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Bianchi type-V string cosmological models in general relativity

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Abstract. Bianchi type-V string cosmological models in general relativity are investigated. To get the exact solution of Einstein's field equations, we have taken some scale transformations used by Camci *et al* [*Astrophys. Space Sci.* **275**, 391 (2001)]. It is shown that Einstein's field equations are solvable for any arbitrary cosmic scale function. Solutions for particular forms of cosmic scale functions are also obtained. Some physical and geometrical aspects of the models are discussed.

Keywords. Cosmology; Bianchi type-V Universe; string theory.

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1. Introduction

The study of Bianchi type-V cosmological models plays an important role in the study of Universe and the study is more interesting as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some point of time. The string theory plays a significant role in the study of physical situation at the very early stages of the formation of the Universe. It is generally assumed that after the Big Bang, the Universe may have undergone a series of phase transitions as its temperature was lowered down below some critical temperature as predicted by grand unified theories [1–6]. At the very early stages of evolution of the Universe, during phase transition, it is believed that the symmetry of the Universe is broken spontaneously. It can give rise to topologically stable defects such as domain walls, strings and monopoles. In these three cosmological structures, cosmic strings are the most interesting [7] because they are believed to give rise to density perturbations which lead to the formation of galaxies [8]. These cosmic strings can be closed like loops or opened like a hair which move through time and trace out a tube or a sheet, according to whether it is closed or open. The string is free to vibrate and its different vibrational modes present different types of particles carrying the force of gravitation. Hence, it is very interesting to study the gravitational effect that arises from strings using Einstein's field equations.

The general relativistic treatment of strings was initially done by Stachel [9] and Letelier [10,11]. Letelier [10] obtained the general solution of Einstein's field equations for a cloud of strings with spherical, plane and a particular case of cylindrical symmetry. Letelier [11] also obtained massive string cosmological models in Bianchi type-I and Kantowski–Sachs space-times. Banerjee *et al* [12] have investigated an axially symmetric Bianchi type-I string dust cosmological model with and without magnetic field. Krori *et al* [13] and Wang [14,15] studied the exact solutions of string cosmology for Bianchi type-II, VI_0, VIII and IX space-times. Bali and Upadhaya [16] have presented LRS Bianchi type-I string dust magnetized cosmological models. Singh and Singh [17] investigated string cosmological models with magnetic field in the context of space-time with G_3 symmetry. Singh [18,19] has studied string cosmology with electromagnetic fields in Bianchi type-II, VIII and IX space-times.

Bianchi V Universes are the natural generalization of FRW models with negative curvature. These open models are favoured by the available evidences for low-density Universes [20]. Heckmann and Schucking [21] studied Bianchi type-V cosmological model where matter moves orthogonally to the hyper-surface of homogeneity. Exact tilted solutions for the Bianchi type-V space-time were obtained by Hawkings [22] and Grishchuk *et al* [23]. Ftaclas and Cohen [24] have investigated LRS Bianchi type-V Universes containing stiff matter with electromagnetic field. Lorentz [25] has investigated LRS Bianchi type-V tilted models with stiff fluid and electromagnetic field. Pradhan *et al* [26] have investigated the generation of Bianchi type-V cosmological models with varying Λ term. Yadav *et al* [27,28] have investigated bulk viscous string cosmological models in different space-times. Bali and Anjali [29] and Bali [30] have obtained Bianchi type-I and type-V string cosmological models in general relativity. The string cosmological models with a magnetic field were discussed by Tikekar and Patel [31], Patel and Maharaj [32]. Ram and Singh [33] obtained some new exact solution of string cosmology with and without a source-free magnetic field for Bianchi type-I space-time in the different basic form considered by Carminati and McIntosh [34]. Yavuz *et al* [35] have examined charged strange quark matter attached to the string cloud in the spherical symmetric space-time admitting one-parameter group of conformal motion. Kaluza-Klein cosmological solutions were obtained by Yilmaz [36] for quark matter attached to the string cloud in the context of general relativity. Recently, Baysal *et al* [37], Kilinc and Yavuz [38], Pradhan [39], Pradhan *et al* [40,41] and Yadav *et al* [42] have investigated some string cosmological models in cylindrically symmetric inhomogeneous Universe. In this paper, we have investigated string cosmological models based on generation technique and obtained a more realistic behaviour of the Universe. This paper is organized as follows. The field equations are presented in §2. In §3, we deal with generation technique for the solution of field equation and finally the results are discussed in §4.

2. Field equations

We consider the Bianchi type-V metric of the form

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2\alpha x}[B^2(t)dy^2 + C^2(t)dz^2], \tag{1}$$

where α is a constant.

The energy–momentum tensor for a cloud of string is given by

$$T_i^j = \rho v_i v^j - \lambda x_i x^j, \tag{2}$$

where v_i and x_i satisfy the condition

$$v^i v_j = -x^i x_j = 1, \quad v^i x_i = 0. \tag{3}$$

Here ρ is the proper energy density of the cloud of string with particle attached to them, λ is the string tension density, v^i is the four velocity of the particles and x^i is the unit space vector representing the direction of strings. If the particle density of the configuration is denoted by $\rho_{\rm p}$, then we have

$$\rho = \rho_{\rm p} + \lambda. \tag{4}$$

For the energy–momentum tensor (2) and Bianchi type-V metric (1), Einstien's field equations

$$R_i^j - \frac{1}{2} R g_{ij} = -8\pi T_{ij} \tag{5}$$

yield the following five independent equations:

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = 0, \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = 0, \tag{7}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -8\pi\lambda, \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} = -8\pi\rho, \tag{9}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \tag{10}$$

Here and in what follows, the overhead dots on the symbols A, B, C denote differentiation with respect to t.

The physical quantities, expansion scalar θ and shear scalar σ^2 have the following expressions:

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{11}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\theta^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC}\right]. \tag{12}$$

Integrating eq. (10) and absorbing the integrating constant into B or C, we obtain

$$A^2 = BC \tag{13}$$

without loss of any generality.

From eqs (6), (7) and (13), we obtain

$$2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 = 2\frac{\ddot{C}}{C} + \left(\frac{\dot{C}}{C}\right)^2 \tag{14}$$

which on integration yields

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k}{(BC)^{3/2}}, \tag{15}$$

where k is the constant of integration. Hence, for the metric function B or C from the above first-order differential eq. (15), some scale transformations permit us to obtain new metric function B or C.

Firstly, under the scale transformation $\mathrm{d}t = B^{1/2}\mathrm{d}T$, eq. (15) takes the form

$$CB_T - BC_T = kC^{-1/2}, \tag{16}$$

where the subscript represents derivative with respect to T. Considering eq. (16) as a linear differential equation for B, where C is an arbitrary function, we obtain

(i)

$$B = k_1 C + kC \int \frac{\mathrm{d}T}{C^{5/2}}, \tag{17}$$

where k_1 is the the constant of integration. Similarly, using the transformation $\mathrm{d}t = B^{3/2}\mathrm{d}\bar{T}$, $\mathrm{d}t = C^{1/2}\mathrm{d}\tilde{t}$ and $\mathrm{d}t = C^{3/2}\mathrm{d}\tau$ in eq. (15) after some algebra we obtain respectively.

(ii)

$$B(\bar{T}) = k_2 C \mathrm{e}^{\left(k \int \frac{\mathrm{d}\bar{T}}{C^{3/2}}\right)}, \tag{18}$$

(iii)

$$C(\tilde{t}) = k_3 B - kB \int \frac{\mathrm{d}\tilde{t}}{B^{5/2}}, \tag{19}$$

(iv)

$$C(\tau) = k_4 B \mathrm{e}^{\left(k \int \frac{\mathrm{d}\tau}{B^{3/2}}\right)}, \tag{20}$$

where k_2, k_3 and k_4 are constants of integration. Thus, choosing any given function B or C in Cases (i), (ii), (iii) and (iv), one can obtain B or C.

3. Generation technique for the solution

We consider the following four cases:

3.1 *Case* (i)*: $C = T^n$ (n is a real number satisfying $n \neq 2/5$)*

Equation (16) leads to

$$B = k_1 T^n + \frac{2k}{2-5n} T^{1-(3n/2)}. \tag{21}$$

From eqs (13) and (21), we obtain

$$A^2 = k_1 T^{2n} + \frac{2k}{2-5n} T^{1-(n/2)}. \tag{22}$$

Hence metric (1) reduces to the following form:

$$\mathrm{d}s^2 = (k_1 T^n + 2lT^{l_1})[\mathrm{d}T^2 - T^n \mathrm{d}x^2] - \mathrm{e}^{2\alpha x}[(k_1 T^n + 2lT^{l_1})^2 \mathrm{d}y^2 + T^{2n}\mathrm{d}z^2], \tag{23}$$

where $l = k/(2-5n)$ and $l_1 = 1-(3n/2)$.

In this case the physical parameters, i.e. the string tension density (λ), the energy density (ρ), the particle density (ρ_{p}) and the kinematical parameters, i.e. the scalar of expansion (θ), shear scalar (σ) and the proper volume (V^3) for model (23) are given by

$$8\pi\lambda = \left[-2k_1^2 n(n-1)T^{2n-2} - k_1 \ln(10-13n)T^{-(l_1+2n)} - \frac{1}{2}l^2(4+4n-11n^2)T^{-3n}\right] \times (k_1 T^n + 2lT^{l_1})^{-3} + \alpha^2 T^{-n}(k_1 T^n + 2lT^{l_1})^{-1}, \tag{24}$$

$$8\pi\rho = \left[3k_1^2 n^2 T^{2n-2} + 3k_1 \ln(2-n)T^{-(l_1+2n)} + \frac{1}{2}l^2(4+4n-11n^2)T^{-3n}\right] \times (k_1 T^n + 2lT^{l_1})^{-3} - 3\alpha^2 T^{-n}(k_1 T^n + 2lT^{l_1})^{-1}, \tag{25}$$

$$8\pi\rho_{\mathrm{p}} = \left[n(5n-2)k_1^2 T^{2n-2} + 16k_1 \ln(1-n)T^{-(l_1+2n)} + l^2(4+4n-11n^2)T^{-3n}\right] \times (k_1 T^n + 2lT^{l_1})^{-3} - 4\alpha^2 T^{n}(k_1 T^n + 2lT^{l_1})^{-1}, \tag{26}$$

$$\theta = 3\left[k_1 n T^{n-1} + \frac{1}{2}l(2-n)T^{-3n/2}\right](k_1 T^n + 2lT^{l_1})^{-3/2}, \tag{27}$$

$$\sigma = \frac{1}{2}kT^{-3n/2}(k_1 T^n + 2lT^{l_1})^{-3/2}, \tag{28}$$

$$V^3 = (k_1 T^{2n} + 2lT^{n+l_1})^{3/2}\mathrm{e}^{2\alpha x}. \tag{29}$$

Equations (27) and (28) lead to

$$\frac{\sigma}{\theta} = \frac{1}{6}k\left[k_1 n T^{n-l_1} + \frac{1}{2}l(2-n)\right]^{-1}. \tag{30}$$

The energy conditions $\rho \geq 0$ and $\rho_{\mathrm{p}} \geq 0$ satisfy for model (23). The conditions $\rho \geq 0$ and $\rho_{\mathrm{p}} \geq 0$ lead to

$$\left[3k_1^2 n^2 T^{3n-2} + 3k_1 \ln(2-n)T^{-(l_1+n)} + \frac{1}{2}l^2(4+4n-11n^2)T^{-2n}\right] \times (k_1 T^n + 2lT^{l_1})^{-2} \geq 3\alpha^2 \tag{31}$$

$$\left[n(2-5n)k_1^2T^{3n-2}+16k_1\ln(n-1)T^{-(l_1+n)}+l^2\left(11n^2-4n-4\right)T^{-2n}\right] \times\left(k_1T^n+2lT^{l_1}\right)^{-2}\geq 4\alpha^2 \tag{32}$$

respectively.

We observe that the string tension density $\lambda\geq 0$, leads to

$$\left[-2k_1^2n(n-1)T^{3n-2}-k_1\ln(10-13n)T^{-(l_1+n)}-\frac{1}{2}l^2\left(4+4n-11n^2\right)T^{-2n}\right] \times\left(k_1T^n+2lT^{l_1}\right)^{-2}\geq-\alpha^2. \tag{33}$$

Generally, model (23) is expanding, shearing and it approaches isotropy at a later time. For $k=0$, the solution represents the shear-free model of Universe. We observe that as $T\to\infty$, $V^3\to\infty$ and $\rho\to 0$, volume increases when T increases and the proper energy density of the cloud of string with particle attached to them decreases, i.e. the proper energy density (ρ) is a decreasing function of time.

3.2 *Case* (ii)*: $C=\bar{T}^n$ (n is a real number satisfying $n\neq 2/3$)*

Equation (18) leads to

$$B=k_2\bar{T}^n\mathrm{e}^{M\bar{T}^{l_1}}. \tag{34}$$

From eqs (13) and (34), we obtain

$$A^2=k_2\bar{T}^{2n}\mathrm{e}^{M\bar{T}^{l_1}}, \tag{35}$$

where $M=k/l_1$. Hence the metric (1) reduces to the form

$$\mathrm{d}s^2=\bar{T}^{\frac{4(1-l_1)}{3}}\left[\bar{T}^{\frac{2(1-l_1)}{3}}\mathrm{e}^{3M\bar{T}^{l_1}}\mathrm{d}\bar{T}^2-\mathrm{e}^{M\bar{T}^{l_1}}\mathrm{d}x^2-\mathrm{e}^{2\alpha x}(\mathrm{e}^{2M\bar{T}^{l_1}}\mathrm{d}y^2+\mathrm{d}z^2)\right]. \tag{36}$$

Here k_2 is a constant and we have taken k_2 as unity without any loss of generality. In this case the physical parameters, i.e. the string tension density (λ), the energy density (ρ), the particle density (ρ_{p}) and the kinematical parameters, i.e. the scalar of expansion (θ), shear scalar (σ) and the proper volume (V^3) for model (36) are given by

$$8\pi\lambda=2n\bar{T}^{2(l_1-2)}+3nk\bar{T}^{3l_1-4}+\frac{1}{2}k^2\bar{T}^{4(l_1-1)}+\alpha^2\bar{T}^{\frac{4(l_1-1)}{3}}\mathrm{e}^{-3M\bar{T}^{l_1}}, \tag{37}$$

$$8\pi\rho=3n^2\bar{T}^{2(l_1-2)}+3nk\bar{T}^{3l_1-4}+\frac{1}{2}k^2\bar{T}^{4(l_1-1)}-3\alpha^2\bar{T}^{\frac{4(l_1-1)}{3}}\mathrm{e}^{-3M\bar{T}^{l_1}}, \tag{38}$$

$$8\pi\rho_{\mathrm{p}}=n(3n-2)\bar{T}^{2(l_1-2)}-4\alpha^2\bar{T}^{\frac{4(l_1-1)}{3}}\mathrm{e}^{3M\bar{T}^{l_1}}, \tag{39}$$

$$\theta=3\left[n\bar{T}^{l_1-2}+\frac{1}{2}k\bar{T}^{2(l_1-1)}\right], \tag{40}$$

$$\sigma = \frac{1}{2} k \bar{T}^{2(l_1-1)} \mathrm{e}^{-3M\bar{T}^{l_1}} \tag{41}$$

$$V^3 = (k_3 \bar{T}^{2n} \mathrm{e}^{M\bar{T}^{l_1}})^{3/2} \mathrm{e}^{2\alpha x}. \tag{42}$$

Equations (40) and (41) lead to

$$\frac{\sigma}{\theta} = \frac{k}{6(n\bar{T}^{-l_1} + \frac{1}{2}k)}. \tag{43}$$

The energy conditions $\rho \geq 0$ and $\rho_{\mathrm{p}} \geq 0$ are satisfied for model (36). The conditions $\rho \geq 0$ and $\rho_{\mathrm{p}} \geq 0$ lead to

$$\mathrm{e}^{3M\bar{T}^{l_1}} \left[3n^2 \bar{T}^{\frac{2l_1-8}{3}} + 3nk\bar{T}^{\frac{5l_1-8}{3}} + \frac{1}{2}k^2 \bar{T}^{\frac{8l_1-8}{3}} \right] \geq 3\alpha^2 \tag{44}$$

$$n(3n-2)\mathrm{e}^{3M\bar{T}^{l_1}} \bar{T}^{\frac{(2l_1-8)}{3}} \geq 4\alpha^2 \tag{45}$$

respectively.

We observe that the string tension density $\lambda \geq 0$, leads to

$$\mathrm{e}^{3M\bar{T}^{l_1}} \left[2n \bar{T}^{\frac{2l_1-8}{3}} + 3nk\bar{T}^{\frac{5l_1-8}{3}} + \frac{1}{2}k^2 \bar{T}^{\frac{8l_1-8}{3}} \right] \geq -\alpha^2. \tag{46}$$

For $l_1 > 2$, model (36) is expanding and for $l_1 < 2$, model starts with Big Bang singularity. Generally, the model is expanding, shearing and it approaches isotropy at a later time. For $k = 0$, the solution represents the shear-free model of the Universe. We observe that as $\bar{T} \to \infty$, $V^3 \to \infty$ and $\rho \to 0$, volume increases when $\bar{T}$ increases and the proper energy density of the cloud of string with particle attached to them decreases, i.e. the proper energy density (ρ) is a decreasing function of time.

3.3 *Case* (iii)*: $B = \tilde{t}^n$ (n is a real number)*

Equation (19) leads to

$$C = k_3 \tilde{t}^n - 2l\tilde{t}^{l_1}. \tag{47}$$

From eqs (13) and (47), we obtain

$$A^2 = k_3 \tilde{t}^{2n} - 2l\tilde{t}^{l_1+n}. \tag{48}$$

Thus the metric (1) reduces to the form

$$\mathrm{d}s^2 = (k_3\tilde{t}^n - 2l\tilde{t}^{l_1})[\mathrm{d}t^2 - \tilde{t}^n \mathrm{d}x^2] - \mathrm{e}^{2\alpha x}[\tilde{t}^{2n}\mathrm{d}y^2 + (k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^2 \mathrm{d}z^2]. \tag{49}$$

In this case the physical parameters, i.e. the string tension density (λ), the energy density (ρ), the particle density (ρ_{p}) and the kinematical parameters, i.e. the scalar of expansion (θ), shear scalar (σ) and the proper volume (V^3) for model (49) are given by

$$8\pi\lambda = \left[-\frac{1}{2}l^2(11n^2 - 4n - 4)\tilde{t}^{-3n} + lk_3 n(13n-10)\tilde{t}^{l_1+n} - 2k_3^2 n(n-1)\tilde{t}^{2n-2} \right]$$
$$\times (k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-3} + \alpha^2 \tilde{t}^{-n}(k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-1}, \tag{50}$$

$$8\pi\rho = \left[-\frac{1}{2}l^2(11n^2-4n-4)\tilde{t}^{-3n} - 3lk_3n(2-n)\tilde{t}^{l_1+n} + 3k_3^2n^2\tilde{t}^{2n-2}\right] \times(k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-3} - 3\alpha^2\tilde{t}^{-n}(k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-1}, \tag{51}$$

$$8\pi\rho_{\rm p} = (k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-3}[n(5n-2)k_3^2\tilde{t}^{2n-2} - lk_3n(12n-8)\tilde{t}^{l_1+n}] -4\alpha^2\tilde{t}^n(k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-1}, \tag{52}$$

$$\theta = 3\left[\frac{1}{2}l(n-2)\tilde{t}^{-3n/2} + k_3n\tilde{t}^{n-1}\right](k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-3/2}, \tag{53}$$

$$\sigma = \frac{1}{2}k\tilde{t}^{-3n/2}(k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-3/2}, \tag{54}$$

$$V^3 = (k_3\tilde{t}^{2n} - 2l\tilde{t}^{l_1+n})^{3/2}{\rm e}^{2\alpha x}. \tag{55}$$

Equations (53) and (54) lead to

$$\frac{\sigma}{\theta} = \frac{k}{6}\left[k_3n\tilde{t}^{-(l_1+n)} + \frac{l(n-2)}{2}\right]^{-1}. \tag{56}$$

The energy conditions $\rho \geq 0$ and $\rho_{\rm p} \geq 0$ are satisfied for model (49). The conditions $\rho \geq 0$ and $\rho_{\rm p} \geq 0$ lead to

$$\left[-\frac{1}{2}l^2(11n^2-4n-4)\tilde{t}^{-2n} - 3lk_3n(2-n)\tilde{t}^{l_1+2n} + 3k_3^2n^2\tilde{t}^{(3n-2)}\right] \times (k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-2} \geq 3\alpha^2 \tag{57}$$

$$(k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-2}[n(5n-2)k_3^2\tilde{t}^{n-2} - lk_3n(12n-8)\tilde{t}_1^{l}] \geq 4\alpha^2 \tag{58}$$

respectively.

We observe that the string tension density $\lambda \geq 0$, leads to

$$\left[-\frac{1}{2}l^2(11n^2-4n-4)\tilde{t}^{-2n} + lk_3(13n-10)\tilde{t}^{l_1+2n} - 2k_3^2n(n-1)\tilde{t}^{3n-2}\right] \times (k_3\tilde{t}^n - 2l\tilde{t}^{l_1})^{-2} \geq -\alpha^2. \tag{59}$$

Thus, we see that model (49) is generally expanding, shearing and it approaches isotropy at a later time. For $k = 0$, the solution represents the shear-free model of the Universe. We observe that as $\tilde{t} \to \infty$, $V^3 \to \infty$ and $\rho \to 0$, volume increases when $\tilde{t}$ increases and the proper energy density of the cloud of string with particle attached to them decreases, i.e. the proper energy density (ρ) is a decreasing function of time.

3.4 *Case* (iv)*:* $B = \tau^n$ *(n is any real number)*

Equation (20) leads to

$$C = k_4 \tau^n \mathrm{e}^{\left(\frac{k}{l_1}\tau^{l_1}\right)}. \tag{60}$$

From eqs (13) and (60), we obtain

$$A^2 = k_4 \tau^{2n} \mathrm{e}^{\left(\frac{k}{l_1}\tau^{l_1}\right)}. \tag{61}$$

Hence the metric (1) reduces to

$$\mathrm{d}s^2 = \tau^{2n} \mathrm{e}^{\left(\frac{k}{l_1}\tau^{l_1}\right)} \left[\tau^n \mathrm{e}^{\left(\frac{2k}{l_1}\tau^{l_1}\right)} \mathrm{d}\tau^2 - \mathrm{d}x^2\right] - \mathrm{e}^{2\alpha x}\left[\mathrm{d}y^2 + \mathrm{e}^{\left(\frac{2k}{l_1}\tau^{l_1}\right)} \mathrm{d}z^2\right]. \tag{62}$$

Here k_4 is a constant and we have taken k_4 as unity without any loss of generality. In this case we see that A, B and C are exponential functions as in Case (ii). Thus, the physical and geometrical properties of model (62) are similar to model (36).

4. Conclusion

If we choose $\alpha = 0$, metric (1) becomes Bianchi type-I metric, studied by several authors in different context. In this paper, we have applied the technique used by Camci *et al* [43] for solving Einstein's field equations and found new solution for string cosmological model. It is shown that the Einstein's field equations are solvable for an arbitrary cosmic scale function. Starting from a particular cosmic function, new classes of spatially homogeneous and anisotropic cosmological models have been investigated for which the string fluid are rotation-free but they do have expansion and shear. It is also observed that in all the cases, the physical and geometrical behaviour of models are similar. Generally, the models are expanding, shearing and nonrotating. All the models are isotropized at a later time.

In Case (i), for $n \geq 1$, model (23) starts expanding with Big Bang singularity and for $n \leq 0$, Bianchi type-V Universe preserve expanding nature as $T \to 0, \theta \to 0$ and $T \to \infty, \theta \to \infty$. It is also observed that for $k = 0$, shear scalar (σ) vanishes and the model becomes isotropic. In Case (ii), for $l_1 < 2$, model (36) starts with Big Bang singularity and expands throughout the evolution of Universe and for $l_1 > 2$, model (36) also preserves the expanding nature as $\bar{T} \to 0, \theta \to 0$ and $\bar{T} \to \infty, \ \theta \to \infty$. From eq. (41), it is clear that when $k = 0$, the shear scalar (σ) vanishes and the model becomes isotropic. In Case (iii), model (49) starts with Big Bang singularity and expands through the evolution of Universe. From eq. (54), it is clear that for $k = 0$, the shear scalar (σ) vanishes and model isotropizes. In Case (iv), it is observed that the physical and geometrical properties of model (62) are similar to model (36), i.e. the Case (ii). We note that in all cases, $\rho(t)$ is a decreasing function of time and it is alway positive. Further, it is observed that for sufficiently large time, ρ_{p} and λ tend to zero. Therefore, the strings disappear from the Universe at a later time (i.e. present epoch). The same is predicted by current observations.

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References

[1] A E Everett, *Phys. Rev*. **24**, 858 (1981)
[2] T W B Kibble, *J. Phys.* **A9**, 1387 (1976)
[3] T W B Kibble, *Phys. Rep*. **67**, 183 (1980)
[4] A Vilenkin, *Phys. Rev*. **D24**, 2082 (1981)
[5] Ya B Zel'dovich, I Yu Kobzarev and L B Okun, *Zh. Eksp. Teor. Fiz*. **67**, 3 (1975)
[6] Ya B Zel'dovich, I Yu Kobzarev and L B Okun, *Sov. Phys.-JETP* **40**, 1 (1975)
[7] A Vilenkin, *Phys. Rep*. **121**, 263 (1985)
[8] Ya B Zel'dovich, *Mon. Mot. R. Astron. Soc*. **192**, 663 (1980)
[9] J Stachel, *Phys. Rev*. **D21**, 2171 (1980)
[10] P S Letelier, *Phys. Rev*. **D20**, 1249 (1979)
[11] P S Letelier, *Phys. Rev*. **D28**, 2414 (1983)
[12] A Banerjee, A K Sanyal and S Chakraborty, *Pramana – J. Phys*. **34**, 1 (1990)
[13] K D Krori, T Chaudhury, C R Mahanta and A Mazumder, *Gen. Relativ. Gravit.* **22**, 123 (1990)
[14] X X Wang, *Chin. Phys. Lett.* **20**, 615 (2003)
[15] X X Wang, *Chin. Phys. Lett.* **20**, 1205 (2003)
[16] R Bali and R D Upadhaya, *Astrophys. Space Sci.* **283**, 97 (2002)
[17] G P Singh and T Singh, *Gen. Relativ. Gravit*. **31**, 371 (1999)
[18] G P Singh, *Nuovo Cimento.* **B110**, 1463 (1995)
[19] G P Singh, *Pramana – J. Phys.* **45**, 189 (1995)
[20] J R Gott *et al*, *Astrophys. J.* **194**, 543 (1974)
[21] O Heckmann and E Schucking, in: *Gravitation: An introduction to current research* (John Wiley & Sons, New York, 1962)
[22] S W Hawking, *Mon. Not. R. Astron. Soc.* **142**, 129 (1969)
[23] L P Grishchuck, A G Doroshkevich and I D Novikov, *Sov. Phys. JETP* **28**, 1214 (1969)
[24] C Ftaclas and J M Cohen, *Phys. Rev*. **D18**, 4373 (1978)
[25] D Lorentz, *Gen. Relativ. Gravit.* **13**, 795 (1981)
[26] A Pradhan, A K Yadav and L Yadav, *Czech. J. Phys.* **55**, 503 (2005)
[27] M K Yadav, A Rai and A Pradhan, *Int. J. Theor. Phys.* **46**, 2677 (2007)
[28] M K Yadav, A Pradhan and S K Singh, *Astrophys. Space Sci.* **311**, 423 (2007)
[29] R Bali and Anjali, *Astrophys. Space Sci.* **302**, 201 (2006)
[30] R Bali, *Electron. J. Theor. Phys.* **5**, 105 (2008)
[31] R Tikekar and L K Patel, *Gen. Relativ. Gravit.* **24**, 397 (1992)
[32] L K Patel and S D Maharaj, *Pramana – J. Phys.* **47**, 1 (1996)
[33] S Ram and T K Singh, *Gen. Relativ. Gravit.* **27**, 1207 (1995)
[34] J Carminati and C B G McIntosh, *J. Phys. A: Math. Gen.* **13**, 953 (1980)
[35] I Yavuz, I Yilmaz and H Baysal, *Int. J. Mod. Phys.* **D14**, 1365 (2005)
[36] I Yilmaz, *Gen. Relativ. Gravit.* **38**, 1397 (2006)
[37] H Baysal, I Yavuz, I Tarhan, U Camci and I Yilmaz, *Turk. J. Phys.* **25**, 283 (2001)
[38] C B Kilinc and I Yavuz, *Astrophys. Space Sci.* **238**, 239 (1996)
[39] A Pradhan, *Fizika B (Zagreb)* **16**, 205 (2007)
[40] A Pradhan, A K Yadav, R P Singh and V K Singh, *Astrophys. Space Sci.* **312**, 145 (2007)
[41] A Pradhan, M K Mishra and A K Yadav, *Rom. J. Phys.* **54**, 747 (2009); arXiv:0705.1765 [gr-qc]
[42] A K Yadav, V K Yadav and L Yadav, *Int. J. Theor. Phys.* **48**, 568 (2009)
[43] U Camci, I Yavuz, H Baysal, I Tarhan and I Yilmaz, *Astrophys. Space Sci.* **275**, 391 (2001)