PRAMANA
— journal of
physics

Vol. 76, No. 6
June 2011
pp. 945–956

Nonlinear stability of pulsational mode of gravitational collapse in self-gravitating hydrostatically bounded dust molecular cloud

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MS received 23 October 2010; revised 23 October 2010; accepted 9 November 2010

Abstract. The pulsational mode of gravitational collapse (PMGC) in a hydrostatically bounded dust molecular cloud is responsible for the evolution of tremendous amount of energy during star formation. The source of free energy for this gravito-electrostatic instability lies in the associated self-gravity of the dispersed phase of relatively huge dust grains of solid matter over the gaseous phase of background plasma. The nonlinear stability of the same PMGC in an infinite dusty plasma model (plane geometry approximation for large wavelength fluctuation in the absence of curvature effects) is studied in a hydrostatic kind of homogeneous equilibrium configuration. By the standard reductive perturbation technique, a Korteweg–de Vries (KdV) equation for investigating the nonlinear evolution of the lowest order perturbed self-gravitational potential is developed in a time-stationary (steady-state) form, which is studied analytically as well as numerically. Different nonlinear structures (soliton-like and soliton chain-like) are found to exist in different situations. Astrophysical situations, relevant to it, are briefly discussed.

Keywords. Jeans instability; dusty plasma.

PACS Nos 52.35.-g; 51.70.+f

1. Introduction

Dark molecular clouds, which appear to have a hierarchy in the structure of nonlinear clumps and subclumps of increasing density and reducing size, are known to be ideal sites of star formation through self-gravitational instability [1–14]. Stellar structures are self-gravitating globes of dusty gas kept in hydrostatic equilibrium by the pressure support of internal origin against self-gravity. The self-gravitational instability [3] leading spontaneously to a lower energy state in the process of stellar structure formation gives rise to a giant source of energy in various forms. The pulsational mode of gravitational collapse (PMGC) of a dusty plasma, arising due to gravito-electrostatic coupling in the presence of partially ionized grains in a self-gravitating dusty plasma system, is similarly responsible for a tremendous amount of energy through star formation processes on gravitational scale of space and time.

The self-gravitational stability (against fluctuations of nonlinear type) of the PMGC under plane geometry model (infinite dusty plasma system) approximation is studied in the present paper. The plane geometry approximation (radial 1D case), instead of spherical geometry (spherical 3D case), is adopted for our convenience. This may, however, be justified under the consideration of radial symmetry assuming that the wavelength of fluctuation is much longer than the grain-to-grain distance. Thus, spherical three degrees of freedom may justifiably be reduced to a single radial degree of freedom for our stability analyses, conveniently. It is also reported [2] that typically, $m_{\mathrm{e}}/m_{\mathrm{d}} < m_{\mathrm{i}}/m_{\mathrm{d}} \sim 10^{-20}$ and $n_{\mathrm{d}}/n_{\mathrm{e}} \sim n_{\mathrm{d}}/n_{\mathrm{i}} \sim 10^{-2}–10^{-3}$. So clearly $m_{\mathrm{e}}n_{\mathrm{e}} < m_{\mathrm{i}}n_{\mathrm{i}} \ll m_{\mathrm{d}}n_{\mathrm{d}}$, where all the symbols have their usual significances. Thus, gravito-electrostatic potential Ψ-fluctuation is mainly because of the relatively massive inertial dust grain-like impurity ions only, except in the point-mass approximation. Self-gravitational effects of plasma thermal electrons and ions are neglected, this ratio being too small to contribute to Ψ-evolution. The grain mass m_{d} is reported to vary, at least typically from 10^{-5} g in the interplanetary space to $10^{-12}–10^{-14}$ g, in the interstellar cloud [2]. Similar situations, in reality, are found to exist in laboratories due to high-Z impurities from tokamak walls in plasma processing and impurity generation in MHD-power generators [2].

To study the nonlinear stability of the dust molecular cloud, the standard and conventional technique of reductive perturbation is applied over our defined equilibrium to derive a well-known Korteweg–de Vries (KdV) equation in terms of the lowest order gravito-electrostatic potential fluctuation. The solution is then analytically and numerically studied. This paper is structurally organized in a simple format as follows. Section 2 describes physical model of the dust molecular cloud, §3 describes the mathematical formulation and derives analytical equations, and §4 presents the results and discussions. Lastly and most importantly, §5 gives the main conclusions along with tentative applicability.

2. Physical model

We consider an ideal situation of a field-free self-gravitating dust molecular cloud under plane geometry approximation (infinite dimension) with global quasi-neutrality. The plane geometry approximation (1D problem) instead of spherical geometry (3D problem) may, however, be justified for very long wavelength fluctuation relative to intergrain separation. Thus, nonplanar geometry is avoided because of the plane-wave approximation of fluctuation. The hydrostatic equilibrium of the dusty plasma system is the static distribution of the multifluid consisting of electrons, ions, neutral gas and neutral dust grains with partial ionization. The dust grains get, as usual, electrically charged due to the plasma environment by statistically random collision processes.

If the contributions of the gravitational and electrostatic forces using usual realistic parameter values are compared for plasma-charged particles, it will be found that they are diverse orders of magnitude leading to negligible gravitational effects. Moreover, if the dust grains are relatively huge, but within the validity limit of point-mass approximation, the gravitational effects will be significant. As the grains are now treated as point masses, the trapping of the plasma thermal ions by them may be ignored. The electric forces generated by electrostatic polarization effects (local charge imbalance) are assumed to be weak [4,5], so that only the lowest-order contributions of the various nonlinear terms in our model are considered.

The distributions of the thermal screening species (parent ions and electrons) are supposed to obey Boltzmann density distributions [4,5] meant for thermodynamic equilibrium on slow dust inertial time scale. This assumption of thermalization of thermal species is valid provided the phase velocity of fluctuations is much smaller than their thermal velocity, i.e., any fluctuation in the electron–ion temperature is instantly smoothened. It is assumed that the neutral gas-dynamic particles form the background which is weakly coupled with the collapsing self-gravitating dust plasma mass. The gravitational decoupling of the background neutral particles may be justified because of the higher inertial mass of the dust grains. To simplify further, complications like the effects of dispersed dust grain rotation, kinetic viscosity, nonthermal energy transport (wave dissipation process) and magnetic field due to convective circulation dynamics are neglected.

3. Mathematical formulation

Assuming all the relatively massive grains as identical point masses, all the charged dust grains will have the same amount of electric charge in a given plasma background (as charge is directly proportional to size). Background plasma electrons and ions may, of course, be assumed to follow the Boltzmann distributions for any physical phenomenon to be investigated on slow dust inertial time scale, with all usual notations [4,5]. Thus, their density distributions (in unnormalized form) are, respectively, as follows:

$$n_{\mathrm{e}} \approx n_{\mathrm{e}0} \mathrm{e}^{e\phi/T_{\mathrm{e}}}, \tag{1}$$

and

$$n_{\mathrm{i}} \approx n_{\mathrm{i}0} \mathrm{e}^{-\sigma e\phi/T_{\mathrm{e}}}, \tag{2}$$

where $\sigma = T_{\mathrm{e}}/T_{\mathrm{i}}$ is electron-to-ion temperature ratio, n_{j0} is the equilibrium density of the jth species and ϕ is the equilibrium electrostatic potential due to polarization.

The dynamical response of the cold neutral dust grains can be well described by the full inertial type without frictional coupling with any other existing fluid in the form of fluid momentum and continuity equations respectively as follows:

$$\frac{\partial v_{\mathrm{dn}}}{\partial t} + v_{\mathrm{dn}} \frac{\partial v_{\mathrm{dn}}}{\partial x} = -\frac{\partial \psi}{\partial x}, \tag{3}$$

and

$$\frac{\partial n_{\mathrm{dn}}}{\partial t} + \frac{\partial}{\partial x} \left(n_{\mathrm{dn}} v_{\mathrm{dn}} \right) = 0, \tag{4}$$

where v_{dn} is the neutral dust fluid flow velocity, n_{dn} is the neutral dust population density and ψ is the unperturbed self-gravitational potential arising due to massive inertial dust grains.

The dynamical response of the charged dust fluid with frictional force term (due to charged–neutral and neutral–neutral dust collisions), with all conventional notations, in the form of momentum and continuity equations, can be described as follows:

$$\frac{\partial v_{\mathrm{dc}}}{\partial t} + v_{\mathrm{dc}} \frac{\partial v_{\mathrm{dc}}}{\partial x} = -\frac{q_{\mathrm{d}}}{m_{\mathrm{d}}} \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x} - \nu_{\mathrm{cn}} \left(v_{\mathrm{dc}} - v_{\mathrm{dn}} \right), \tag{5}$$

and

$$\frac{\partial n_{\mathrm{dc}}}{\partial t}+\frac{\partial}{\partial x}\left(n_{\mathrm{dc}} v_{\mathrm{dc}}\right)=0, \tag{6}$$

where v_{dc} is the charged dust fluid flow velocity, n_{dc} is the charged dust population density, q_{d} is the grain charge and ν_{cn} is the collision frequency between charged and neutral dust fluids.

Lastly, Poisson's equations for the distribution of electrostatic potential (ϕ) and gravitational potential (ψ) close all the basic governing equations as a coupled dynamical self-gravitating system. These equations with usual notations [4,5] are set as

$$\frac{\partial^2 \phi}{\partial x^2}=4\pi e\left(n_{\mathrm{e}}-n_{\mathrm{i}}+q_{\mathrm{d}} n_{\mathrm{dc}}/e\right) \tag{7}$$

and

$$\frac{\partial^2 \psi}{\partial x^2} = 4\pi G m_{\mathrm{d}}\left(n_{\mathrm{dc}}+n_{\mathrm{dn}}-n_{\mathrm{d0}}\right), \tag{8}$$

where G represents the universal gravitational constant, and $n_{\mathrm{d0}}=n_{\mathrm{dc0}}+n_{\mathrm{dn0}}$ models the Jeans swindle of the equilibrium unipolar gravitational force field which is, in fact, a kind of local approximation for equilibrium mass distribution similar to that of equilibrium electric charge distribution [4,5]. It is worth mentioning that eq. (8) here is in the limit of Newtonian gravity, i.e., without involving general relativity.

Applying the standard methodology of conventional reductive perturbation technique [11] over the coupled equations (1)–(8), a nonlinear dynamical evolution equation is derived to study the nonlinear stability of the multifluid plasma system. Thus the independent variables with all usual notations [4,5] are stretched into a new space defined by the transformations $\xi=\epsilon^{1/2}(x-\lambda t)$ and $\tau=\epsilon^{3/2}t$. In the new space, the differential operators get transformed as $\partial/\partial x \equiv \epsilon^{1/2}\partial/\partial\xi$, $\partial^2/\partial x^2 \equiv \epsilon\partial^2/\partial\xi^2$ and $\partial/\partial t \equiv \epsilon^{3/2}\partial/\partial\tau-\lambda\epsilon^{1/2}\partial/\partial\xi$, where λ is the Mach number of the perturbation and ϵ is a minor parameter characterizing the strength of nonlinearity and dispersion.

We now carry out nonlinear perturbative analyses in the absence of any kind of spatial inhomogeneities for simplicity. The relevant dependent plasma variables like density, velocity, potential, etc. in the basic set of eqs (1)–(8), however, are now expanded nonlinearly (in ϵ-powers) around the respective equilibrium values as follows:

$$\begin{pmatrix} n_{\mathrm{e}} \\ n_{\mathrm{i}} \\ n_{\mathrm{dn}} \\ n_{\mathrm{dc}} \\ v_{\mathrm{dn}} \\ v_{\mathrm{dc}} \\ \phi \\ \psi \end{pmatrix} = \begin{pmatrix} n_{\mathrm{e0}} \\ n_{\mathrm{i0}} \\ n_{\mathrm{dn0}} \\ n_{\mathrm{dc0}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \epsilon \begin{pmatrix} n_{\mathrm{e1}} \\ n_{\mathrm{i1}} \\ n_{\mathrm{dn1}} \\ n_{\mathrm{dc1}} \\ v_{\mathrm{dn1}} \\ v_{\mathrm{dc1}} \\ \phi_1 \\ \psi_1 \end{pmatrix} + \epsilon^2 \begin{pmatrix} n_{\mathrm{e2}} \\ n_{\mathrm{i2}} \\ n_{\mathrm{dn2}} \\ n_{\mathrm{dc2}} \\ v_{\mathrm{dn2}} \\ v_{\mathrm{dc2}} \\ \phi_2 \\ \psi_2 \end{pmatrix} + \cdots\cdot. \tag{9}$$

The expansion (9) is used in eqs (1)–(8) for systematic order-by-order analyses. Equating similar terms in various powers of ϵ from both sides of eq. (3), one gets

$$\epsilon^{3/2}:\quad \lambda\frac{\partial v_{\mathrm{dn1}}}{\partial\xi} = \frac{\partial\psi_1}{\partial\xi}, \tag{10}$$

$$\epsilon^{5/2}:\quad \frac{\partial v_{\mathrm{dn1}}}{\partial\tau} - \lambda\frac{\partial v_{\mathrm{dn2}}}{\partial\xi} + v_{\mathrm{dn1}}\frac{\partial v_{\mathrm{dn1}}}{\partial\xi} = -\frac{\partial\psi_2}{\partial\xi}, \ldots. \tag{11}$$

Similarly, equating the like terms in various powers in ϵ from eq. (4), one gets

$$\epsilon^{3/2}:\quad -\lambda\frac{\partial n_{\mathrm{dn1}}}{\partial\xi} + n_{\mathrm{dn0}}\frac{\partial v_{\mathrm{dn1}}}{\partial\xi} = 0, \tag{12}$$

$$\epsilon^{5/2}:\quad \frac{\partial n_{\mathrm{dn1}}}{\partial\tau} - \lambda\frac{\partial n_{\mathrm{dn2}}}{\partial\xi} + \frac{\partial}{\partial\xi}(n_{\mathrm{dn1}}v_{\mathrm{dn1}}) + n_{\mathrm{dn0}}\frac{\partial v_{\mathrm{dn2}}}{\partial\xi} = 0, \ldots. \tag{13}$$

Applying the same methodology, the order-by-order analysis in various powers in ϵ from eq. (5) similarly yields

$$\epsilon^{3/2}:\quad -\lambda\frac{\partial v_{\mathrm{dc1}}}{\partial\xi} + \frac{q_{\mathrm{d}}}{m_{\mathrm{d}}}\frac{\partial\phi_1}{\partial\xi} + \frac{\partial\psi_1}{\partial\xi} = 0, \tag{14}$$

$$\epsilon^{5/2}:\quad \frac{\partial v_{\mathrm{dc1}}}{\partial\tau} - \lambda\frac{\partial v_{\mathrm{dc2}}}{\partial\xi} + v_{\mathrm{dc1}}\frac{\partial v_{\mathrm{dc1}}}{\partial\xi} + \frac{q_{\mathrm{d}}}{m_{\mathrm{d}}}\frac{\partial\phi_2}{\partial\xi} + \frac{\partial\psi_2}{\partial\xi} = 0, \ldots. \tag{15}$$

The same order-by-order analysis in various powers in ϵ from eq. (6) yields

$$\epsilon^{3/2}:\quad -\lambda\frac{\partial n_{\mathrm{dc1}}}{\partial\xi} + n_{\mathrm{dn0}}\frac{\partial v_{\mathrm{dc1}}}{\partial\xi} = 0, \tag{16}$$

$$\epsilon^{5/2}:\quad \frac{\partial n_{\mathrm{dc1}}}{\partial\tau} - \lambda\frac{\partial n_{\mathrm{dc2}}}{\partial\xi} + \frac{\partial}{\partial\xi}(n_{\mathrm{dc1}}v_{\mathrm{dc1}}) + n_{\mathrm{dn0}}\frac{\partial v_{\mathrm{dc2}}}{\partial\xi} = 0, \ldots. \tag{17}$$

Similarly, equating the like terms in various powers in ϵ from eq. (7), one gets

$$\epsilon^{1}:\quad n_{\mathrm{e1}} - n_{\mathrm{i1}} + \left(\frac{q_{\mathrm{d}}}{e}\right)n_{\mathrm{dc1}} = 0, \tag{18}$$

$$\epsilon^{2}:\quad \frac{\partial^2\phi_1}{\partial\xi^2} = 4\pi e\left[n_{\mathrm{e2}} - n_{\mathrm{i2}} + \left(\frac{q_{\mathrm{d}}}{e}\right)n_{\mathrm{dc2}}\right], \tag{19}$$

$$\epsilon^{3}:\quad \frac{\partial^2\phi_2}{\partial\xi^2} = 4\pi e\left[n_{\mathrm{e3}} - n_{\mathrm{i3}} + \left(\frac{q_{\mathrm{d}}}{e}\right)n_{\mathrm{dc3}}\right], \ldots. \tag{20}$$

Finally, the order-by-order analysis in various powers in ϵ from eq. (8) gives

$$\epsilon^{1}:\quad n_{\mathrm{dc1}} + n_{\mathrm{dn1}} = 0, \tag{21}$$

$$\epsilon^{2}:\quad \frac{\partial^2\psi_1}{\partial\xi^2} = 4\pi G m_{\mathrm{d}}[n_{\mathrm{dc2}} + n_{\mathrm{dn2}}], \tag{22}$$

$$\epsilon^{3}:\quad \frac{\partial^2\phi_2}{\partial\xi^2} = 4\pi G m_{\mathrm{d}}[n_{\mathrm{dc3}} + n_{\mathrm{dn3}}], \ldots. \tag{23}$$

We apply the systematic method of elimination and simplification in eqs (10)–(23) to get the following form of Korteweg–de Vries (KdV) equation [6] for the lowest order self-gravitational potential fluctuation ψ_1 with $\alpha = 8\pi G m_{\mathrm{d}} n_{\mathrm{dn0}}/\lambda^4$ as

$$\alpha\lambda \frac{\partial \psi_1}{\partial \tau} + \alpha\left(\frac{n_{\mathrm{dn0}}}{n_{\mathrm{dc0}}} - 1\right)\psi_1 \frac{\partial \psi_1}{\partial \xi} - \frac{\partial^3 \psi_1}{\partial \xi^3} = 0\cdot \tag{24}$$

We are interested in time-stationary solutions and eq. (24) is transformed into an ordinary differential equation (ODE) with Galilean type of transformation $x = (\xi - \lambda\tau)$ so that $\partial/\partial\tau \equiv -\lambda\partial/\partial x$ and $\partial/\partial\xi \equiv \partial/\partial x$. Equation (24), therefore, with equilibrium neutral-to-charged dust density ratio $\delta_0 = n_{\mathrm{dn0}}/n_{\mathrm{dc0}}$ and perturbed self-gravitational potential $\psi_1 = \Psi$ gets transformed into the following stationary form:

$$\alpha\lambda^2 \frac{\partial \Psi}{\partial x} + \alpha(1 - \delta_0)\,\Psi \frac{\partial \Psi}{\partial x} + \frac{\partial^3 \Psi}{\partial x^3} = 0\cdot \tag{25}$$

The present form of KdV equation (25) has already been analytically integrated [6] by imposing appropriate boundary conditions, viz. $\Psi \to 0$, $\partial\Psi/\partial x \to 0$, $\partial^2\Psi/\partial x^2 \to 0$ at $x \to \pm\infty$ (asymptotically), for localized nonlinear perturbations. The approximate solution of eq. (25) is, therefore, directly written with all usual notations as follows:

$$\Psi(\xi, \tau) = \Psi_0 \,\mathrm{sech}^2\left[(\xi - \lambda\tau)/\Delta_s\right], \tag{26}$$

which represents a KdV soliton of amplitude $\Psi_0 = 3\lambda^3/(1 - \delta_0)$ and width $\Delta_s = \sqrt{4/\alpha\lambda^3}$. This immediately implies that our self-gravitating dusty plasma system supports soliton structures in terms of the lowest-order self-gravitational potential fluctuation.

Equation (25) can, in addition, be numerically integrated to understand the basic features of the lowest-order fluctuations in more detail. It is seen that the KdV coefficients in eq. (25) are characteristics of our defined self-gravitating dusty plasma system in terms of dust grain mass (m_{d}) and Mach flow (λ). It is the mathematical construct contributed by the collective dynamics of the self-gravitating massive dust grains amidst the integrated interplay of diverse nonlinear (hydrodynamic in origin) and dispersive (self-gravitational in origin) effects.

4. Results and discussions

The nonlinear stability of a self-gravitating dust molecular cloud with partial ionization is studied with multifluid approach. It is found that the lowest-order self-gravitational potential fluctuation evolves dynamically like a KdV soliton through gravito-electrostatic interaction. The self-gravitational fluctuation arises due to the perturbation dynamics of the relatively massive inertial dust grains (in point-mass approximation) only. This, however, is quite idealistic as plasma thermal species such as electrons and ions are assumed to be inertialess (Boltzmann density distributions). In a time-stationary form for plane-wave

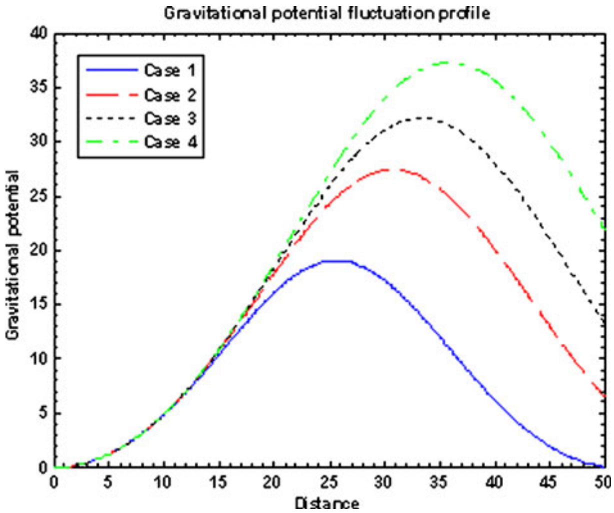

Figure 1. Variation of gravito-electrostatic potential fluctuation (on the units of plasma thermal potential) with distance (on the units of Jeans length) for different Mach numbers corresponding to Case (1) $\lambda = 0.10$, Case (2) $\lambda = 0.12$, Case (3) $\lambda = 0.13$ and Case (4) $\lambda = 0.14$. The other values kept fixed are $x_{\rm i} = 0.01\lambda_J$, $n_{\rm dc0} = 10^3$ m^{-3}, $n_{\rm dn0} = 10^5$ m^{-3} and $m_{\rm d} = 10^{-5}$ kg.

fluctuation, we study the steady state evolution of the gravito-acoustic potential perturbation analytically and numerically. The resulting numerical profiles for different plasma input conditions are presented in figures 1–6.

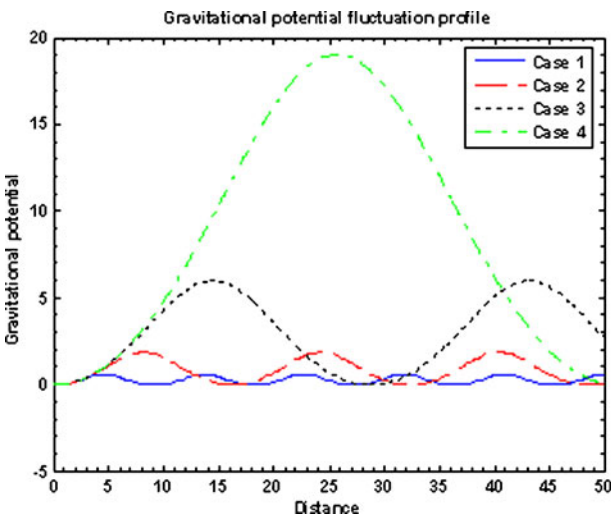

Figure 2. Same as figure 1 for Case (1) $n_{\rm dc0} = 1$ m^{-3}, Case (2) $n_{\rm dc0} = 10$ m^{-3}, Case (3) $n_{\rm dc0} = 10^2$ m^{-3} and Case (4) $n_{\rm dc0} = 10^3$ m^{-3}. The other values kept fixed are $\lambda = 0.1$, $x_{\rm i} = 10^{-2}\lambda_J$, $n_{\rm dn0} = 10^5$ m^{-3} and $m_{\rm d} = 10^{-5}$ kg.

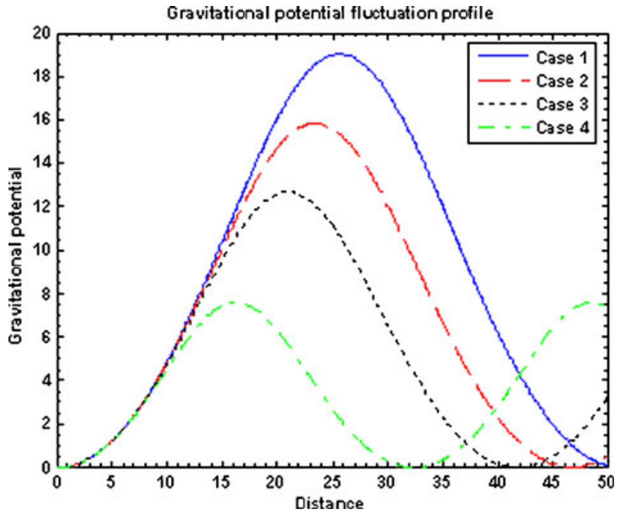

Figure 3. Same as figure 1 for Case (1) $n_{\mathrm{dn0}} = 10^5\ \mathrm{m}^{-3}$, Case (2) $n_{\mathrm{dn0}} = 1.2 \times 10^5\ \mathrm{m}^{-3}$, Case (3) $n_{\mathrm{dn0}} = 1.5 \times 10^5\ \mathrm{m}^{-3}$ and Case (4) $n_{\mathrm{dn0}} = 2.5 \times 10^5\ \mathrm{m}^{-3}$, respectively. The other values kept fixed are $\lambda = 0.1$, $n_{\mathrm{dc0}} = 10^3\ \mathrm{m}^{-3}$, $x_{\mathrm{i}} = 10^{-2}\lambda_J$ and $m_{\mathrm{d}} = 10^{-5}$ kg.

Figure 1 shows the graphical variation of gravito-electrostatic potential fluctuation (on the units of plasma thermal potential) with distance (on the units of Jeans length) for different Mach numbers of gravito-electrostatic fluctuations corresponding to Case (1) $\lambda = 0.10$, Case (2) $\lambda = 0.12$, Case (3) $\lambda = 0.13$ and Case (4) $\lambda = 0.14$. The other values kept fixed are $x_i = 0.01\lambda_J$, $n_{\mathrm{dc0}} = 10^3\,\mathrm{m}^{-3}$, $n_{\mathrm{dn0}} = 10^5\,\mathrm{m}^{-3}$, and $m_{\mathrm{d}} = 10^{-5}\,\mathrm{kg}$. It shows

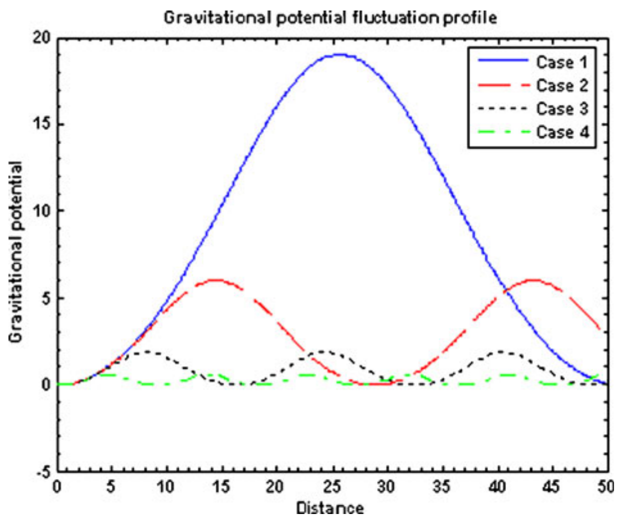

Figure 4. Same as figure 1 for Case (1) $m_{\mathrm{d}} = 10^{-5}$ kg, Case (2) $m_{\mathrm{d}} = 10^{-4}$ kg, Case (3) $m_{\mathrm{d}} = 10^{-3}$ kg and Case (4) $m_{\mathrm{d}} = 10^{-2}$ kg, respectively. The other values kept fixed are $\lambda = 0.1$, $n_{\mathrm{dc0}} = 10^3\ \mathrm{m}^{-3}$, $n_{\mathrm{dn0}} = 10^5\ \mathrm{m}^{-3}$ and $x_{\mathrm{i}} = 10^{-2}\lambda_J$.

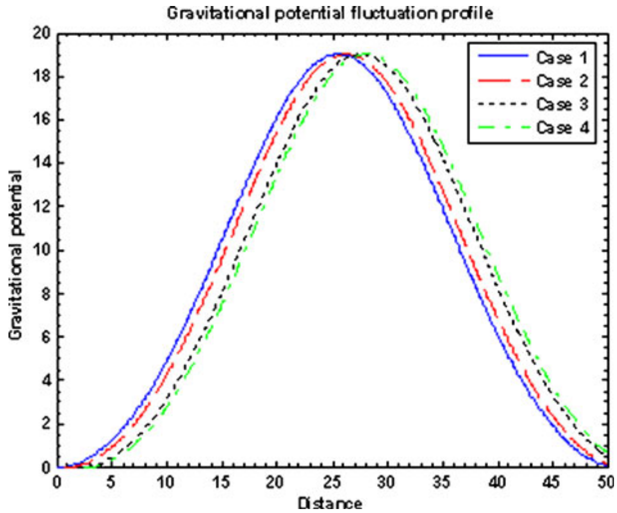

Figure 5. Same as figure 1 for Case (1) $x_i = 0.01\lambda_J$, Case (2) $x_i = 0.7\lambda_J$, Case (3) $x_i = 2.0\lambda_J$ and Case (4) $x_i = 2.5\lambda_J$. The other values kept fixed are $\lambda = 0.1$, $n_{dc0} = 10^3$ m^{-3}, $n_{dn0} = 10^5$ m^{-3} and $m_d = 10^{-5}$ kg.

extended soliton-like structures of different amplitudes in terms of the lowest order self-gravitational potential fluctuation contributed by the self-gravitational interaction of the relatively massive dust grains. It is clear that the fluctuation amplitude appears to increase with increase in flow velocity and vice versa.

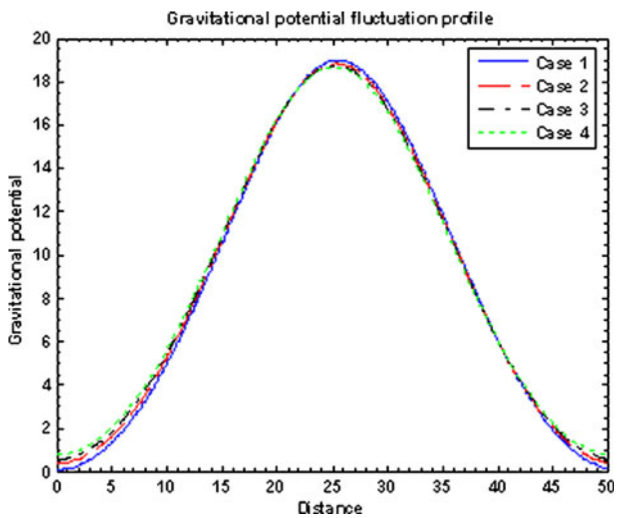

Figure 6. Same as figure 1 for Case (1) $\Psi_i = 0.01$, Case (2) $\Psi_i = 0.4$, Case (3) $\Psi_i = 0.6$ and Case (4) $\Psi_i = 0.8$, respectively. The other values kept fixed are $x_i = 10^{-2}\lambda_J$, $\lambda = 0.1$, $n_{dc0} = 10^3$ m^{-3}, $n_{dn0} = 10^5$ m^{-3} and $m_d = 10^{-5}$ kg.

Figure 2 depicts the same as figure 1 for Case (1) $n_{\rm dc0} = 1$ m^{-3}, Case (2) $n_{\rm dc0} = 10$ m^{-3}, Case (3) $n_{\rm dc0} = 10^2$ m^{-3} and Case (4) $n_{\rm dc0} = 10^3$ m^{-3}. The other values kept fixed are $\lambda = 0.1$, $x_{\rm i} = 10^{-2}\lambda_J$, $n_{\rm dn0} = 10^5$ m^{-3} and $m_{\rm d} = 10^{-5}$ kg. It shows extended soliton-like structures as well as soliton chain of different amplitudes. It is interesting to note here that the fluctuation amplitude decreases with increase in equilibrium-charged dust population density and vice versa. This physically means that electrostatic repulsive pressure among the like charged grains increases with their population density against self-gravitational attractive pressure. This, in turn, decreases the fluctuation level of the self-gravitational interaction among them. This is in accordance with the basic principle of gravito-electrostatic coupling in a self-gravitating dusty plasma model, as reported earlier [2–5].

Figure 3 shows the same as figure 1 for Case (1) $n_{\rm dn0} = 10^5$ m^{-3}, Case (2) $n_{\rm dn0} = 1.2 \times 10^5$ m^{-3}, Case (3) $n_{\rm dn0} = 1.5\times 10^5$ m^{-3} and Case (4) $n_{\rm dn0} = 2.5 \times 10^5$ m^{-3}. The other values kept fixed are $\lambda = 0.1$, $n_{\rm dn0} = 10^3$ m^{-3}, $x_{\rm i} = 10^{-2}\lambda_J$ and $m_{\rm d} = 10^{-5}$ kg. It shows extended soliton-like structures as well as chain of solitons of different amplitudes of 8–19 units (in terms of the plasma thermal potential). The amplitude decreases with increase in neutral dust population density, thereby the charged dust population density also increases due to collision processes in plasma. Thus the electrostatic repulsion among the charged dust grains, as in figure 2, decreases the fluctuation amplitude against self-gravitational attraction among them.

Figure 4 shows the same as figure 1 for Case (1) $m_{\rm d} = 10^{-5}$ kg, Case (2) $m_{\rm d} = 10^{-4}$ kg, Case (3) $m_{\rm d} = 10^{-3}$ kg and Case (4) $m_{\rm d} = 10^{-2}$ kg. The other values kept fixed are $\lambda = 0.1$, $n_{\rm dc0} = 10^3$ m^{-3}, $n_{\rm dc0} = 10^5$ m^{-3} and $x_{\rm i} = 10^{-2}\lambda_J$. It shows extended soliton-like structures as well as chain of solitons of different amplitudes of 1–20 units (on the units of plasma thermal potential). It is interesting to notice that the fluctuation amplitude gradually decreases with increase in equilibrium value of grain mass and vice versa. This basically happens because of the electrostatic repulsion among the charged dust grains in spite of their relatively larger inertial masses.

Figure 5 shows the same as figure 1 for Case (1) $x_{\rm i} = 0.01\lambda_J$, Case (2) $x_i = 0.7\lambda_J$, Case (3) $x_{\rm i} = 2.0\lambda_J$ and Case (4) $x_{\rm i} = 2.5\lambda_J$. The other values kept fixed are $\lambda = 0.1$, $n_{\rm dc0} = 10^3$ m^{-3}, $n_{\rm dn0} = 10^5$ m^{-3} and $m_{\rm d} = 10^{-5}$ kg. It shows extended soliton-like structures of amplitude of 18 units (on plasma thermal potential) at a distance of 25 units (on Jeans-scale) from the centre of the dusty plasma mass distribution. The fluctuation amplitude level, however, structurally remains almost the same with variation of input position values except undergoing some localized translations as usually expected.

Figure 6 is similar to figure 1 for Case (1) $\Psi_{\rm i} = 0.01$, Case (2) $\Psi_{\rm i} = 0.4$, Case (3) $\Psi_{\rm i} = 0.6$ and Case (4) $\Psi_{\rm i} = 0.8$. The other values kept fixed are $x_{\rm i} = 10^{-2}\lambda_J$, $\lambda = 0.1$, $n_{\rm dc0} = 10^3$ m^{-3}, $n_{\rm dn0} = 10^5$ m^{-3} and $m_{\rm d} = 10^{-5}$ kg. Here too, observations very similar to figure 5, are noticed. The main results thus arrived at, both analytically as well as numerically, are summarized in brief as follows.

(1) A nonlinear model for self-gravitational fluctuation study in a self-gravitating multi-fluid dusty plasma system under hydrostatic kind of homogeneous equilibrium configuration is developed.
(2) The dynamical evolution of the lowest-order self-gravitational fluctuations, arising mainly due to relatively massive dust grains (within point-mass approximation), in a self-gravitating dust molecular cloud is governed by a KdV type of equation.

(3) Solitary waves, solitons and soliton chains of self-gravitational origin are found to exist. This is in qualitative agreement with other reported results with different model methodologies [1,7,9,13].
(4) Various nonlinear structures are contributed mainly due to the collective dynamics of the massive dust grains' self-gravity fluctuation under the interplay of diverse nonlinear (hydrodynamic source) and dispersive (self-gravitational source) effects of internal origin in the plasma system. Self-gravitational inflows (contraction due to gravitational attraction) and outflows (expansion due to electrostatic repulsion) of the dust grains are responsible for the periodic nature of the soliton chains (figures 2–4).
(5) The additional presence of non-thermal (fast) ions may, under multi-ion temperature model, also allow compressive and rarefactive solitonic structures of self-gravitational origin to co-exist. As a result, one species is compressed whereas the other is rarefied, allowing the system to reach a mass neutral point outside equilibrium in a gas-dynamic description. This may, in addition, be of astrophysical importance to electrostatic solitonic structures too, as observed by Freja satellite and Viking spacecraft [6,13].
(6) However, it is well known that self-gravitationally sensitive dust grain-like impurity ions (DGIIs) of different scale size are found to exist in a dispersed phase (low density) even in the solar plasma system. Thus plasma-based gravito-electrostatic sheath (GES) model [12,15] for the steady-state description of solar wind plasma (SWP) in the presence of various dispersed DGIIs may also be extended for further studies as in plasma situations, but not so ideal.
(7) Lastly, by incorporating the lowest order inertial correction of plasma thermal species [11] in the presence of rotation and magnetic field asymptotically, we may have different characteristics of the modified fluctuation structures. Moreover, by applying electron inertial correction on this Jeans gravitational instability [14] with and without the presence of electromagnetic forces, we may get a more realistic stability picture.

5. Conclusions

By applying the basic methodology of the standard reductive perturbation technique over a well-defined equilibrium in a self-gravitating dusty plasma system with partial ionization of astrophysical interest, a KdV equation is developed to study nonlinear stability of the self-gravitational dusty plasma system. The main conclusions drawn from our present analyses are as follows:

(1) Nonlinear gravitational fluctuations in the self-gravitating dust molecular cloud are governed by a KdV type of equation obtained by multifluid model, perturbatively.
(2) Various nonlinear structures, like extended solitons and soliton chains, are found to be supported in such a system.
(3) The structures are contributed by the collective dynamics of the massive inertial dust grains' self-gravity fluctuation amidst an integrated interplay of diverse nonlinear (hydrodynamic cause) and dispersive (self-gravitational cause) effects.

The reductive perturbation method is, however, not very popular as a mathematically rigorous perturbation method, even conditionally. Nevertheless, it is a convenient, approximate and easy method to produce certain mathematically interesting paradigm of nonlinear equations. By the free ordering, we may get almost any explicit form of results as expected. In particular, the ordering required to get, for example, a soliton solution is now reached. Numerical solutions show that there are many other soliton solutions that do not satisfy the 'required ordering'. In fact, although solitons are frequently found experimentally, a soliton that satisfies the KdV ordering has never been found, whether in neutral fluid, lattice, or plasma. Thus, analytically, it provides a new mathematical stimulus scope for future interest to derive analytical results with greater accuracy with newer mathematical techniques so as to get more detailed picture of self-gravitational fluctuations.

These mathematical analyses may be extended for further investigation of fluctuation and stability with more realistic assumptions like grain rotations, spatial inhomogeneities, different gradient forces, etc., taken into account in other astrophysical and space environments. These calculations, although tentative for any concrete application to any sharply specified stellar formation mechanism, may be extensively useful in the study of fluctuation-induced dynamics with electrostatic charge fluctuation of dust grains in astrophysical environment of dusty plasmas in the complex form of self-gravitationally collapsing dust cloud.

Acknowledgements

The financial support received from the University Grants Commission (UGC) of New Delhi (India), through the research project F. No. 34-503/2008 (SR), is thankfully acknowledged for carrying out this work. In addition, the valuable comments and suggestions by the anonymous referee, to refine the original manuscript into the present improved form, are also gratefully acknowledged.

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