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Quantum ion-acoustic solitary waves in weak relativistic plasma

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Abstract. Small amplitude quantum ion-acoustic solitary waves are studied in an unmagnetized two-species relativistic quantum plasma system, comprised of electrons and ions. The one-dimensional quantum hydrodynamic model (QHD) is used to obtain a deformed Korteweg–de Vries (dKdV) equation by reductive perturbation method. A linear dispersion relation is also obtained taking into account the relativistic effect. The properties of quantum ion-acoustic solitary waves, obtained from the deformed KdV equation, are studied taking into account the quantum mechanical effects in the weak relativistic limit. It is found that relativistic effects significantly modify the properties of quantum ion-acoustic waves. Also the effect of the quantum parameter H on the nature of solitary wave solutions is studied in some detail.

Keywords. Ion-acoustic solitary waves; relativistic plasma.

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1. Introduction

Quantum effects in plasma has been studied recently by a number of authors [1]. This study is of considerable physical importance as it has applications in many aspects of plasma like quantum echo [2], dense plasma, particularly in astrophysics and cosmological studies [3–6], quantum plasma instabilities in Fermi gases [7], quantum Landau damping [8] etc. There are various models to study the quantum effects in plasma, for example, the Wigner–Poisson system [9–11] which involves an integro-differential system and the popular QHD model. The QHD model [12–16] can be considered as an extension of the usual fluid model of plasma. The extension is incorporated by adding an extra term in the equation of motion, the extra term being referred as Bohm potential or quantum potential [12]. According to the Bohm interpretation of quantum mechanics, the wave function Ψ can be written as $\Psi = R\ \exp(iS/\hbar)$. Use of Schrödinger equation then results in a modified Hamilton–Jacobi equation for S which differs from the usual classical one by having an extra term which is called the Bohm potential. In ultra-small electronic device, the QHD model describes negative differential resistance in resonant tunnelling diodes and ultra-small high electron mobility transistors [17,18]. Quantum corrections are clearly contained within the Bohm potential, which can be obtained from the moments of the nonrelativistic Wigner function [19]. But a correct treatment of quantum effects should rely on the moments of a relativistic Wigner function such as the one described in refs [20,21]. So it is relevant to deal with the relativistic regime with the QHD model. Bohm's nonrelativistic approach to quantum mechanics can be generalized to the relativistic domain. The nonrelativistic concept of Bohm potential transcribed into the corresponding quantum force. This quantum force can be deduced from the theory of relativistic Schrödinger equations in a similar way as the original quantum potential is deduced from the ordinary Schrödinger equation. The leading term of the new quantum force turns out to be the well-known classical Lorentz force of electrodynamics, but modified by some quantum corrections. Haas *et al* [22] used the QHD model to study quantum ion-acoustic waves in the weakly nonlinearized theory and obtained a deformed Korteweg–de Vries (dKdV) equation involving the parameter H, proportional to the Planck's constant $\hbar$. They observed several characteristic features of pure quantum origin for the linear, weakly nonlinear and fully nonlinear waves. Garcia *et al* [23] derived modified Zakharov equations for plasmas with a quantum correction. They described the nonlinear interaction between quantum Langmuir waves and quantum ion-acoustic waves. Recently, Stenflo *et al* [24] observed two new low-frequency electrostatic modes in ultra-cold unmagnetized quantum dusty plasmas. Ali and Shukla [25] studied dust-acoustic solitary waves in quantum plasma. Taibany and Wadati [26] used the QHD model for plasmas to study the properties of the nonlinear quantum dust-acoustic waves in a nonuniform ultra-cold Fermi dusty gas composed of inertialess electrons, and ions as well as negatively charged inertial dust grains. Marklund and Brodin [27] have investigated the significance of the spin contribution in quantum plasma. Recently, Sahu and Roychoudhury [28] investigated quantum acoustic solitary waves and shock waves in planar and nonplanar geometries. But, to the best of my knowledge, the study of quantum ion-acoustic waves in relativistic plasma has not yet been done. Relativistic effect cannot be neglected [29] in the formation of solitary waves when the speed of particles is comparable to that of light. In that case, relativistic effects may significantly modify the solitary wave's behaviour. For example, ions with very high speed are frequently observed in the solar atmosphere, in interplanetary space and also in dense plasmas, where quantum effect is significant and relativistic effects are of importance [30]. Relativistic plasmas occur in a variety of situations, e.g., in space–plasma phenomena [31], laser–plasma interactions [32], plasma sheet boundary layer of Earth's magnetosphere [33], in the Van Allen radiation belts [34] etc. Streaming ions with energies ranging from 0.1 to 100 MeV are observed in solar atmosphere and interstellar space. In the present paper we have studied the quantum ion-acoustic solitary waves in an unmagnetized weak relativistic plasma. It must be mentioned that most of the studies in relativistic nonquantum plasma did not take into account the relativistic component in the continuity equation, as was recently pointed out by Lee and Choi [35]. The organization of the paper is as follows. In §2 we present the QHD equations and dispersion relation for two-species relativistic quantum plasma system, comprising electrons and ions. In §3 we derive the deformed KdV equations in relativistic plasma along with its stationary solutions and numerical results, valid for small-amplitude waves in the weak relativistic limit, and §4 provides the conclusion.

2. Governing equations and linear waves

We considered an unmagnetized relativistic quantum plasma system comprising electrons and ions and investigated the nonlinear propagation of ion-acoustic solitary waves. The one-dimensional quantum hydrodynamic mode consists of the continuity and momentum balance equations for both electrons and ions in relativistic plasma together with the Poisson's equation for the self-consistent potential. The nonlinear dynamics of the ion-acoustic waves in relativistic quantum plasma system is governed by

$$\frac{\partial}{\partial t}(\gamma_e n_e) + \frac{\partial}{\partial x}(\gamma_e n_e u_e) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(\gamma_i n_i) + \frac{\partial}{\partial x}(\gamma_i n_i u_i) = 0, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + u_e\frac{\partial}{\partial x}\right)\gamma_e u_e = \frac{e}{m_e}\frac{\partial \phi}{\partial x} - \frac{1}{m_e n_e}\frac{\partial p_e}{\partial x} + \frac{\hbar^2}{2m_e^2}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_e}/\partial x^2}{\sqrt{n_e}}\right), \tag{3}$$

$$\left(\frac{\partial}{\partial t} + u_i\frac{\partial}{\partial x}\right)\gamma_i u_i = -\frac{e}{m_i}\frac{\partial \phi}{\partial x} + \frac{\hbar^2}{2m_i^2}\frac{\partial}{\partial x}\left(\frac{\partial^2\sqrt{n_i}/\partial x^2}{}\sqrt{n_i}\right), \tag{4}$$

$$\frac{\partial^2\phi}{\partial x^2} = \frac{e}{\varepsilon_0}(\gamma_e n_e - \gamma_i n_i), \tag{5}$$

where $\gamma_e = \left(1 - \frac{{u_e}^2}{c^2}\right)^{-1/2} \simeq 1 + \frac{{u_e}^2}{2c^2}$, $\gamma_i = \left(1 - \frac{{u_i}^2}{c^2}\right)^{-1/2} \simeq 1 + \frac{{u_i}^2}{2c^2}$ (in the weak relativistic limit); $n_e, u_e, m_e, -e$ (n_i, u_i, m_i, e) are the electron (ion) density field, velocity field, mass and charge, respectively; and ε_0 and $\hbar$ are the dielectric and Planck constant divided by 2π, respectively. ϕ is the electrostatic wave potential, p_e is the pressure effects for electrons, μ_e and μ_i are the electron and ion kinematic viscosity respectively. Pressure effects for ions are neglected for simplicity. Also the so-called Bohm potential term is left undisturbed. We assumed the electrons to obey the equation of state pertaining to a one-dimensional zero temperature Fermi gas [7],

$$p_e = \frac{m_e v_{\mathrm{Fe}}^2}{3n_0^2} n_e^3\,, \tag{6}$$

where n_0 is the equilibrium density for both electrons and ions, and v_{Fe} is the electronic Fermi velocity connected to the Fermi temperature T_{Fe} by $m_e v_{\mathrm{Fe}}^2/2 = k_{\mathrm{B}}T_{\mathrm{Fe}}$, k_{B} is the Boltzmann's constant. Now we introduce the following normalization:

$$\bar{x} = \omega_{pi}x/t, \quad \bar{t} = \omega_{pi}t, \quad \bar{n}_e = n_e/n_0, \quad \bar{n}_i = n_i/n_0\,,$$

$$\bar{u}_e = u_e/c_s, \quad \bar{u}_i = u_i/c_s, \quad \bar{\phi} = e\phi/(2k_{\mathrm{B}}T_{\mathrm{Fe}})\,, \tag{7}$$

where ω_{pe} and ω_{pi} are the corresponding electron and ion plasma frequencies,

$$\omega_{pe} = \left(\frac{n_0 e^2}{m_e \varepsilon_0}\right)^{1/2}, \quad \omega_{pi} = \left(\frac{n_0 e^2}{m_i \varepsilon_0}\right)^{1/2},$$

c_s is the quantum ion-acoustic velocity given by

$$c_s = \left(\frac{2k_{\mathrm{B}} T_{\mathrm{Fe}}}{m_i}\right)^{1/2}.$$

We have denoted nondimensional quantum parameter

$$H = \frac{\hbar \omega_{pe}}{2k_{\mathrm{B}} T_{\mathrm{Fe}}} (>0),$$

where H is a measure of the quantum diffraction effects. Physically, H is the ratio between the electron plasmon energy and the electron Fermi energy. Notice that H^2 is proportional to the r_s parameter of the electron gas, which is the Wigner–Seitz radius in units of the Bohr radius; r_s takes on values in the range 2–6 for metallic electrons.

Using the above normalization we obtain from eqs (3) and (4) (dropping bars)

$$\frac{m_e}{m_i}\left[\left(\frac{\partial}{\partial t} + u_e \frac{\partial}{\partial x}\right)\gamma_e u_e\right] = \frac{\partial \phi}{\partial x} - n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_e}/\partial x^2}{\sqrt{n_e}}\right), \quad (8)$$

$$\left(\frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x}\right)\gamma_i u_i = -\frac{\partial \phi}{\partial x} + \frac{m_e}{m_i}\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{\partial^2 \sqrt{n_i}/\partial x^2}{\sqrt{n_i}}\right). \quad (9)$$

Due to small electron inertia ($m_e/m_i \ll 1$), integrating eq. (8) once and assuming the boundary conditions $n_e = 1$, $\phi = 0$ at infinity, we get

$$\phi = -\frac{1}{2} + \frac{n_e^2}{2} - \frac{H^2}{2\sqrt{n_e}}\frac{\partial^2}{\partial x^2}\sqrt{n_e}\,. \quad (10)$$

This equation gives the electrostatic potential in terms of electron density and its derivatives. In the ion momentum equation (9), the quantum diffraction term may be neglected due to $m_e/m_i \ll 1$ because the de Broglie wavelength is inversely proportional to mass. The larger the de Broglie wavelength in comparison with the typical dimension of the system, the larger are the quantum diffraction effects.

Now, the continuity eqs (1) and (2), momentum equation (9) and Poisson's equations become

$$\frac{\partial}{\partial t}(\gamma_e n_e) + \frac{\partial}{\partial x}(\gamma_e n_e u_e) = 0, \quad (11)$$

$$\frac{\partial}{\partial t}(\gamma_i n_i) + \frac{\partial}{\partial x}(\gamma_i n_i u_i) = 0, \quad (12)$$

$$\left(\frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x}\right)\gamma_i u_i = -\frac{\partial \phi}{\partial x}, \quad (13)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \gamma_e n_e - \gamma_i n_i. \quad (14)$$

Equations (11)–(14) and (10) are the five basic equations with five unknown quantities n_i, n_e, u_i, u_e and ϕ.

2.1 *Linear waves*

To investigate the linear properties of quantum ion-acoustic waves in relativistic plasma, we introduced the linear perturbations such as $n_e = 1+\epsilon n_e^{(1)}$, $n_i = 1+\epsilon n_i^{(1)}$, $u_i = u_{0i}+\epsilon u_i^{(1)}$, $u_e = u_{0e}+\epsilon u_e^{(1)}$, $\phi = \epsilon\phi^{(1)}$ in eqs (10)–(14). Then the normalized equations are

$$\phi^{(1)} = n_e^{(1)} - \frac{H^2}{4}\frac{\partial^2 n_e^{(1)}}{\partial x^2}, \tag{15}$$

$$\frac{\partial}{\partial t}\left(\beta_{1i} n_i^{(1)} + \frac{u_{0i}}{c^2}u_i^{(1)}\right) + \frac{\partial}{\partial x}\left(\beta_{1i}u_i^{(1)} + u_{0i}\beta_{1i}n_i^{(1)} + \frac{u_{0i}^2}{c^2}u_i^{(1)}\right) = 0, \tag{16}$$

$$\frac{\partial}{\partial t}\left(\beta_{1e} n_e^{(1)} + \frac{u_{0e}}{c^2}u_e^{(1)}\right) + \frac{\partial}{\partial x}\left(\beta_{1e}u_e^{(1)} + u_{0e}\beta_{1e}n_e^{(1)} + \frac{u_{0e}^2}{c^2}u_e^{(1)}\right) = 0, \tag{17}$$

$$\beta_i\frac{\partial u_i^{(1)}}{\partial t} + u_{0i}\beta_i\frac{\partial u_i^{(1)}}{\partial x} = -\frac{\partial\phi^{(1)}}{\partial x}, \tag{18}$$

$$\frac{\partial^2\phi^{(1)}}{\partial x^2} = \beta_{1e}n_e^{(1)} - \beta_{1i}n_i^{(1)} + \frac{u_{0e}^2}{2c^2}(1+2u_e^{(1)}) - \frac{u_{0i}^2}{2c^2}(1+2u_i^{(1)}), \tag{19}$$

where

$$\beta_{1i} = 1 + \frac{u_{0i}^2}{2c^2}, \quad \beta_i = 1 + \frac{3u_{0i}^2}{2c^2},$$

$$\beta_{1e} = 1 + \frac{u_{0e}^2}{2c^2}, \quad \beta_e = 1 + \frac{3u_{0e}^2}{2c^2}.$$

Assuming the plane-wave solution of the form $\exp[i(kx-\omega t)]$, where k is the wave number and ω is the frequency, we obtain the dispersion relation as

$$\begin{aligned} k^2\left(1+\frac{H^2k^2}{4}\right) = {} & \frac{\left(\beta_{1i}k^2 - \frac{ku_{0i}}{c^2}(\omega - ku_{0i})\right)(1+\frac{H^2k^2}{4})}{\beta_i(\omega-ku_{0i})^2} - \beta_{1i} \\ & + \frac{\frac{ku_{0i}}{c^2}(1+\frac{H^2k^2}{4})}{\beta_i(\omega-ku_{0i})} - \frac{\frac{u_{0e}}{c^2}\beta_{1e}(\omega-ku_{0e})}{\beta_{1e}k - \frac{u_{0e}}{c^2}(\omega-ku_{0e})}. \end{aligned} \tag{20}$$

From eq. (20) it is seen that the phase velocity is dependent on the quantum correction and relativistic effect. In the absence of relativistic effect, we retrieve the dispersion relation of [22] for quantum ion-acoustic waves. But, it is evident from eq. (20) that wave dispersion effects appear due to the inclusion of both relativistic and Bohm potential effects of electrons in quantum plasmas. The dispersion effect due to Bohm potential in quantum plasmas depend upon the value of quantum diffraction parameter H and the relativistic factor (u_{0i}/c). Figure 1 shows the linear variation of normalized wave frequency ω as a function of normalized wave number k for different values of u_{0i}/c. It is seen that the wave frequency of oscillation increases with u_{0i}/c. Therefore, it seems important to study the effects of relativistic streaming factor on quantum ion-acoustic solitary waves.

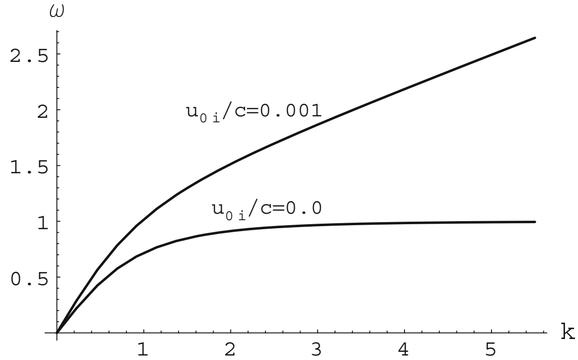

Figure 1. Plot of dispersion relation (20) for several values of u_{0i}/c (with $c = 300$), where $u_{0e}/c = 0.01$, $H = 0.5$.

3. Nonlinear small-amplitude quantum ion-acoustic waves

To study the small-amplitude quantum ion-acoustic waves in relativistic plasma, we introduce the stretched coordinates

$$\xi = \varepsilon^{1/2}(x - Vt), \quad \tau = \varepsilon^{3/2}t$$

and expand n_i, n_e, u_i and ϕ in a power series of ε as

$$n_i = 1 + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \cdots, \tag{21}$$

$$n_e = 1 + \varepsilon n_e^{(1)} + \varepsilon^2 n_e^{(2)} + \cdots, \tag{22}$$

$$u_i = u_{0i} + \varepsilon u_i^{(1)} + \varepsilon^2 u_i^{(2)} + \cdots, \tag{23}$$

$$u_e = u_{0e} + \varepsilon u_e^{(1)} + \varepsilon^2 u_e^{(2)} + \cdots, \tag{24}$$

$$\phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \cdots. \tag{25}$$

Substituting eqs (21)–(25) into eqs (10)–(14), we obtain from the lowest order in ε,

$$n_e^{(1)} = \phi^{(1)}, \quad n_i^{(1)} = \frac{\beta_i - \frac{Vu_{0i}}{c^2}}{(V - u_{0i})^2 \beta_i \beta_{1i}} \phi^{(1)},$$

$$u_i^{(1)} = \frac{\phi^{(1)}}{(V - u_{0i})\beta_i}, \quad u_e^{(1)} = \frac{(V - u_{0e})\beta_{1e}}{\beta_e - \frac{Vu_{0e}}{c^2}} \phi^{(1)}$$

and

$$\frac{\beta_i - \frac{Vu_{0i}}{c^2}}{(V-u_{0i})^2\beta_i} + \frac{\frac{u_{0i}}{c^2}}{(V-u_{0i})\beta_i} = \beta_{1e} + \frac{\frac{u_{0e}}{c^2}(V-u_{0e})\beta_{1e}}{\beta_e - \frac{Vu_{0e}}{c^2}}.$$

Using the usual reductive perturbation technique we get a modified deformed Korteweg–de Vries equation for quantum ion-acoustic solitary waves

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{26}$$

where

$$A = \frac{q}{p} \quad \text{and} \quad B = \frac{\left\{1 - \left(\frac{\beta_i - \frac{Vu_{0i}}{c^2}}{(V-u_{0i})^2\beta_i} + \frac{\frac{u_{0i}}{c^2}}{(V-u_{0i})\beta_i}\right)(H^2/4)\right\}}{p} \tag{27}$$

with

$$p = \left\{\frac{\beta_i - \frac{Vu_{0i}}{c^2}}{(V-u_{0i})^2\beta_i} + \frac{u_{0i}/c^2}{(V-u_{0i})\beta_i}\right\}\left\{\frac{2}{V-u_{0i}} + \frac{u_{0e}/c^2}{\beta_e - (Vu_{0e}/c^2)}\right\}$$

and

$$\begin{aligned} q = &\left\{\frac{\beta_i - \frac{Vu_{0i}}{c^2}}{(V-u_{0i})^2\beta_i} + \frac{\frac{u_{0i}}{c^2}}{(V-u_{0i})\beta_i}\right\}\left\{1 + \frac{1}{(V-u_{0i})\beta_i} - \frac{3\frac{u_{0i}}{c^2}}{(V-u_{0i})\beta_i^2}\right. \\ &\left. + \frac{2(\beta_i - \frac{Vu_{0i}}{c^2})}{(V-u_{0i})^2\beta_i\beta_{1i}}\right\} - \frac{(V-u_{0e})^2\beta_{1e}^2}{c^2(\beta_e - \frac{Vu_{0e}}{c^2})^2}\left\{1 - \frac{\frac{u_{0e}}{c^2}(3u_{0e}-V)}{\beta_e - \frac{Vu_{0e}}{c^2}}\right\} \\ &+ \frac{1}{c^2(V-u_{0i})^2\beta_i^2}\left\{1 + \frac{3u_{0i}-V}{V-u_{0i}}\right\}. \end{aligned}$$

The stationary wave solution of the deformed KdV equation (26) is obtained by transforming the independent variable ξ and τ into the new variable $\eta = \xi - V_0\tau$ and $\tau' = \tau$, where V_0 is a constant speed normalized by c_s. For localized solutions, we imposed appropriate boundary conditions, namely $\phi^{(1)} \to 0$, $\frac{\partial \phi^{(1)}}{\partial \eta} \to 0$, $\frac{\partial^2 \phi^{(1)}}{\partial \eta^2} \to 0$ at $\eta \to \pm\infty$. The stationary solution of eq. (26) is

$$\phi^{(1)} = \phi_m \,\mathrm{sech}^2\left(\frac{\eta}{\Delta}\right) \tag{28}$$

with amplitude

$$\phi_m = \frac{3V_0}{A} \tag{29}$$

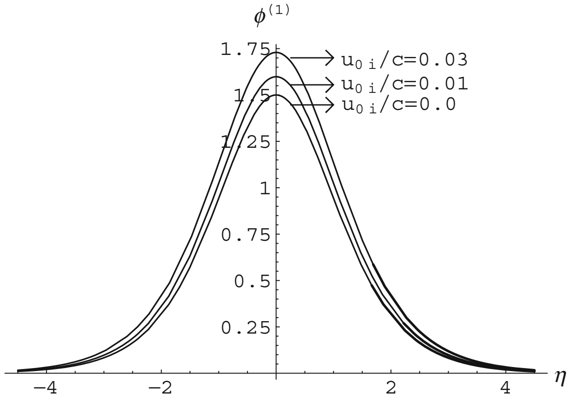

Figure 2. Variation of electrostatic potential $\phi^{(1)}$ against $\eta\,(=\xi - V_0\tau)$ for different values of u_{0i}/c (with $c = 300$), where $u_{0e}/c = 0.01$, $H = 0.1$, $V_0 = 1$.

and the width

$$\Delta = \sqrt{\frac{4B}{V_0}}\,. \tag{30}$$

Please note that the coefficients of nonlinear and dispersive terms A and B are modified because of the presence of quantum effects as well as relativistic effect. These terms contribute to the amplitude and width of the soliton, as is obvious from (29) and (30). So the formation of quantum ion-acoustic solitary waves are affected by quantum as well as relativistic effect. It is to be noted that in the nonrelativistic limit with $V = 1$, $u_{0i} = 0$, the critical value of H is 2. At this critical value the soliton solution collapses. In the presence of relativistic effect, this critical value differs slightly from 2 and is dependent on the parameter u_{0i}. Quantum diffraction affects the nature and characteristic of the soliton in a novel way, as the coefficient of dispersive term in eq. (26) is H-dependent. The nature of soliton solutions is studied for various values of H both below and above the critical

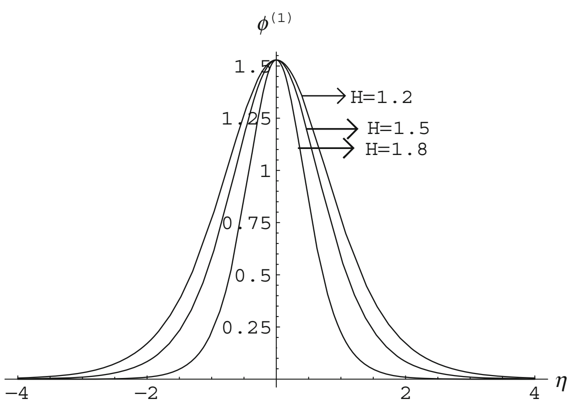

Figure 3. Variation of electrostatic potential $\phi^{(1)}$ against $\eta\,(= \xi - V_0\tau)$ for several values of H (< 2), where $u_{0i}/c = 0.01$, $u_{0e}/c = 0.01$ (with $c = 300$), $V_0 = 1$.

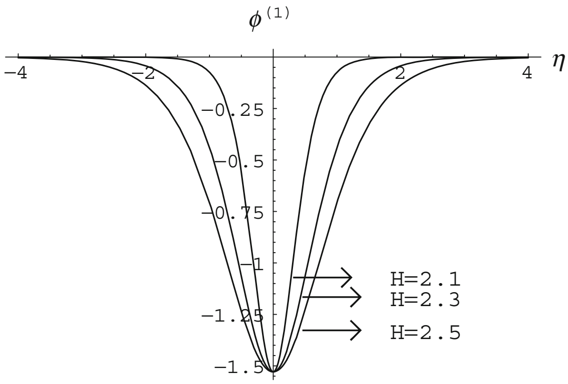

Figure 4. Variation of electrostatic potential $\phi^{(1)}$ against $\eta\,(=\,\xi\,-\,V_0\tau)$ for several values of H (>2), where $u_{0i}/c = 0.01$, $u_{0e}/c = 0.01$ (with $c = 300$) and $V_0 = -1$.

value. In figure 2 the electrostatic potential $\phi^{(1)}$ is plotted against $\eta(= \xi - V_0\tau)$ for several values of u_{0i}/c. It is clear that the relativistic effect plays an important role in the soliton formation. It is seen that both the amplitude and width of the solitary waves increase with the increase of u_{0i}/c. This indicates that relativistic streaming factor energizes the soliton. For $0 \leq H < 2$, the velocity V_0 must be positive, whereas for $H > 2$, V_0 must be negative; otherwise the soliton solution will be nullified. In figure 3 electrostatic potential $\phi^{(1)}$ is plotted against η for several values of $H(0 \leq H < 2)$, for given u_{0i}/c. It is seen that the amplitude of the solitary waves remains fixed but width decreases with the increase of H. In figure 4 electrostatic potential $\phi^{(1)}$ is plotted against η for several values of $H(>2)$, for given u_{0i}/c, where V_0 is taken to be negative for the existence of soliton. So, figures 3 and 4 show that quantum effects have no influence on the absolute amplitude of the soliton. This is expected as the amplitude of the electrostatic potential involves the nonlinear coefficient A that is independent of the quantum diffraction and the coefficient of dispersive term is H-dependent. Also, it is to be noted that both rarefactive and compressive solitary waves

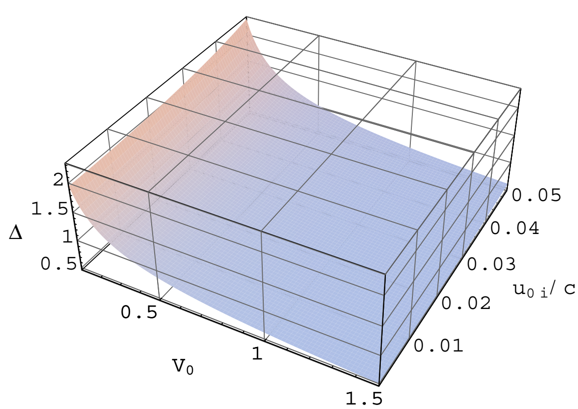

Figure 5. Plot of soliton's width with the combined effect of constant velocity V_0 and u_{0i}/c for a fixed value of $H = 1.8$ and $u_{0e}/c = 0.01$.

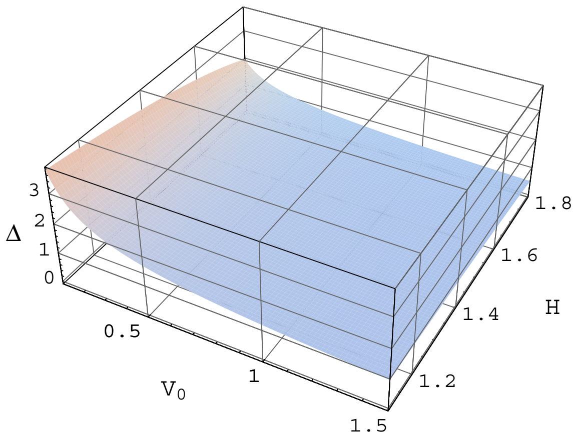

Figure 6. Plot of soliton's width with the combined effect of constant velocity V_0 and H for a fixed value of $u_{0i}/c = 0.01$ and $u_{0e}/c = 0.01$.

can be produced. There is a critical value of H (which depends upon other parameters) for which the soliton collapses. In particular, if we take $u_{0i}/c = 0.02$ and other parameters same as in figure 2, then the critical value of $H = 1.9998$. In figures 5 and 6, the variation of soliton's width with the combined effect of constant velocity V_0 and u_{0i}/c for a fixed value of H and the combined effect of constant velocity V_0 and H are depicted, respectively. It is observed that soliton's width decreases with the increase of V_0, but increases with the increase of u_{0i}/c. From figure 6 it is seen that the width decreases with the increase of both V_0 and H. The variation of width with the combined effect of u_{0i}/c and H is displayed in figure 7, which indicate that the width increases with the increase of u_{0i}/c, but decreases with the increase of H. This is because coefficient of nonlinear term in eq. (26) is dependent on relativistic streaming factor, but not on H, whereas the coefficient of dispersive term is dependent on relativistic streaming factor as well as on H.

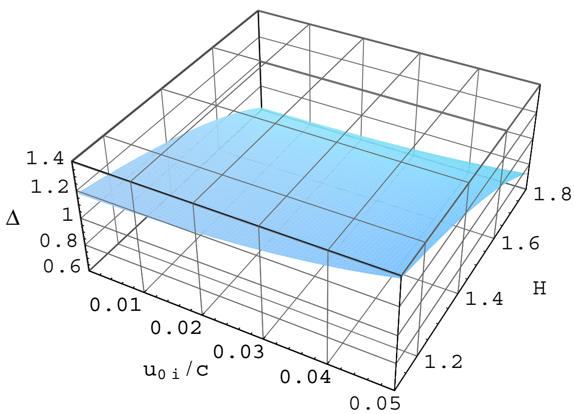

Figure 7. Plot of soliton's width with the combined effect of constant velocity u_{0i}/c and H for a fixed value of $V_0 = 1$ and $u_{0e}/c = 0.01$.

4. Conclusion

We have derived dKdV equation for quantum ion-acoustic waves in an unmagnetized two-species quantum plasma system, comprising electrons and ions in a relativistic plasma, in the weak relativistic limit. The standard reductive perturbation technique was used to derive dKdV equations. The dispersion relation was obtained by the plane-wave analysis of the basic set of equation. In the absence of relativistic streaming factor, we retrieved the dispersion relation of quantum ion-acoustic wave in electron–ion plasmas. The relativistic effect is clearly discernable in soliton formation. Another important point is that for small values of u_{0i}/c the relativistic effect is prominent in the dispersion relation as shown by figure 1. From analytical and numerical results, we have found that both the amplitude and the width of the nonlinear quantum ion-acoustic wave are affected by the quantum correction and relativistic streaming factor. To summarize, the present investigation shows that the weak relativistic effect plays a significant role in the propagation of small-amplitude quantum ion-acoustic solitary waves.

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