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Bianchi type-I massive string magnetized barotropic perfect fluid cosmological model in bimetric theory

S D KATORE[1], R S RANE[2] and K S WANKHADE[2,*]
[1]Department of Mathematics, S.G.B. Amravati University, Amravati 444 602, India
[2]Department of Mathematics, Y.C. Science and Arts College, Mangrulpir 444 403, India
*Corresponding author. E-mail: wankhade.kishor@rediffmail.com

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Abstract. Bianchi type-I massive string cosmological model for perfect fluid distribution in the presence of magnetic field is investigated in Rosen's [*Gen. Relativ. Gravit.* **4**, 435 (1973)] bimetric theory of gravitation. To obtain the deterministic model in terms of cosmic time, we have used the condition $A = (BC)^n$, where n is a constant, between the metric potentials. The magnetic field is due to the electric current produced along the x-axis with infinite electrical conductivity. Some physical and geometrical properties of the exhibited model are discussed and studied.

Keywords. Bimetric theory; magnetic field; massive string; cosmology.

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1. Introduction

Bimetric theory proposed by Rosen [1] refers to a class of modified Einstein's theories of gravity in which two metric tensors are used instead of one. These two metric tensors are the Riemannian metric tensor g_{ij} and the background flat space-time metric tensor f_{ij}. Often the second metric tensor is introduced at high energies with the implication that speed of light may be energy-dependent. The metric tensor g_{ij} describes the Riemannian geometry of a curved space-time which plays the same role as given in the Einstein's general theory of relativity. The background metric tensor f_{ij} refers to inertial forces. This metric tensor f_{ij} has no direct physical significance but appears in the field equations. Hence f_{ij} describes a space-time of constant curvature. Moreover, the bimetric theory also satisfies the covariance and equivalence principles. The theory agrees with the present observational facts pertaining to general relativity.

The field equations of bimetric theory of gravitation proposed by Rosen [1] are

$$N_i^j - \frac{1}{2} N \delta_i^j = -8\pi k T_i^j, \tag{1}$$

where

$$N_i^j = \frac{1}{2} f^{ab} (g^{hj} g_{hi|a})_{1b} \quad \text{and} \quad k = \sqrt{\frac{g}{f}},$$

together with $g = \det(g_{ij})$ and $f = \det(f_{ij})$. The vertical bar (|) indicates covariance differentiation with respect to f_{ij} and T_i^j is the energy–momentum tensor of the matter field.

The study of cosmic string cosmological models has generated a considerable interest as these strings are believed to play a significant role during the early stages of the evolution of Universe [2]. These strings generate density fluctuations which lead to galaxy formation [3]. Cosmic strings are one-dimensional topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early Universe. The gravitational effects of cosmic strings, both in general relativity and in the alternative theories of gravitation, have been extensively studied [4–11].

Several aspects of bimetric theory of gravitation were studied in [1,12–19], and in particular, a few have established [20–25] the non-existence of spatially homogeneous and isotropic cosmological model of Bianchi types and Kantowski–Sachs in bimetric theory of gravitation when the field of gravitation is governed by either a perfect fluid or a cosmic string. Reddy *et al* [26] have established the non-existence of axially symmetric cosmological model with domain walls and cosmic string. Sahoo [27] has shown that the spherically symmetric cosmological model did not exist in Rosen's bimetric theory of gravitation when the source of gravitation was either perfect fluid or massive string whereas the model did exist in scalar meson field. Rao *et al* [28] have studied Bianchi type-I string cosmological models in bimetric theory of gravitation. Recently, Sahoo [29] has discussed inhomogeneous plane symmetric string cosmological models in bimetric theory of gravitation.

The influence of intergalactic magnetic fields on cosmological evolution has been studied for over four decades from theoretical and observational points of view [30,31]. Melvin [32] suggested, in the cosmological solution for dust and electromagnetic field, that during the evolution of the Universe, the matter was in highly ionized state and smoothly coupled with electromagnetic field and consequently formed a neutral matter as a result of the expansion of the Universe. Hence, in string dust Universe the presence of magnetic field is not unrealistic. Banerjee *et al* [33] have investigated an axially symmetric Bianchi type-I string dust cosmological model in the presence and absence of magnetic field using the condition $a = \alpha\beta$ between metric potentials α and β, where α and β are functions of t and a is a constant. Wang [34] has investigated Bianchi types II, VIII and IX string cosmological models generalizing the results obtained by Krori *et al* [8]. The string cosmological models with electromagnetic field are investigated extensively, both in general relativity and in the alternative theories of gravitation [35–41]. Katore and Rane [42] have investigated Bianchi type-III magnetized cosmological model when the field of gravitation is governed by either a perfect fluid or a cosmic string in bimetric theory of gravitation. Bali and Pradhan [41] have investigated Bianchi type-III string cosmological model with time-dependent bulk viscosity.

In view of the importance of Maxwell's electromagnetic field interactions with a perfect fluid or a cosmic string and there is a lot of interest in alternative theories of gravitation, due to some lacunas in general relativity, we have discussed Bianchi type-I massive string magnetized cosmological model in Rosen's bimetric theory of gravitation. The magnetic field is due to an electric current produced along the x-axis with infinite electrical conductivity. Some of the physical and geometrical aspects of the exhibited model are discussed and studied in §4.

2. Metric and field equations

We consider Bianchi type-I metric of the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{2}$$

where A, B and C are functions of cosmic time t only.

The background flat space-time corresponding to the metric (2) is

$$\mathrm{d}\sigma^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2. \tag{3}$$

The energy–momentum tensor for a cloud of massive string and perfect fluid distribution with electromagnetic field is given by Letelier [6,43] as

$$T_i^j = (\rho + p)u_i u^j + pg_i^j - \lambda x_i x^j + E_i^j \tag{4}$$

together with

$$u_i u^i = -x_i x^i = -1 \quad \text{and} \quad u^i x_i = 0, \tag{5}$$

where p is the isotropic pressure, ρ is the rest energy density of the system of the string with massive particle attached to them, λ is the tension density of the string and E_i^j is that for magnetic field. As pointed out by Letelier [6], λ may be positive or negative, u^i represents the four-velocity vector and x^i represents the anisotropic direction, i.e. direction of the string.

We consider

$$\rho = \rho_{\mathrm{p}} + \lambda,$$

where ρ_{p} is the particle energy density attached to the string.

From eqs (2) and (5) we write

$$u_i u^i = (0, 0, 0, 1) \tag{6}$$

and x^i can be taken parallel to any of the directions $\partial/\partial x$, $\partial/\partial y$, $\partial/\partial z$.

We choose x^i parallel to $\partial/\partial x$ so that $x^i = (A^{-1}, 0, 0, 0\,)$.

The electromagnetic field E_{ij} is given by

$$E_{ij} = \frac{1}{4\pi}\left[-F_{is}F_j^s + \frac{1}{4}g_{ij}F_{sp}F^{sp}\right]. \tag{7}$$

In co-moving coordinates, the magnetic field is taken along x-direction so that $h_1 \neq 0$ and $h_2 = h_3 = h_4 = 0$. We assume that the only non-vanishing component of the electromagnetic field tensor F_{ij} is F_{23}. A cosmological model which contains a global magnetic field is necessarily anisotropic because the magnetic field specifies a preferred spatial direction. Bronnikov *et al* [44] have studied the evolution of Bianchi type-I space-time with a global unidirectional electromagnetic field (F_{ij}).

The first set of Maxwell's equation

$$F_{[ij,k]} = 0,$$

leads to the result $F_{23} =$ constant $= H$. Here $F_{14} = 0 = F_{24} = F_{34}$, due to the assumption of infinite electrical conductivity [45].

Now the non-vanishing components of E_{ij} corresponding to the line element (2) is given by

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = \frac{-H^2}{8\pi B^2 C^2} = -\eta \text{ (say)}. \tag{8}$$

Here u^i is the flow vector satisfying

$$g_{ij} u^i u^j = -1. \tag{9}$$

Using eq. (8), the non-zero components of energy–momentum tensor T_i^j yield

$$T_1^1 = p - \lambda - \eta, \quad T_2^2 = p + \eta = T_3^3 \quad \text{and} \quad T_4^4 = -\rho - \eta. \tag{10}$$

The Rosen's field equations (eq. (1)) of bimetric theory of gravitation for Bianchi type-I metric (2), with the help of eqs (3) and (10) can be written as

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = 16\pi k(p - \lambda - \eta), \tag{11}$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi k(p + \eta), \tag{12}$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = -16\pi k(p + \eta), \tag{13}$$

and

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 16\pi k(\rho + \eta), \tag{14}$$

where suffix 4 following an unknown function denotes an ordinary differentiation with respect to time.

3. Solution of field equations

Equations (11)–(14) are the four equations connecting six unknowns A, B, C, λ, p and ρ. For the complete determination of these field equations, we require certain plausible physical condition to obtain determinate solutions, hence we assume the relation between metric potentials as

$$A = (BC)^n, \tag{15}$$

where n ($\neq 1$) is a constant. Katore and Rane [42] obtained a magnetized cosmological model in bimetric theory of gravitation with the above condition. A more general relationship between the rest energy density ρ and string tension density λ is taken to be

$$\rho = \gamma\lambda,$$

where γ is an arbitrary constant which can take both positive and negative values. For barotropic cosmic fluid, the pressure p is considered to be directly proportional to the energy density, i.e.

$$p = \varepsilon_0 \rho, \quad \varepsilon_0 \in [0 \le \varepsilon_0 \le 1].$$

The combined effect of barotropic cosmic fluid and string tension density can be expressed as

$$p = \varepsilon_0 \gamma \lambda$$

without loss of generality take $\gamma = 1/\varepsilon_0$.

Hence the barotropic cosmic fluid containing one-dimensional string satisfying the general equation of state is

$$p = \lambda. \tag{16}$$

From eqs (12) and (13), we get

$$\left(\frac{B_4}{B}\right)_4 = \left(\frac{C_4}{C}\right)_4 \tag{17}$$

and using eqs (16) and (17) in (11) we obtain

$$\left(\frac{A_4}{A}\right)_4 - 2\left(\frac{B_4}{B}\right)_4 = -16\pi k\eta. \tag{18}$$

Using eq. (17) in (15) we obtain

$$\left(\frac{A_4}{A}\right)_4 = 2n\left(\frac{B_4}{B}\right)_4. \tag{19}$$

From eqs (18) and (19), we get

$$\left(\frac{B_4}{B}\right)_4 = \frac{H^2(BC)^{n-1}}{(1-n)}, \tag{20}$$

by setting

$$BC = \alpha \quad \text{and} \quad \frac{B}{C} = \beta. \tag{21}$$

Equations (20) and (17) give

$$\left(\frac{\alpha_4}{\alpha}\right)_4 + \left(\frac{\beta_4}{\beta}\right)_4 = \frac{2H^2\alpha^{n-1}}{(1-n)} \tag{22}$$

and

$$\left(\frac{\beta_4}{\beta}\right)_4 = 0. \tag{23}$$

From eqs (22) and (23) we obtain

$$\alpha = \left[\left(\frac{1-n}{2}\right)(at+b)\right]^{2/(1-n)}, \tag{24}$$

where

$$a = 2H\sqrt{\frac{1}{(1-n)\,(n-1)}}.$$

Integrating eq. (23), we get

$$\beta = d\mathrm{e}^{ct}, \tag{25}$$

where b, c and d are constants of integration.

Using eqs (24) and (25), from eqs (21) and (15) we obtain

$$B^2 = \alpha\beta = d\left[\left(\frac{1-n}{2}\right)(at+b)\right]^{2/(1-n)} \mathrm{e}^{ct},$$

$$C^2 = \frac{\alpha}{\beta} = \frac{1}{d}\left[\left(\frac{1-n}{2}\right)(at+b)\right]^{2/(1-n)} \mathrm{e}^{-ct},$$

and

$$A^2 = \left[\left(\frac{1-n}{2}\right)(at+b)\right]^{4n/(1-n)}.$$

For $n = \frac{1}{2}$

$$A^2 = \left[\frac{1}{4}\,(at+b)\right]^4,$$

$$B^2 = \left[\frac{1}{4}\,(at+b)\right]^4 \mathrm{e}^{t},$$

and

$$C^2 = \left[\frac{1}{4}\,(at+b)\right]^4 \mathrm{e}^{-t}.$$

Hence Bianchi type-I massive string magnetized barotropic perfect fluid cosmological model in bimetric theory of gravitation becomes

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \left[\frac{1}{4}\,(at+b)\right]^4 \left[\mathrm{d}x^2 + \mathrm{e}^{t}\mathrm{d}y^2 + \mathrm{e}^{-t}\mathrm{d}z^2\right]. \tag{26}$$

This model can be transformed through a proper choice of coordinates and constants to

$$\mathrm{d}s^2 = -\frac{4}{a}\mathrm{d}T^2 + T^4\left[\mathrm{d}X^2 + \mathrm{e}^{4T/a}\mathrm{d}Y^2 + \mathrm{e}^{-4T/a}\mathrm{d}Z^2\right]. \tag{27}$$

It is interesting to note that model (27) is free from singularity at $T = 0$, but it does not seem to be a flat model.

4. Some physical and geometrical aspects of the model

The physical parameters λ (string tension density), ρ (energy density) and p (isotropic pressure) of the string are given by

$$\rho = -\frac{(3+H^2)}{8\pi T^8},$$

$$\lambda = p = \frac{(1-H^2)}{8\pi T^8},$$

and

$$\rho_p = -\frac{1}{2\pi T^8}.$$

To gain further insight into the behaviour of the Universe (eq. (27)), we write the kinematical parameters for the model (27) as follows:

Spatial volume: $V^3 = (-g)^{1/2} = T^6$.

It can be observed that the volume is increasing as time increases, which shows that the expansion of Universe with time is inevitable.

Shear scalar: $\sigma^2 = \frac{1}{2}\sigma_{ij}\,\sigma^{ij} = \frac{2}{a}\sqrt{2}$.

Expansion scalar: $\theta = u^4_{;4} = \frac{6}{T}$.

Deceleration parameter: $q = -\frac{V_{44}V}{V_4^2} = -\frac{5}{6} < 0$.

The model (27) has no initial singularity at $T = 0$. From the above parameters, we observed that ρ, p, λ and expansion scalar (θ) decrease as time increases. At the singularity stage $T \to 0$, $V^3 \to 0$ and ρ, λ and θ are infinitely large, whereas at $T \to \infty$, ρ, λ and θ all vanish which represent an empty Universe. But all these parameters remain finite and physically significant for all $T > 0$. We also see that magnetic field does not have any affect in the expansion of the Universe. Moreover, as $\lim_{T\to\infty}(\sigma/\theta) \neq 0$, the model does not approach isotropy for large values of T. The shear scalar (σ) is constant throughout the evolution of the Universe for all values of T. The role of the deceleration parameter q seems to specify the expansion of the Universe. In the last decades, the standard cosmological model favoured a presently matter-dominated Universe expanding in a decelerated fashion. The positive value of the deceleration parameter q indicates that the model decelerates in the standard way. But the recent observation seems to be a negative value of this parameter which shows that the model inflates in the present case.

The scale factor R is given by

$$R^3 = ABC = \frac{1}{64}T^6.$$

Thus R increases as T increases.

5. Conclusion

There has been a lot of interest in cosmological model on the basis of Rosen's bimetric theory of gravitation. The purpose of Rosen's bimetric theory of gravitation is to get rid of the singularities that occur in general relativity that was appearing in the big-bang in cosmological models. Our investigation reveals that the bimetric theory of gravitation does not admit singularities which are of physical nature. Thus the difficulty faced in gravitation theory of relativity as regards the physical singularities does not exist in bimetric theory of gravitation. Hence, Rosen's basic aim to build the bimetric theory of relativity was fully fulfilled in the present work.

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