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Beyond the Chandrasekhar limit: Structure and formation of compact stars

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Abstract. The concept of limiting mass, introduced by Chandrasekhar in case of white dwarfs, plays an important role in the formation and stability of compact objects such as neutron stars and black holes. Like white dwarfs, neutron stars have their own mass limit, and a compact configuration would progress from one family to the next, more dense one once a mass limit is crossed. The mass limit of neutron stars depends on the nature of nuclear forces at very high density, which has so far not been determined conclusively. This article reviews how observational determinations of the properties of neutron stars are starting to impose significant constraints on the state of matter at high density.

Keywords. S Chandrasekhar; limiting mass; neutron star; equation of state.

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1. Introduction

One of the most well known contributions of Chandrasekhar is the concept and the estimate of the mass limit of white dwarf stars. When this was first suggested [1,2], the neutron was yet to be discovered and the possibility of stable stellar configurations at densities higher than that of white dwarfs was unknown. The existence of such a mass limit therefore sparked the thought that the ultimate fate of more massive stars might be infinitely compact configurations which we now call Black Holes. This was an uncomfortable thought at that time, and the suggestion of the mass limit, correct though it was, required a long time to gain acceptance in the astronomical community.

Today, the existence of neutron stars is well established; neutron stars of mass well above the Chandrasekhar limit for white dwarfs are known, and it is clear that the routes to black holes pass through such neutron-rich configurations. What is still not accurately known, however, is at what mass black hole formation would occur – namely, what is the upper mass limit for neutron stars. The pressure in a neutron star arises primarily from the nuclear forces between its constituent particles. At very high (supranuclear) densities the nature of this force is still unknown, uncovering which is one of the outstanding challenges in fundamental physics today. Particle accelerators of progressively higher energies are making incremental progress towards the understanding of the nature of nuclear forces, and are laying the grounds for more sophisticated theoretical models. On the other hand, astronomical measurements of the properties of neutron stars are advancing to such degree as to significantly constrain the equation of state of high-density matter. This article will primarily be devoted to a discussion of the present state of astronomical constraints on the nature of dense matter.

2. The mass limit

We begin this discussion with a review of the physics that determines the mass limit of compact stars. Compact stars represent the final products of stellar evolution. In the absence of energy generation, they derive their balance against gravity from sources other than thermal pressure. In the case of white dwarfs, the pressure is contributed by electron degeneracy and in the case of neutron stars, by a combination of neutron degeneracy and nuclear forces. Existence of yet another family of stars, called strange stars, have been proposed, the constituents of which are deconfined up, down and strange quarks [3]. In these the degeneracy pressure of quarks balances the configuration against the combined action of attractive interquark forces and gravity.

The general nature of these forces are such that more massive configurations are also more dense. With increasing density the Fermi momentum of the constituent particles rises, and at high enough densities they become relativistic. The change in the equation of state accompanying this is the reason behind the existence of the mass limit of white dwarfs which are configurations supported by electron degeneracy pressure.

At low densities, when the electron Fermi momentum $p_{\mathrm{F}} \ll m_{\mathrm{e}}c$ (m_{e} is the electron mass and c is the speed of light), the equation of state (dependence of pressure P on density ρ) takes the form $P \propto \rho^{5/3}$. But, as the particles become relativistic ($p_{\mathrm{F}} \gg m_{\mathrm{e}}c$) this relation changes to $P \propto \rho^{4/3}$. As shown by Chandrasekhar [1,4] a polytropic stellar configuration with this latter equation of state has a unique mass, the limiting mass, which numerically evaluates to $5.76 M_{\odot}/\mu_{\mathrm{e}}^2$. Here μ_{e} is the 'mean molecular weight per electron', defined as $\rho/n_{\mathrm{e}}m_{\mathrm{p}}$, where n_{e} is the number density of electrons and m_{p} is the mass of the proton. In his 1931 publication Chandrasekhar used a value of $\mu_{\mathrm{e}} \approx 2.5$ and obtained a limiting mass of $0.91 M_{\odot}$. Later, a more realistic value of $\mu_{\mathrm{e}} = 2$ has been commonly used [4], giving a value of $1.44 M_{\odot}$ to the upper mass limit of white dwarfs, now referred to as the 'Chandrasekhar limit'.

Being the maximum mass of configurations supported by electron degeneracy, the Chandrasekhar limit also plays a vital role in the formation of more compact configurations. At the centres of stars massive enough to undergo all stages of nuclear fusion, inert iron cores form which are supported by electron degeneracy pressure. Nuclear fusion in the surrounding shells continues to add mass to this core. Once its mass exceeds the Chandrasekhar limit, a rapid gravitational collapse ensues, forming a neutron star and expelling the surrounding envelope in a supernova explosion. Some mass from the envelope may fall back on the neutron star, increasing its mass further. If the total mass of the final remnant exceeds the limiting mass of a neutron star, then a black hole would form.

This physics of the mass limit, however, does not bear a straightforward extension from the case of white dwarf to that of a neutron star. First, the neutron star is a much more compact configuration, and so explicit inclusion of general relativity becomes essential. Oppenheimer and Volkoff [5] investigated configurations supported by pure degeneracy pressure of neutrons using the full relativistic treatment, and showed that the mass limit in this case comes down to just $0.7M_{\odot}$. If this were indeed the true mass limit of neutron stars then, given the formation route outlined above, hardly any neutron star would be expected to exist – all cores exceeding Chandrasekhar mass would directly collapse into black holes. Nevertheless, neutron stars do exist, and it is the repulsive nuclear force at short distance that is responsible for raising the neutron star mass limit significantly above the value obtained for pure neutron degeneracy. Observational determination of the neutron star mass limit, as well as the relation between the mass and the radius of neutron stars, therefore convey important information regarding the nature of short-range nuclear forces. Since this regime of high density is still beyond the reach of terrestrial experiments, constraining the nature of nuclear forces via neutron star studies has become one of the holy grails of modern astrophysics.

Based on extrapolations from laboratory knowledge and different theoretical models, a large number of different equations of state (EoS) have been proposed for matter at high

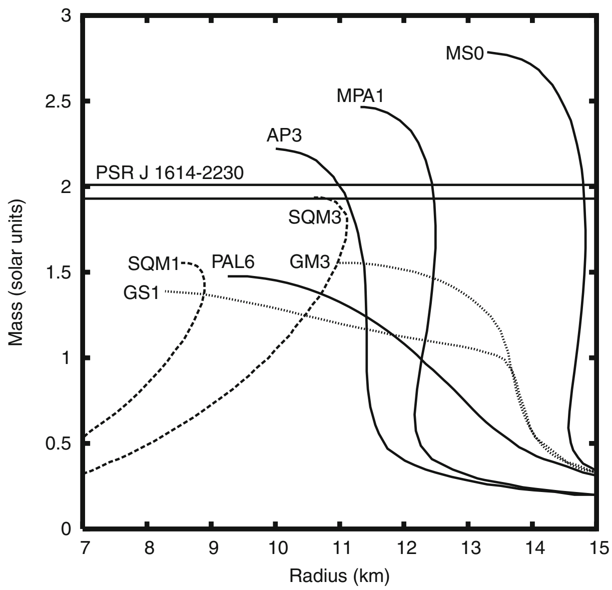

Figure 1. Mass–radius relation of neutron stars in several representative equations of state. The labels indicating the equation of state models are the same as in [14]. Solid lines represent nuclear matter EoS and dashed lines those for pure strange matter. Dotted lines show equations of state involving exotic matter, such as kaon condensate or hyperons, at high densities. The horizontal lines represent the bounds on the mass of PSR J1614 – 2230. Existence of this heavy neutron star rules out most equations of state except some of nuclear matter family (after [13]).

densities. These predict different mass limits for neutron stars, as well as different mass–radius relations. It is hoped that accurate astrophysical measurements of mass and radius of neutron stars will help one choose among this great diversity of EoS models. Figure 1 displays the predicted mass–radius relation of neutron stars (and strange matter stars) in a few representative EoS models.

3. Measuring the mass of neutron stars

Neutron stars in binary systems provide us an opportunity to measure their masses. To date, over 60 mass estimates have been made with varying degrees of accuracy (see [6] for a compilation). The most precise measurements come from some of the binary radiopulsar systems where, in addition to the Keplerian orbital elements, other relativistic parameters such as the precession of periastron, gravitational redshift, Shapiro delay or orbital decay due to gravitational radiation can be measured. A few of the best measured neutron star masses are listed in table 1.

As can be observed from table 1, the well-measured masses of neutron stars lie in the range 1.25–2.0$M_\odot$. At the lower end of the mass spectrum lie neutron stars which have acquired relatively little matter by either supernova fall-back or subsequent accretion from the binary companion. It is to be noted that an $\sim 1.4 M_\odot$ pre-collapse core would result in an $\sim 1.25 M_\odot$ neutron star after the collapse, the difference being accounted for by gravitational binding energy. Of particular interest, however, is the most massive neutron star reported, PSR J1614 − 2230. Its mass is already above the maximum mass expected in several proposed families of EoS models. In particular, all known models of strange stars are excluded, and all equations of state that hypothesize exotic matter such as kaon condensate or hyperons in the neutron star core are ruled out (see figure 1). All known EoS models that are compatible with this measurement are of pure nuclear matter [13].

The mass–radius relations shown in figure 1 belong to non-rotating configurations. Only the stable part of the sequence, up to the maximum mass, is displayed for each EoS. These maximum mass configurations also represent the highest density at which stable equilibrium

Table 1. Selected list of measured masses of neutron stars.

System	$M (M_\odot)$	Reference
PSR B 1913 + 16	1.4398 ± 0.0002	[7]
Companion NS	1.3886 ± 0.0002	
PSR B 1534 + 12	1.333 ± 0.001	[8]
Companion NS	1.345 ± 0.001	
PSR J 0737 − 3039A	1.337 ± 0.005	[9]
PSR J 0737 − 3039B	1.250 ± 0.005	
PSR J 1141 − 6545	1.27 ± 0.01	[10]
PSR J 1903 + 0327	1.667 ± 0.021	[11]
PSR J 1909 − 3744	1.438 ± 0.024	[12]
PSR J 1614 − 2230	1.97 ± 0.04	[13]

can be obtained for the corresponding EoS models. At densities higher than this, radial instability sets in, leading to a collapse of the configuration.

4. Rotating neutron stars

Many neutron stars spin very rapidly. Some could be born spinning fast, but the fastest spinning objects among the known neutron star population have acquired their spin by accreting matter from their binary companions. Rotation of the neutron star alters the internal equilibrium. A given configuration can support a maximum amount of rotation, beyond which mass begins to be shed due to centrifugal force. Cook *et al* [15] computed hydrostatic equilibrium of fully relativistic neutron star models in the presence of rotation, and determined the mass shed limits for various EoS, as a function of the mass of the star. They also determined the radial instability limit of these models, and showed that the maximum mass is somewhat increased in the presence of rapid rotation. Figure 2 displays, in an angular speed vs. mass diagram, the mass shed and radial instability limits for two representative nuclear equations of state – a soft (FPS) and a stiff one (PS). Also indicated in the figure is the position of the most massive neutron star known, PSR J1614 − 2230 [13], and the fastest spin of a neutron star measured so far (PSR J1748 − 2446ad, spin frequency 716 Hz [16]). It is clear that while the highest measured mass rules out the soft equation

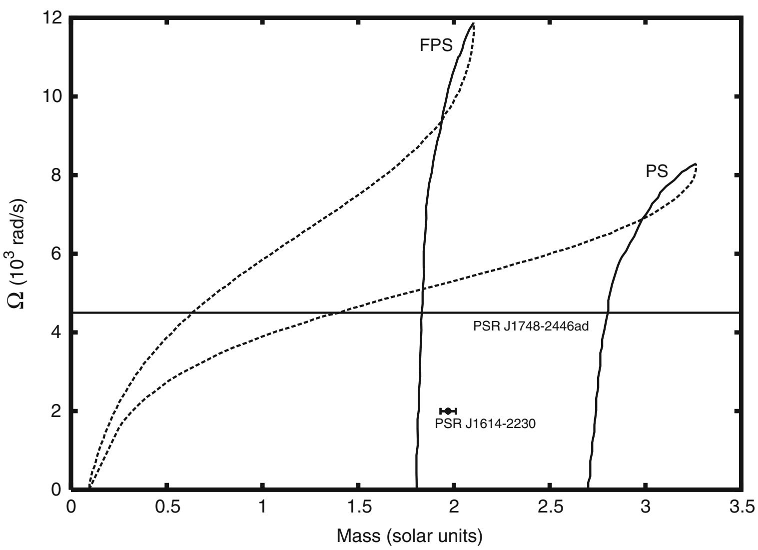

Figure 2. Limiting mass and angular velocity of spinning neutron star configurations. Results from Cook *et al* [15] are displayed for two equations of state, one soft and one stiff. Solid lines are the radial instability limits and dashed lines are mass shed limits. For each equation of state, configurations are allowed within the region bounded by these two lines, below their point of crossing. The positions of the most massive neutron star reported, PSR J1614 − 2230 and the angular speed of the fastest rotating neutron star known, PSRJ1748 − 2446ad are also shown.

of state, the constraints placed by measured spins of neutron stars are so far not very significant. Neutron stars spinning at sub-millisecond periods, if found, would be of great interest from this point of view.

There may, however, be an important barrier to forming neutron stars spinning at sub-millisecond periods. As mentioned, the fast spinning neutron stars we observe are in fact spun-up by accretion in binary systems. The matter accreting on the star brings in some of the orbital angular momentum and adds it to the neutron star's spin. As the neutron star is spun up, however, unstable non-radial modes are excited, which radiate away angular momentum in gravitational waves [17–19]. As a result the spin-up may never proceed to sub-millisecond periods.

5. Measuring the radius

As discussed in §3, the recent measurement of the mass of PSR J1614 − 2230 provides one of the strongest observational constraints on the equation of state so far. However this still leaves a wide variety of nuclear equations of state to choose from, as can be easily seen from figure 1. Further refinement of the constraints could come from the measurement of both mass and radius of a few neutron stars, which can help isolate which mass–radius relation the neutron stars actually follow. At present, accurate and reliable measurements of neutron star radii are not yet available, but serious efforts are on in this direction.

It is to be noted that for nuclear matter EoS models that allow neutron stars as heavy as $\sim 2M_{\odot}$ to exist, the stellar radius is nearly independent of mass (see figure 1), except in regions close to the lower and upper mass limits. As a result, a precision measurement of radius could in itself be of significant help in discriminating between these equations of state, even when the mass estimate is relatively crude.

One of the traditional methods for estimating radius of the neutron star involves the use of thermonuclear X-ray bursts seen in low-mass X-ray binaries. In X-ray bursters, matter accreting on the neutron star accumulates for a while and then ignites in a thermonuclear flash, causing the X-ray luminosity to suddenly rise. The luminosity then declines over the next minute or so. Spectral measurements during the cooling phase of the burst allow the estimation of colour temperature, which can then be combined with the measured luminosity to yield an estimate of the radius of the star. The normalization of a black body spectral fit is proportional to the quantity [20]

$$K \equiv \left(\frac{R_{\rm app}}{D}\right)^2 = \frac{R^2}{f_{\rm c}^4 D^2}\left(1 - \frac{2GM}{Rc^2}\right)^{-1},$$

where D is the distance to the star, R its radius and M its mass. The 'colour factor' $f_{\rm c}$ accounts for the correction to be applied to the derived temperature, because of the non-Planckian nature of the spectrum. It is assumed that in the tail of the burst, where this method is applied, the entire surface of the neutron star radiates uniformly.

While this concept appears simple, the method suffers from many systematic uncertainties, particularly in the estimate of the colour factor. Continued improvement in spectral measurements as well as in theoretical modelling have contributed to significant progress in the past few years, but a consensus is yet to be achieved. Özel *et al* [21], using a carefully selected set of X-ray burst sources and a class of neutron star atmosphere models, derive fairly tight constraints on the masses and radii of three neutron stars. These estimates appear to favour neutron stars being slightly more compact than that predicted by EoS models that are consistent with the mass of PSR J1614 − 2230. However, Suleimanov *et al* [22], using a different set of atmosphere models, obtained constraints which were far less restrictive. More work is clearly needed to refine the models and reconcile these differences, but this remains one of the most promising methods to constrain neutron star radii in the near future.

Another approach that has been proposed involves the modelling of pulse profiles of X-ray pulsars. A neutron star, being very compact with radius only a few times its gravitational radius, contributes to strong light bending of radiation emanating from its surface. Even if the emitting region is a point on the surface, a distant observer is able to view it over a wide range of rotation phases. As a result, the degree of modulation of pulsed radiation is reduced, and the amount of reduction is a function of the radius of the neutron star. This effect can be used to put a lower limit on the ratio R/M. With high-precision measurement of the pulse profile, one can resort to detailed modelling to strongly constrain this ratio [23]. However, observations of precision adequate enough to use this method effectively will be possible only with next-generation, highly sensitive X-ray observatories.

Yet another method of putting an upper limit on the stellar radius derives from the fluctuation spectrum of X-ray intensity arising from the accretion disk around a neutron star in an X-ray binary. Strongest constraints from this method result when the accretion rate is high, and the magnetic field of the neutron star is weak, allowing the disk to extend very close to the neutron star surface. The intensity fluctuation spectrum of such systems show distinct peaks of quasiperiodic oscillations (QPO) of near-kilohertz frequency, called 'kHz QPOs' [24]. The highest frequency peak in the intensity fluctuation spectrum is attributed to the Keplerian frequency at the inner edge of the accretion disk, and hence provides a measure of $R_{\rm in}$, the inner radius of the disk. This also serves as an upper limit on the radius R of the neutron star. The maximum kHz QPO frequency reported till date is 1330 Hz [25] which constrains R to be less than 10–15 km for masses in the range 0.5–2$M_{\odot}$, but still accommodates most of the EoS models under discussion.

A measure of the inner radius $R_{\rm in}$ of the disk can also be provided by iron K shell emission lines observed in the X-ray spectra of such systems. The extreme red wing of the observed lines are shaped by the relativistic Doppler shift and the gravitational redshift in the innermost parts of the accretion disk. Thus a measurement of such a line profile with high signal-to-noise ratio can be used to constrain the inner disk radius. Recent observations, particularly with the *Suzaku* X-ray mission, are beginning to provide good measurements of the line shape. While the resulting constraints in the neutron star M–R plane are not yet discriminating enough, this method appears to hold much promise for the future [26].

6. Seismology

In the wake of the extremely powerful X-ray/γ-ray hyperflare of the strongly magnetized neutron star SGR 1806 − 20 on 27 December 2004, the declining X-ray intensity showed periodic, 7.5-s oscillations corresponding to the rotation period of the star. Superposed on this periodic variation, quasiperiodic oscillations of various frequencies (18, 26, 29, 92, 150, 625, 720, 976, 1840 and 2384 Hz) were discovered [27,28]. This neutron star belongs to a class of objects called 'magnetars', whose radiation is thought to be powered primarily by the release of stored internal magnetic free energy [29]. This energy appears to be released in bursts of varying magnitudes, most probably accompanied by cracking and rearrangement of the crust. The 27 December 2004 flare is so far the strongest known from any such object, and is likely to have excited a host of seismic oscillations in the neutron star. The observed quasiperiodic oscillations are believed to be the signatures of such seismic oscillations. Interpreted as pure crustal modes, these constrain the crust thickness to about 10–13% of the stellar radius [28]. Constraints on the equation of state can be derived from those on the crust thickness, although what is obtained from the present data is not yet very strong. Accumulation of more such observations in the future, particularly of a wider spectrum of seismic oscillations, would be very valuable in providing an independent set of constraints on the equation of state. Apart from magnetar flares, it may also be possible to use future gravitational wave observations to derive seismic signatures of neutron stars [30].

7. Concluding remarks

The past decade has seen a very significant progress in the observational determination of the fundamental properties of neutron stars such as their masses and radii. Important constraints on the equation of state of neutron stars are already beginning to emerge from these observations, and these constraints are gradually becoming stronger and more refined. The trend is set to continue with new radio millisecond pulsar surveys and timing programmes, more sensitive studies of X-ray burst spectra and improved modelling, continued X-ray timing over wider energy bands with forthcoming missions such as Astrosat (India) and LOFT (ESA). Atomic lines from neutron star surface are likely to be detected with highly sensitive missions like the proposed International X-ray Observatory, leading to new constraints on surface gravity. Neutron star seismology is also an exciting new prospect on the horizon, particularly with new-generation gravitational wave observatories.

Soon after introducing the concept of mass limit in the literature of stellar physics, Chandrasekhar [31] remarked:

> We may conclude that great progress in the analysis of stellar structure is not possible before we can answer the following fundamental question:
>
> *Given an enclosure containing electrons and atomic nuclei, (total charge zero) what happens if we go on compressing the material indefinitely?*

Today, one is beginning to be able to turn this around and probe the nature of ultradense matter using astronomical observations themselves.

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