted BH, coordinate conditions to prevent slices from crashing into singularities and spatial coordinates from falling into BH as evolution proceeds, Cactus computational toolkit, modern adaptive mesh refinement and finite difference and multidomain spectral infrastructure for NR [39]. In 2005, Pretorius [40] produced the first simulation with a large number of orbits through merger using modified harmonic coordinates, compactification of numerical domain at spatial infinity, singularity excision and damping of constraints. With this amazing breakthrough in NR, one has reliable waveforms for the late inspiral and merger parts of the binary evolution which can be used for constructing templates. This led to a renaissance of numerical relativity with BBH. In eight months, other groups at University of Texas (Brownesville), NASA GSFC, using other methods like BSSN equations and moving puncture methods followed with similar results for the equal mass case. In 2006, came results for unequal mass BBH and recoil [41,42], merger of spinning BH, long waveforms (seven orbits), systematic parameter study (up to four orbits before forming common apparent horizon), astrophysics implications, energy in GW, spin of merged BH, recoil velocities of BH and their distribution and finally EM signatures from final merger to answer whether BH mergers which are expected to be `loud` would also be `bright`? For non-spinning BBH, aligned spinning BBH and antialigned spinning BBH, the energy in GW corresponds to $0.04M$, $0.07M$ and $0.02M$ respectively, the final spin corresponding to 0.69, 0.89 and 0.44 respectively. The GW luminosity of $L \sim 10^{23} L_{\odot}$ is greater than the total luminosity of all stars in the visible Universe. For stellar mass BH, the event lasts for a few milliseconds, for MBH it lasts for several minutes. For references to the original papers, see ref. [39].

10.1 *Recoil, kicks, superkicks*

In addition to energy and angular momentum, GW carry away linear momentum. Since the smaller mass body in the binary moves faster, its wave emission undergoes more forward beaming than for the more massive body. Instantaneously, this gives a net flux of linear momentum parallel to the velocity of the smaller body and opposite recoil or kick at the centre of mass. The direction of kick changes with the BH orbit. Since the BH is inspiralling, instantaneous kicks do not cancel but accumulate causing the final merged BH to have the net recoil in the orbital plane of magnitude 100–200 km/s depending on the mass ratio. In 2007, superkicks or recoils exceeding 1000 km/s [43,44] were discovered. Superkicks are possible for spins antialigned and lying in the orbital plane for equal mass systems and $(a/M)_1 = (a/M)_2$. The values can go up to 4000 km/s for the maximum spinning BH [39].

10.2 *Confronting numerical relativity with PNA*

The waveforms obtained by numerical simulations are calibrated and interpreted by the PN inspiral results. The progress is exciting in matching the PN waveforms to the numerical relativity ones [41,42]. They provide access to accurate knowledge of the waveform emitted during late inspiral and merger. However, such results are available only for a sparse sample of BBH systems. BBH simulations are time-consuming. NR simulations are currently too expensive to solely build the required bank of GW templates and densely fill multidimensional space of BBH physical parameters (masses and spins). The computational cost of accurate equal mass non-spinning 8-orbit BBH simulation is 200000 CPU hours. For a mass ratio of 10, it would be at least 2 million CPU hours. NR BBH groups require few million CPU hours. There is thus a need for analytical waveforms that represent the best numerical waveforms and EOB NR or phenomenological WF are geared to meet this need. Improving the match of the EOB waveforms at the final stages with the results from NR, requires improvement of radiation reaction and hence the accuracy of the waveforms of inspiral and plunge (figures 2 and 3).

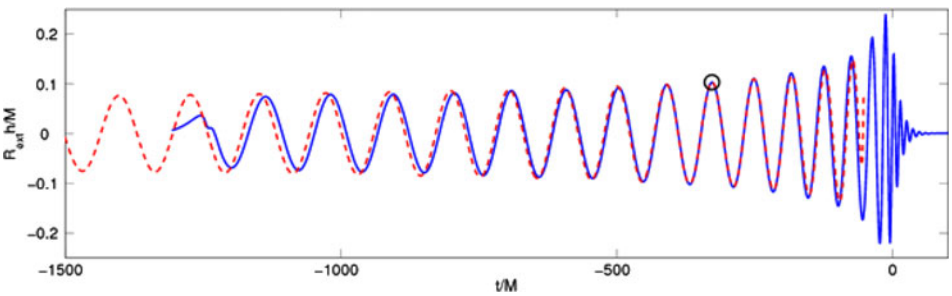

Figure 2. Comparison of a NR waveform (red line) with a PN waveform (blue dashed) [41].

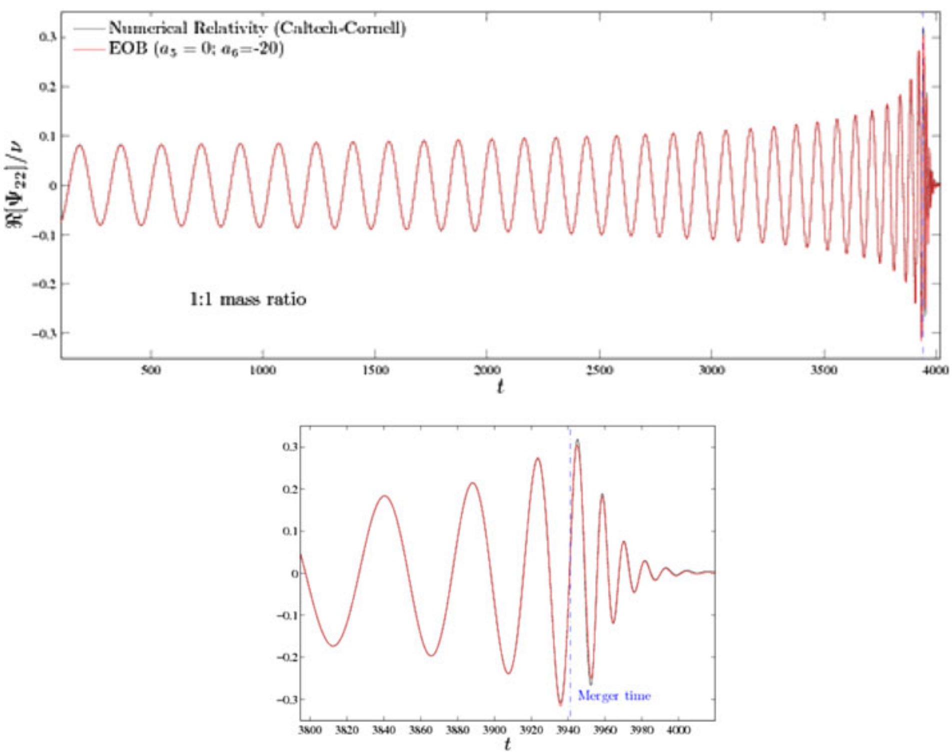

Figure 3. Improved EOB-based (red) and NR (black) for $\ell = m = 2$ metric waveforms [42].

11. Concluding remarks

Regarding the endeavour to detect GW, a few years ago, Peter Saulson said [45]: "Any experimental physicist marvels at the audacity of the attempt to detect GW. Detection of GW is nearly *impossible*. Involves technological challenges that appear *insurmountable*. That we are close to detecting it is remarkable. When we succeed it will be truely wonderful." The field stresses the symbiotic relation between basic sciences and applied technology on one hand and theory, experiment, computation on the other. The experiment on the other hand is driving the theory – PN computations for ICB, the self-force problem for EMRI, numerical relativity results for binary black holes or PN–NR comparison. The two-body problem in GR appears like the Lamb shift calculation in QED stretching the limits of the beautiful analytical results in GR. The key to the success is not a mechanical adaptation of Newtonian techniques but as emphasized by Damour [46] the insight that "It is not sufficient to transplant in Einstein's theory the technical steps of Newton's theory but one needs to transmute within Einstein's conceptual framework the ideas that underlie the technical developments."

In view of the limited scope of the article, the presentation has not been exhaustive and I was not able to discuss other related and equally important approaches. These include the work in ADM coordinates [47] related to 3PN and 3.5PN EOM, direct integration of relaxed Einstein equations (DIRE) [3,4] related to 2PN EOM and fluxes, strong field point particle limit [48] for 3PN EOM in which each star is considered as an extended object and limit taken to set radius zero in a very specific manner and EOM derived by surface integral approach *à la* Einstein–Infeld–Hoffman (EIH), effective field theory techniques [49] for 2PN EOM, self-force approaches for EMRI's to go beyond the test particle limit, Padè resummation [24] and effective-one-body [25] to go beyond PN approaches, numerical relativity [39] for BBH and BNS, self force–PN comparisons [50] and phenomenological waveforms [51] stitching together PN and NR results.

Four hundred years after Galileo's telescope launched optical astronomy, a major revolution in astronomy using GW is around the corner. It is a wonderful tribute in the past year of astronomy to Galileo from hundreds of brave GW experimenters struggling over decades undaunted in their belief that *impossible is nothing!* One is left speculating, if the first discovery of gravitational waves would be from a binary black hole system and Chandra would be doubly right about astronomy being the natural home of general relativity!!

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