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Introduction to quantum chromodynamics at hadron colliders

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Abstract. A basic introduction to the application of QCD at hadron colliders is presented. I briefly review the phenomenological and theoretical origins of QCD, and then discuss factorization and infrared safety, parton distributions, the computation of hard scattering amplitudes and applications of perturbative QCD.

Keyword. Quantum chromodynamics.

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1. Introduction

In this talk I present a basic introduction to the application of QCD at hadron colliders. I shall start by briefly reviewing the origins of QCD in the quark model, the parton model, the development of non-Abelian gauge theory and the discovery of asymptotic freedom, and then discuss infrared safety, the factorization theorem and perturbative QCD.

2. Hadron classification and the quark model

The quark model [1,2] arose out of the effort to classify the many hadronic states, mesons and baryons, that were discovered in cosmic rays and at the new accelerators that were built after World War II. The fact that many of these particles decayed by the weak interaction implied that they were not mere excited states of known particles. While the particles could be classified into isospin multiplets, the 'strange' particles, K, Λ, Σ did not fit in as expected.

2.1 *Strangeness*

The configuration of isospin multiplets was explained by the introduction of the strangeness quantum number, S [3,4], which was conserved by the strong interaction, but violated by the weak interactions. Now classified by both strangeness and isospin, it was observed that the charge of a hadronic state was governed by the Gell-Mann–Nishijima relation,

$$Q = T_3 + \frac{1}{2}\left(B + S\right), \tag{1}$$

where T_3 is the eigenvalue of the diagonal isospin generator, B is the baryon number (± 1 for baryons, 0 for mesons) and S is the strangeness number. This has the interesting property that a linear combination of the quantum number of two approximate symmetries, isospin and strangeness, gives the electric charge which is the quantum number of an exact symmetry.

2.2 *SU*(3)

As more mesons were discovered, they continued to be classified into a bewildering array of isospin multiplets. Building on the Sakata model [5], which postulated that the proton, neutron and Lambda hyperon were the fundamental constituents of all hadrons, Gell-Mann and Ne'eman [6–8] pointed out the symmetry group obtained by combining isospin and strangeness was $SU(3)$. Discarding for the time the notion of fundamental constituents, they proposed $SU(3)$, in the abstract, as the governing principle of hadron classification and postulated that all the hadronic states could be classified into $SU(3)$ multiplets. The adjoint representation of $SU(3)$ is an octet. It was postulated that all hadronic states would fit into $SU(3)$ multiplets that were either octets or formed from combinations of octets. This model was therefore called 'the eightfold way'. While the scalar and vector mesons and the spin-$\frac{1}{2}$ baryons all fit into octet representations of $SU(3)$, the spin-$\frac{3}{2}$ baryons did not. There were nine known states, fitting into three isospin multiplets, the $\left(\Delta^-, \Delta^0, \Delta^+, \Delta^{++}\right)$, the $\left(\Sigma^{*-}, \Sigma^{*0}, \Sigma^{*+},\right)$ and the $\left(\Xi^{*-}, \Xi^{*0}\right)$. When two octet representations are combined in $SU(3)$, the resulting representations are: **27**, **10**, $\overline{\mathbf{10}}$, **8**, **8**, and **1**. The only possibilities were for the spin-$\frac{3}{2}$ baryons to fit into a decuplet or a 27-plet. In 1962, Gell-Mann and others predicted that multiplet was a decuplet and that the missing hyperon, which he called the Ω^-, would have strangeness $S = -3$, a mass of $\sim$1680 MeV, and would decay by the weak interaction. Just two years later, Samios and collaborators found the Ω^- [9], firmly establishing the value of the $SU(3)$ classification.

2.3 *The quark model*

The quark model [1,2] built upon the success of $SU(3)$ by reintroducing fundamental constituents for the hadrons. This time, however, instead of being the known particles of the Sakata model (p, n, Λ), which were now known to be parts of an $SU(3)$ octet, the new constituents were proposed to be fractionally charged fermions lying in the fundamental (triplet) representation of $SU(3)$ that had not been observed outside of hadronic bound states. Zweig, who called his constituents 'Aces', (A_1, A_2, A_3), took them to be physical quanta and called for a search to be made for them. Gell-Mann, who called his constituents 'Quarks', (u, d, s), insisted that they were mere book-keeping devices to aid in the $SU(3)$ classification and did not represent physical quanta. While Gell-Mann's nomenclature was adopted, the notion of quarks as physical quanta remained popular despite his objections.

2.4 *The introduction of colour*

The quark model described mesons as the combination of a quark and an antiquark. As quarks lie in the fundamental and antiquarks in the conjugate fundamental representations of $SU(3)$, their combination forms an octet and a singlet,

$$\mathbf{3} \times \overline{\mathbf{3}} \rightarrow \mathbf{8} + \mathbf{1}. \tag{2}$$

Baryons are described as the combination of three quarks,

$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} \to \mathbf{10} + \mathbf{8} + \mathbf{8} + \mathbf{1}. \tag{3}$$

Assuming the quarks to have spin-$\frac{1}{2}$, a careful analysis of the meson and baryon wave functions indicates that they have the wrong statistics. For instance, in the baryon decuplet, the $SU(3)$ part of the wave function is symmetric, because the total spin is $\frac{3}{2}$, the spin part of the wave function is also symmetric. However, the total wave function of a fermionic system must be antisymmetric. Independently, Greenberg, Han and Nambu introduced a three-valued internal quantum number (or parastatistics), which came to be called colour [10–12], to provide the antisymmetry. There was no proposal for internal dynamics and the force carriers which bound the quarks (if mentioned at all) were generally taken to be colourless.

3. The structure of the proton and the parton model

Although the proton was known to have a much larger magnetic moment than one would expect from a fundamental Dirac particle, there was no direct evidence of proton structure until the mid-1950s. The crucial experiment involved the scattering of high-energy electrons off of proton targets. If the proton were simply a static (infinitely massive) spin-$\frac{1}{2}$ point charge, elastic electron scattering would be given by the Mott scattering formula [13],

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)}. \tag{4}$$

The proton, of course, has a finite mass and an anomalously large magnetic moment, $\kappa_p = (g_p - 2)/2 \sim 1.79$. If the proton were a point particle, the scattering formula would be

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right]_{\mathrm{Mott}} \frac{E'^3}{E^3} \left(1 + \kappa_p^2 + \frac{Q^2}{2M^2}(1+\kappa_p)^2 \tan^2(\theta/2)\right), \tag{5}$$

where $E' = \frac{E}{1+2E/M \sin^2(\theta/2)}$ and $Q^2 = 4EE' \sin^2(\theta/2)$. If the proton is instead a diffuse object, the scattering can be parametrized in terms of form factors $F_1(Q^2)$ and $F_2(Q^2)$,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right]_{\mathrm{Mott}} \frac{E'^3}{E^3} \Bigg(F_1^2(Q^2) + \frac{\kappa_p^2 Q^2}{4M^2} F_2^2(Q^2) + \frac{Q^2}{2M^2}(F_1(Q^2) + \kappa_p F_2(Q^2))^2 \tan^2(\theta/2)\Bigg). \tag{6}$$

Hofstadter and McAlister [14,15] measured a clear deviation from point-like behaviour and were able to measure the effective charge radius of the proton.

3.1 *Deep inelastic scattering*

As the energy of the scattering electrons is increased, much of the cross-section is inelastic, so that the scattering angle and the electron energy loss are independent variables. The momentum transfer Q^2 is determined by measuring both.

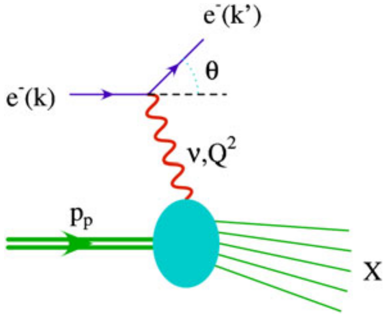

Figure 1. Deep inelastic scattering.

The kinematics of deep inelastic scattering (DIS) is as follows (see figure 1):

$$e(k) + p(p) \to e(k') + X; \quad q^\mu = k^\mu - k'^\mu; \quad Q^2 = -q^2;$$
$$\nu = \frac{p \cdot q}{m_p} = E_k - E_{k'}; \quad x = \frac{Q^2}{2m_p\nu}; \quad y = \frac{p \cdot q}{p \cdot k};$$
$$W^2 = m_X^2 = m_p^2 + Q^2\frac{1-x}{x}. \tag{7}$$

The DIS cross-section may be written as the contraction of a leptonic tensor with a hadronic tensor,

$$\mathrm{d}\sigma^{\mathrm{DIS}} = \frac{1}{2\hat{s}}\frac{\mathrm{d}^3k'}{k'^0}\frac{\alpha^2}{Q^4}L^{\mu\nu}(k,q)W_{\mu\nu}(p,q),$$
$$L^{\mu\nu}(k,q) = \frac{1}{2}\mathrm{Tr}\left[\not{k}\gamma^\mu(\not{k} - \not{q})\gamma^\nu\right],$$
$$W_{\mu\nu}(p,q) = \frac{1}{8\pi}\sum_X \langle P(p)|j_\mu^*(0)|X\rangle\langle X|j_\nu(0)|P(p)\rangle. \tag{8}$$

Symmetries restrict the structure of $W_{\mu\nu}$ so that it may be parametrized in terms of two structure functions,

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) + \frac{(p_\mu + xq_\mu)(p_\nu + xq_\nu)}{m_p\nu}F_2(x,Q^2). \tag{9}$$

Through a complicated analysis of current algebra, Bjorken predicted that the structure functions would become scale invariant at asymptotically large energies [16],

$$\lim_{\nu,Q^2\to\infty} F_{1,2}(x,Q^2) \to F_{1,2}(x); \quad x = \frac{Q^2}{2\,m_p\nu}. \tag{10}$$

The scale invariance of $F_{1,2}$ is referred to as scaling, and x is the scaling variable.

In the late 1960's, the linear accelerator at SLAC began producing 20 GeV electron beams and the SLAC-MIT experiment measured deep inelastic scattering off of protons [17–19]. The experiment produced a number of surprising results. In particular, it found that the DIS cross-section fell much more slowly with increasing Q^2 than did elastic scattering, and observed 'precocious' scaling. Far from being an asymptotic property, scaling, without any apparent violation, was observed over the entire available range of Q^2 (see figure 2).

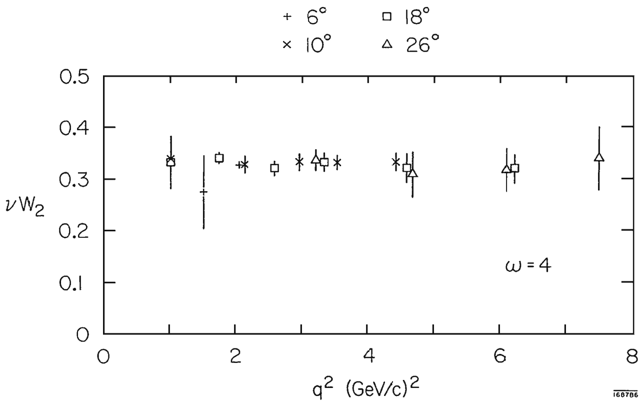

Figure 2. Observation of precocious scaling (figure is taken from [19]).

3.2 *The parton model*

Feynman realized that the DIS measurements could be explained by assuming that the electrons were actually undergoing elastic scattering off of constituent objects inside the proton, which he called partons, that behaved as approximately free particles. This was the parton model [20,21] and took 'asymptotic freedom', the property that partons respond to high-energy impulses as if they were free particles, as an axiom. While the parton model clearly invited a quark interpretation, there was no evidence yet that the partons were fermionic. Still, the parton model was not the quark model! It envisioned the proton as a rich spectrum of virtual partons and antipartons, not as three quarks rattling around in a box. Furthermore, there was still no theory of what bound the partons inside of the proton and provided the dynamics of asymptotic freedom.

The parton model gave a prescription for calculating the DIS cross-section,

$$\frac{\mathrm{d}\sigma^{ep}(p,q)}{\mathrm{d}E_k\,\mathrm{d}\Omega} = \sum_f \int_0^1 \mathrm{d}\xi\, \frac{\mathrm{d}\sigma^{ef}_{\mathrm{Born}}(\xi p, q)}{\mathrm{d}E_k\,\mathrm{d}\Omega}\phi_{f/p}(\xi)\,, \tag{11}$$

where f represents the type or 'flavour' of parton, ξ represents the parton momentum fraction and $\phi_{f/p}(\xi)$ is the density of parton f with momentum fraction ξ in the proton. Because electromagnetic corrections are small and there was no theory of the strong interaction built into the model, the electron–parton cross-section was computed in the Born approximation.

If one adopted the quark interpretation of the partons, the parton densities would obey sum rules such that momentum was conserved; the proton charge was exactly one, the isospin was equal to $\frac{1}{2}$ and the net strangeness was 0. A careful analysis found that the momentum sum rule could only be satisfied by assuming that the strange and antistrange partons carried most of the protons energy, or that there were partons that did not interact with photons. (These turned out to be the gluons of QCD.)

Callan and Gross pointed out that one could measure the spin of the partons [22] by measuring the Q^2 dependence of the structure functions $F_{1,2}(x)$. If the partons are bosonic,

$\lim_{Q^2\to\infty} F_1(x) \to 0$, while if they are fermions, $\lim_{Q^2\to\infty} F_2(x) \to 2xF_1(x)$. The measurements showed no evidence that $F_1(x)$ was vanishing at high Q^2 [19], strongly favouring the quark interpretation of the partons.

Given its success in describing DIS, it was natural to try to extend the parton model to describe other high-energy hadronic processes, including $e^+e^- \to$ hadrons [25,26] and $pp \to \ell^+\ell^- + X$ [23,24]. At leading order, the ratio $R = \frac{\sigma_{e^+e^-\to \text{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$ has a simple interpretation in terms of the number and charges of the parton flavours. The Drell–Yan process, $pp \to \ell^+\ell^- + X$, requires knowledge of the parton densities and high energy. Initial studies of this process did not seem to work well.

4. Non-Abelian gauge theory

Despite the advances of the quark and parton models, there was still no fundamental theory of the strong interactions. The answer finally came from an unlikely source, the non-Abelian gauge theory. Non-Abelian gauge theory [27] was invented by Yang and Mills, who were trying to describe the strong interaction as a gauged isospin symmetry. It was a beautiful idea, but it did not work on a number of levels. Among its most glaring problems was that gauge bosons must be massless and yet ‘clearly’ they were not or they would have been seen already. Other problems were of a more practical nature; the self-interactions of the gauge bosons made calculation cumbersome and obscured the renormalizability of the theory. As spontaneous symmetry breaking came to be understood, people returned to the idea [43] of non-Abelian gauge theories with broken symmetries to make the gauge bosons massive, but without success.

In fact, many theorists had given up on gauge theories, and even field theories, as viable descriptions of particle physics [42]. The spectacular success of QED was seen by many as an anomaly amidst widespread failure. The first real breakthrough for applying non-Abelian gauge theories to the strong interaction came when ’t Hooft proved the renormalizability of both unbroken and spontaneously broken theories [28,29]. There was a surge of interest in gauge theories, and the machinery of renormalization and more efficient computation was quickly developed [30].

The real key to establishing a theory of the strong interactions, however, was asymptotic freedom. In a field theory, asymptotic freedom is related to the renormalization group [31–34] flow of the coupling constant. For a theory to be asymptotically free, the effective coupling of the theory must get smaller (and asymptotically vanish) as the energy transfer increases. At infinite momentum transfer, the particles in an asymptotically free theory are effectively free particles. In the language of the renormalization group, the ‘β’ function, $\beta(g(\mu)) = \mu\frac{\partial g}{\partial\mu}$, is negative for an asymptotically free theory.

Gross was engaged in a program to prove that the strong interactions could not be described by any field theory. There were two parts to the argument – scaling demands asymptotic freedom; no field theory is asymptotically free [42]. In 1973, only non-Abelian gauge theories had not been rejected, but they had been left for last because they were so cumbersome to work with. Instead, Gross and Wilczek [35] and simultaneously Politzer [36], who were studying dynamical symmetry breaking in non-Abelian gauge theories, discovered that the β function was negative, meaning that non-Abelian gauge theories are asymptotically free!

Gross and Wilczek immediately suggested that a non-Abelian gauge theory could be a viable model of the strong interactions [37,38], Gell-Mann and collaborators renewed a proposal of an $SU(3)$ symmetric gauge theory [39,41] (though still clinging to the idea that quarks were 'fictitious' quanta), and Weinberg proposed an $SU(3)$ gauge theory as an element of a product group of gauge interactions of the strong, weak and electromagnetic forces [40].

The notion that quarks are fictitious quanta persisted for some time, although the discovery of the charm quark helped to change this view [44–46]. The study of the charmonium spectrum and its description as resulting from an asymptotically linear potential [47,48] helped to explain confinement: the property that quarks could not be observed directly, but only as parts of hadronic bound states.

5. Quantum chromodynamics – The theory of the strong interaction

With the discovery of asymptotic freedom, $SU(3)$ Yang–Mills theory became a serious contender as a fundamental theory of the strong interactions. However, all of the prescriptions and hand-waving arguments of the parton model had to be made rigorous. In particular, one had to specify what could be calculated, identify rules for performing perturbative QCD calculations, derive a factorization theorem in deep inelastic scattering and in hadron–hadron scattering, and define the parton densities. It took many years before all of these properties were well established.

The guiding principle of applying perturbative QCD is 'infrared safety'. Infrared safe quantities do not depend on the long-distance behaviour of QCD. In particular, they are finite in the limit of vanishing masses, so that

$$\sigma\left(\frac{s_{ij}}{\mu^2}, \frac{m_i^2}{\mu^2}, \alpha_s(\mu)\right) = \sigma\left(\frac{s_{ij}}{\mu^2}, 0, \alpha_s(\mu)\right)\left(1 + \mathscr{O}\left(\frac{m_i^2}{\mu^2}\right)\right), \tag{12}$$

where Q^2 is a scale characteristic of the larger invariants s_{ij}. Renormalization group invariance then says that

$$\sigma\left(\frac{s_{ij}}{\mu^2}, 0, \alpha_s(\mu)\right) = \sigma\left(\frac{s_{ij}}{Q^2}, 0, \alpha_s(Q)\right). \tag{13}$$

The proof of infrared safety comes from the KLN theorem [49,50], which states that fully inclusive measurements, which sum over all degenerate initial and final states, are free from infrared singularities. This means that the short-distance physics of parton scattering does not interfere with the long-distance process that turns partons into hadrons.

The theorem can be extended to cover differential cross-sections by understanding the origin of infrared divergences. Sterman [51] showed that all infrared divergences are related to either soft or collinear momentum configurations and that as long as a measurement is 'sufficiently inclusive' that it sums over the soft and collinear configurations, it will be infrared safe.

As an example, consider a higher-order calculation of an observable $\mathscr{J}$ using a measurement function $\mathscr{S}$,

$$\sigma^{(m)}(\mathscr{J}) = \int \mathrm{d}\Omega_n \frac{\mathrm{d}\sigma^{(m)}(\mathscr{J})}{\mathrm{d}\Omega_n} \mathscr{S}_n(p_1, \ldots, p_n)$$
$$+ \int \mathrm{d}\Omega_{n+1} \frac{\mathrm{d}\sigma^{(m-1)}(\mathscr{J})}{\mathrm{d}\Omega_{n+1}} \mathscr{S}_{n+1}(p_1, \ldots, p_n, p_{n+1}) + \cdots . \quad (14)$$

Infrared safety requires that

$$\mathscr{S}_{n+1}(p_1, \ldots, \lambda p_n, (1-\lambda) p_n) = \mathscr{S}_n(p_1, \ldots, p_n); \quad 0 \leq \lambda \leq 1,$$
$$\mathscr{S}_{n+1}(p_1, \ldots, p_n, 0) = \mathscr{S}_n(p_1, \ldots, p_n). \quad (15)$$

5.1 *The factorization theorem*

The factorization theorem of QCD [52,53] is the field theory realization of the parton model. For DIS, the factorization theorem states that

$$F_1(x, Q^2) = \sum_i \int_0^1 \frac{\mathrm{d}\xi}{\xi} C_1^{(i)}(x/\xi, Q^2/\mu^2, \alpha_s(\mu)) \phi_{i/p}(\xi, \mu),$$
$$F_2(x, Q^2) = \sum_i \int_0^1 \mathrm{d}\xi \, C_2^{(i)}(x/\xi, Q^2/\mu^2, \alpha_s(\mu)) \phi_{i/p}(\xi, \mu), \quad (16)$$

where C_i is the hard-scattering function (or Wilson coefficient) and $\phi_{i/p}(\xi, \mu)$ is the parton distribution function. The factorization theorem also justifies the extension of the parton model to hadron–hadron scattering,

$$\sigma_{A+B\to\mathscr{J}} = \sum_{a,b} \int_0^1 \mathrm{d}\xi_A \, \mathrm{d}\xi_B \, \hat{\sigma}_{ab\to\mathscr{J}} \left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q, \ldots \right) \phi_{a/A}(\xi_A, \mu) \phi_{b/B}(\xi_B, \mu)$$
$$+ \mathscr{O}(1/Q^2). \quad (17)$$

The key departure from the simple parton model picture is that factorization works only to leading order in $1/Q^2$. This result stems from the fact that soft particle exchanges between incoming hadrons cancel only at the leading order in $1/Q^2$. Unsuppressed power corrections at low Q^2 explain why early Drell–Yan measurements did not support the parton model.

5.2 *Parton distribution functions*

The fundamental aspect of the factorization theorem is the separation of long-distance and short-distance effects. The actual factorization scale, μ, at which this separation takes place is arbitrary. The short-distance physics is contained in the hard scattering function $\hat{\sigma}$, which is computed in perturbation theory using the usual Feynman rules for QCD. All long-distance physics is contained in the parton distribution functions, $\phi_{a/A}(\xi_A, \mu)$, $\phi_{b/B}(\xi_B, \mu)$.

Unlike the parton model, QCD is a fundamental theory of the strong interactions. Therefore, there must be a rigorous definition of the parton distributions. This is done by defining them in terms of renormalized operators in QCD [54]. For example, for a hadron, h of momentum p,

$$\phi_{q/h}(x,\mu) = \frac{1}{4\pi}\int \mathrm{d}y^- \, \mathrm{e}^{-ixp^+y^-} \langle p|\bar{\psi}_q(y^-)\gamma^+ \mathbf{W}(y^-,0)\psi_q(0)|p\rangle_R \,, \tag{18}$$

where $\mathbf{W}$ is a Wilson line,

$$\mathbf{W}(y^-,0) = \mathbf{P}\exp\left(ig\int_0^{y^-} \mathrm{d}s^- \, A_a^+(s^-)\, t^a\right). \tag{19}$$

Parton distribution functions (PDFs) have many important properties. They involve matrix elements of the proton wave function and are therefore nonperturbative objects. They cannot be calculated in perturbation theory, but must be extracted from experimental measurements. PDFs are ultraviolet singular. Renormalization spoils their interpretation as number densities, as in the parton model. However, treated as distributions, they satisfy all the sum rules expected of the parton densities of the parton model. PDFs are renormalized and obey renormalization group equations, known as the DGLAP equations [55], and evolve in Q^2. Finally, PDFs are universal and process-independent. A set of parton distributions determined from measurements of DIS can be used to compute cross-sections for hadron–hadron collisions.

6. Conclusions

In this talk I have described the origins of QCD. I have not tried to discuss modern methods and applications, but have instead focussed on what QCD is, where it came from and why we believe it.

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