PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 76, No. 3
March 2011
pp. 489–500

Self-field effects on small-signal gain in two-stage free-electron lasers

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MS received 9 December 2009; revised 9 May 2010; accepted 26 August 2010

Abstract. Self-field effects, induced by charge and current densities of the electron beam, on gain in two-stage free-electron laser with nonuniform guide magnetic field is presented. The gain equation for small-signal has been derived analytically. The results of numerical calculations show a gain decrement for group I orbits and a gain enhancement for group II orbits, due to the self-field effects. The wiggler-induced self-magnetic field has a diamagnetic effect for group I orbits, whereas for group II, it has a paramagnetic effect. It is also found that using a nonuniform guide field, rather than a uniform one, causes the gain to increase.

Keywords. Relativistic electron beam; self-field effects; electromagnetic wiggler.

PACS Nos 41.60.Cr; 41.85.Ja; 41.60.Ap

1. Introduction

The free electron laser (FEL) is an extremely adaptable light source which can produce high power, coherent radiation over an extensive range of wavelengths. Considerable interest has been generated recently in utilizing an electromagnetic pumped field for a FEL. A model for two-stage FEL has been illustrated by Elias [1] and Sprangle *et al* [2], in which the first stage uses a static magnetic field for the pump, the output of which is used as an electromagnetic pump wave for the second stage. When the electron beam energy and wiggler period are constrained, the electromagnetic wigglers become attractive alternatives to magnetostatic wigglers for the production of shorter wavelengths [3]. With the present trend towards application of increasingly high current in FELs, it is of interest to consider the effects of the self-field, i.e., the fields generated by the electron beam itself. The electron beam and therefore the self-fields can exhibit a complex structure at the entrance to the wiggler. Self-field can produce chaos in FELs particulary in the vicinity of gyroresonance [4–6]. The self-field can either enhance or reduce the external pump field, depending on the latter's phase velocity and strength of the longitudinal guide magnetic field [7].

The FEL is saturated at a certain energy level due to the phase trapping of the electrons in the troughs of the ponderomotive wave formed by the beating of the wiggler and the radiation fields. After the trapping, the FEL radiation and electron keep exchanging the energy, which prevents the radiation from increasing further. One approach for improving the efficiency is to apply energy detuning, so that a little higher energy electrons can sustain in the resonance longer [8,9]. The side-band instability in FELs amplifies the shot noise in the incident electron beam in side bands separated by multiples of the synchrotron frequency from the resonance frequency determined by the wiggler period and the incident electron energy [10]. The instability is undesirable because it severely limits not only the spectral purity of the electromagnetic field but also the efficiency of the conversion of the electron energy into electromagnetic energy. It is shown that the use of tapered wigglers offers a way to overcome these problems, and hence the efficiency can be considerably increased by optimizing the tapering [11]. In the tapered undulator, the resonant electron energy decreases along the undulator length to enhance gain and efficiency in strong optical fields. Axially nonuniform magnetic fields are important for enhancing the extraction efficiency in free-electron lasers. The experimental observation of efficiency enhancement by linearly tapering the last quarter of the 10 m NISUS undulator at the NSLS-SDL was reported by Watanabe *et al* [12]. A different concept of achieving high extraction efficiency using a stair-step tapered magnetic field in a FEL was illustrated by Nguyen and Freund [13]. Levush *et al* [14] found that in a tapered FEL a tunability of about 25% can be achieved without significant reduction in the output power. This is particularly important for applications in which the radiation source is required to operate at high efficiency. For example, the tunability requirement for some fusion applications is of the order of 5% for a source with multi-megawatt output power at frequencies around 150 GHz. This can be met using a tapered FEL amplifier. An alternative efficiency enhancement scheme for electromagnetic wave wigglers which employs a nonuniform guide magnetic field was described by Freund *et al* [15,16] and Mehdian *et al* [17]. In this paper, the specific configuration to be investigated consists of the propagation of an electron beam counter to that of a circularly polarized electromagnetic wave. The electromagnetic wave acts as a wiggler which induces an undulatory motion on the beam [18]. Mutual influence of the electron velocity and self-magnetic field is considered to account for the total self-magnetic field.

The purpose of this paper is to study the influence of diamagnetic and paramagnetic, generated by the effects of self-fields, for gain in electromagnetically pumped free-electron laser with nonuniform guide magnetic field. The organization of this paper is as follows: the derivations of self-magnetic field by employing the self-consistent method and the equations of motion are presented in §2. A small-signal gain equation is derived analytically in §3. In §4 numerical studies and discussion are given. Section 5 gives a summary and conclusions.

2. Electron trajectories

Consider a beam with uniform azimuthally symmetric profiles in both density and velocity. In such a case, the beam density $n_{\mathrm{b}}(r) = n_{\mathrm{b}} = \text{const.}$ for $r \leq r_{\mathrm{b}}$ and $n_{\mathrm{b}}(r) = 0$ for $r > r_{\mathrm{b}}$, where n_{b} is the number density of the electrons and r_{b} is the beam radius.

The self-electric field induced by steady-state charge density of the non-neutral electron beam can be obtained by solving Poisson's equation, $\mathbf{E}_s = -2\pi e n_\mathrm{b}(x\hat{e}_x + y\hat{e}_y)$. The lowest-order representation for the self-magnetic field is obtained under the assumption that the beam propagates paraxially with $\mathbf{v} = v_\parallel \hat{e}_z$ for $r \le r_\mathrm{b}$ and is zero otherwise. In this case, the self-magnetic field is determined by Ampere's law, $\mathbf{B}_s = 2\pi e n_\mathrm{b}\beta_\parallel(y\hat{e}_x - x\hat{e}_y)$. The configuration of interest consists of a relativistic electron beam propagating through a nonuniform guide magnetic field, $\mathbf{B}_0(z) = B_0(1 + k_0 z)\hat{e}_z$, and a backward propagating electromagnetic wave described by

$$\mathbf{B}_\mathrm{w} = B_\mathrm{w}[\hat{e}_x \cos(k_\mathrm{w} z + \omega_\mathrm{w} t) + \hat{e}_y \sin(k_\mathrm{w} z + \omega_\mathrm{w} t)], \tag{1}$$

$$\mathbf{E}_\mathrm{w} = -\frac{\omega_\mathrm{w}}{ck_\mathrm{w}} B_\mathrm{w}[\hat{e}_x \sin(k_\mathrm{w} z + \omega_\mathrm{w} t) - \hat{e}_y \cos(k_\mathrm{w} z + \omega_\mathrm{w} t)], \tag{2}$$

where B_w denotes the amplitude of the wiggler magnetic field, $(\omega_\mathrm{w}, k_\mathrm{w})$ describe the frequency and wave number, and $k_0 = \mathrm{d}\ln B_0(z)/\mathrm{d}z$ is the inverse scale length for variation of the magnetic field. The equations of motion of the electron in the wiggler frame can be written as

$$\begin{aligned}\frac{\mathrm{d}\beta_1}{\mathrm{d}\tau} &= \omega_\mathrm{b}^2(1 - \beta_3^2 - \beta_1^2)\chi_1 + \beta_2[\beta_3 + \beta_\mathrm{p}(1 + \Omega_\mathrm{w}\beta_1) \\ &\quad - \Omega_0(1 + \kappa_0\chi_3) - \omega_\mathrm{b}^2\beta_1\chi_2],\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\mathrm{d}\beta_2}{\mathrm{d}\tau} &= \omega_\mathrm{b}^2(1 - \beta_3^2 - \beta_1^2)\chi_2 - \beta_1[\beta_3 + \beta_\mathrm{p} - \Omega_0(1 + \kappa_0\chi_3) + \omega_\mathrm{b}^2\beta_2\chi_1] \\ &\quad - \Omega_\mathrm{w} + \beta_\mathrm{p}\Omega_\mathrm{w}(\beta_2^2 - \beta_3),\end{aligned} \tag{4}$$

$$\frac{\mathrm{d}\beta_3}{\mathrm{d}\tau} = \Omega_\mathrm{w}\beta_2(1 + \beta_3\beta_\mathrm{p}), \tag{5}$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\tau} = -\gamma[\beta_2(\Omega_\mathrm{w}\beta_\mathrm{p} - \omega_\mathrm{b}^2\chi_2) - \beta_1\omega_\mathrm{b}^2\chi_1], \tag{6}$$

where β_1, β_2 and β_3 are the components of normalized velocity ($\beta_i \equiv v_i/c$), τ is the normalized time ($\tau \equiv ck_\mathrm{w}t$), $\beta_\mathrm{p} \equiv \omega_\mathrm{w}/ck_\mathrm{w}$ denotes the normalized phase velocity for the electromagnetic wiggler, $\Omega_{0,\mathrm{w}} \equiv eB_{0,\mathrm{w}}/m\gamma c^2 k_\mathrm{w}$ is the normalized axial guide and wiggler magnetic field frequencies, $\omega_\mathrm{b} \equiv (2\pi e^2 n_\mathrm{b}/m\gamma c^2 k_\mathrm{w}^2)^{1/2}$ is the normalized beam frequency, $\chi_i \equiv k_\mathrm{w} x_i$, and $\kappa_0 \equiv k_0/k_\mathrm{w}$. The steady-state orbits for this configuration are found by considering $\gamma = \text{const}$. This implies that $\beta_2 = 0$ and $\beta_3 = \beta_\parallel$. Hence, if we write $\beta_1 = \beta_{0\mathrm{w}}$, we find that

$$\beta_{0\mathrm{w}} = \frac{\Omega_\mathrm{w}(\beta_\parallel + \beta_\mathrm{p})^2}{\Omega_0(1 + \kappa_0\chi_3)(\beta_\parallel + \beta_\mathrm{p}) - (1 - \beta_\parallel^2)\omega_\mathrm{b}^2 - (\beta_\parallel + \beta_\mathrm{p})^2}. \tag{7}$$

The normalized displacement components are given by $\chi_1 = 0$, $\chi_2 = -\beta_{0\mathrm{w}}/(\beta_\parallel + \beta_\mathrm{p})$ and $\chi_3 = \beta_\parallel\tau$. The self-magnetic field induced by transverse velocity, generated by the wiggler magnetic field, is known as wiggler-induced self-magnetic field and is denoted by

$\mathbf{B}_{\rm sw}$. Assuming that the wiggler-induced self-magnetic field is proportional to the wiggler magnetic field, i.e., $\mathbf{B}_{\rm sw}^{(1)} = \lambda^{(1)}\mathbf{B}_{\rm w}$, we can find $\lambda^{(1)}$ from the Ampere's law as

$$\lambda^{(1)} \equiv \zeta = \frac{2\omega_{\rm b}^2(\beta_\parallel + \beta_{\rm p})^2}{\Omega_0(1+\kappa_0\chi_3)(\beta_\parallel + \beta_{\rm p}) - (1-\beta_\parallel^2)\omega_{\rm b}^2 - (\beta_\parallel + \beta_{\rm p})^2}. \tag{8}$$

$\mathbf{B}_{\rm sw}^{(1)}$ also produces a transverse velocity called wiggler-induced transverse velocity and its magnitude may be given by $\beta_{\rm w}^{(1)} = (1+\zeta)\beta_{0\rm w}$. Consequently, this velocity generates a new self-magnetic field, $\mathbf{B}_{\rm sw}^{(2)} = \lambda^{(2)}\mathbf{B}_{\rm w}$, and so on. Using Ampere's law, we can write a geometric series as $\lambda^{(n)} = \zeta(1+\zeta+\zeta^2+\cdots+\zeta^{n-1})$, $n = 1, 2, 3, \ldots$. For $n \to \infty$ this series is convergent, if $|\zeta| < 1$. Thus, the total wiggler-induced self-magnetic field can be written as

$$\begin{aligned}\mathbf{B}_{\rm sw} &= \sum_{i=0}^{\infty} \zeta^{i+1}\mathbf{B}_{\rm w} \\ &= \frac{2\omega_{\rm b}^2(\beta_\parallel + \beta_{\rm p})^2}{\Omega_0(1+\kappa_0\chi_3)(\beta_\parallel + \beta_{\rm p}) - (1-\beta_\parallel^2)\omega_{\rm b}^2 - (1+2\omega_{\rm b}^2)(\beta_\parallel + \beta_{\rm p})^2}\mathbf{B}_{\rm w}.\end{aligned} \tag{9}$$

The magnitude of the normalized wiggler-induced transverse electron velocity can be written as

$$\begin{aligned}\beta_{\rm w} &= \left(1 + \sum_{i=0}^{\infty} \zeta^{i+1}\right)\beta_{0\rm w} \\ &= \frac{\Omega_{\rm w}(\beta_\parallel + \beta_{\rm p})^2}{\Omega_0(1+\kappa_0\chi_3)(\beta_\parallel + \beta_{\rm p}) - (1-\beta_\parallel^2)\omega_{\rm b}^2 - (1+2\omega_{\rm b}^2)(\beta_\parallel + \beta_{\rm p})^2}.\end{aligned} \tag{10}$$

Equation (10) shows a resonant enhancement. Setting the denominator equal to zero, gives the characterization of two classes of trajectories called group I and group II orbits defined as

$$\Omega_0(1+\kappa_0\chi_3) < \frac{(1-\beta_\parallel^2\omega_{\rm b}^2)}{(\beta_\parallel + \beta_{\rm p})^2} + (1+2\omega_{\rm b}^2)(\beta_\parallel + \beta_{\rm p}) \quad \text{for group I orbits,} \tag{11}$$

$$\Omega_0(1+\kappa_0\chi_3) > \frac{(1-\beta_\parallel^2\omega_{\rm b}^2)}{(\beta_\parallel + \beta_{\rm p})^2} + (1+2\omega_{\rm b}^2)(\beta_\parallel + \beta_{\rm p}) \quad \text{for group II orbits.} \tag{12}$$

If we assume that the beam is monoenergetic and has uniform density in the transverse direction, then $\beta_{\rm p}$ will be determined by

$$\beta_{\rm p}^2 - 1 + \frac{2\omega_{\rm b}^2(\beta_\parallel + \beta_{\rm p})^2}{\Omega_0(1+\kappa_0\chi_3)(\beta_\parallel + \beta_{\rm p}) - (1-\beta_\parallel^2)\omega_{\rm b}^2 - (1+2\omega_{\rm b}^2)(\beta_\parallel + \beta_{\rm p})^2} = 0. \tag{13}$$

3. Small-signal gain

Consider an electron beam accompanied by its electromagnetic radiation propagating through a free-electron laser. The electric and magnetic fields of radiation in the wiggler frame may be written as

$$\begin{pmatrix} \mathbf{E}_\mathrm{r} \\ \mathbf{B}_\mathrm{r} \end{pmatrix} = E_\mathrm{r} \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \hat{e}_1 \\ \hat{e}_2 \end{pmatrix}, \tag{14}$$

where $\psi = (k_\mathrm{r} + k_\mathrm{w})z + (\omega_\mathrm{w} - \omega_\mathrm{r})t + \phi$ is the phase, k_r and ω_r are the wave number and angular frequency, $\hat{e}_1 = \hat{e}_x \cos(k_\mathrm{w}z + \omega_\mathrm{w}t) + \hat{e}_y \sin(k_\mathrm{w}z + \omega_\mathrm{w}t)$, and $\hat{e}_2 = -\hat{e}_x \sin(k_\mathrm{w}z + \omega_\mathrm{w}t) + \hat{e}_y \cos(k_\mathrm{w}z + \omega_\mathrm{w}t)$ are the unit vectors in the wiggler frame. Assuming $\tilde{E}_\mathrm{r} \equiv eE_\mathrm{r}/\gamma mc^2 k_\mathrm{w}$, eqs (3)–(5), by adding the effects of electromagnetic radiation, can be modified as

$$\frac{\mathrm{d}\beta_1}{\mathrm{d}\tau} = \omega_\mathrm{b}^2(1 - \beta_3^2 - \beta_1^2)\chi_1 + \beta_2[\beta_3 + \beta_\mathrm{p}(1 + \Omega_\mathrm{w}\beta_1) - \Omega_0(1 + \kappa_0\chi_3) - \omega_\mathrm{b}^2\beta_1\chi_2] - \tilde{E}_\mathrm{r}(1 - \beta_3)\cos\psi, \tag{15}$$

$$\frac{\mathrm{d}\beta_2}{\mathrm{d}\tau} = \omega_\mathrm{b}^2(1 - \beta_3^2 - \beta_1^2)\chi_2 - \beta_1[\beta_3 + \beta_\mathrm{p} - \Omega_0(1 + \kappa_0\chi_3) + \omega_\mathrm{b}^2\beta_2\chi_1] - \Omega_\mathrm{w} + \beta_\mathrm{p}\Omega_\mathrm{w}(\beta_2^2 - \beta_3) + \tilde{E}_\mathrm{r}(1 - \beta_3)\sin\psi, \tag{16}$$

$$\frac{\mathrm{d}\beta_3}{\mathrm{d}\tau} = \Omega_\mathrm{w}\beta_2(1 + \beta_3\beta_\mathrm{p}) + \tilde{E}_\mathrm{r}(1 - \beta_3)[\beta_2\sin\psi - \beta_1\cos\psi]. \tag{17}$$

Considering $\beta_\parallel \simeq 1$, the last terms in eqs (15)–(17) can be safely neglected in comparison with the other terms. As a result, the solutions of the above equations yield the same results as the steady-state eqs (3)–(5). The normalized electron axial velocity can be expanded for $\gamma \gg 1$ as

$$\beta_3 \simeq 1 - \frac{1}{2\gamma^2} \times \left[1 + \frac{\gamma^2\Omega_\mathrm{w}^2(\beta_\parallel + \beta_\mathrm{p})^4}{(\Omega_0(1 + \kappa_0\chi_3)(\beta_\parallel + \beta_\mathrm{p}) - (1 - \beta_\parallel^2)\omega_\mathrm{b}^2 - (1 + 2\omega_\mathrm{b}^2)(\beta_\parallel + \beta_\mathrm{p})^2)^2}\right]. \tag{18}$$

Employing the method used by Mehdian *et al* [19], i.e., $z(t) = c\beta_\parallel t + c\int_0^t \int_0^{t''} \dot{\beta}_z(t')\mathrm{d}t'\mathrm{d}t''$, we can write

$$z(t) = c\beta_\parallel t + \frac{c^2k_\mathrm{w}F}{2}t^2 + \frac{c^2k_\mathrm{w}\Lambda}{\Omega}\left[\cos(\Omega t + \phi) - \cos\phi + \Omega t\sin\phi\right], \tag{19}$$

where

$$F = \frac{\kappa_0\beta_\parallel\Omega_0\Omega_{\rm w}^2(\beta_\parallel+\beta_{\rm p})^5}{[(\Omega_0+\Omega_0\kappa_0\beta_\parallel)(\beta_\parallel+\beta_{\rm p})-(1-\beta_\parallel^2)\omega_{\rm b}^2-(1+2\omega_{\rm b}^2)(\beta_\parallel+\beta_{\rm p})^2]^3}, \tag{20}$$

$$\Lambda = \frac{\tilde{E}_{\rm r}}{\Omega\gamma^2}\beta_{\rm w} \times\left[1-\frac{\beta_{\rm w}^2\gamma^2(\beta_\parallel+\beta_{\rm p})^2}{(\Omega_0+\Omega_0\kappa_0\beta_\parallel)(\beta_\parallel+\beta_{\rm p})-(1-\beta_\parallel^2)\omega_{\rm b}^2-(1+2\omega_{\rm b}^2)(\beta_\parallel+\beta_{\rm p})^2}\right], \tag{21}$$

$$\Omega = [(k_{\rm r}+k_{\rm w})c\beta_\parallel+(\omega_{\rm w}-\omega_{\rm r})]/ck_{\rm w}. \tag{22}$$

The energy exchange of the electron with the radiation field is given by $\dot{\gamma} = -\beta_{\rm w}\gamma\tilde{E}_{\rm r}\cos\psi$. Inserting eq. (19) into $\dot{\gamma}$, we may get

$$\dot{\gamma} = -ck_{\rm w}\gamma\tilde{E}_{\rm r}\beta_{\rm w}\cos\left[c^2k_{\rm w}(k_{\rm r}+k_{\rm w})\frac{\Lambda}{\Omega}(\cos(\Omega t+\phi)-\cos\phi+\Omega t\sin\phi) +\Omega t+\phi+\frac{c^2k_{\rm w}}{2}F(k_{\rm w}+k_{\rm r})t^2\right]. \tag{23}$$

Taking the average of energy exchange from over all phases ϕ, we obtain

$$\langle\dot{\gamma}\rangle_\phi = \frac{c^3k_{\rm w}^2}{2}\gamma\tilde{E}_{\rm r}\beta_{\rm w}(k_{\rm r}+k_{\rm w}) \times\left[\frac{\Lambda}{\Omega}(\Omega t\cos\Omega t-\sin\Omega t)-\frac{F}{2}\sqrt{\frac{\pi}{2}}C\left(\sqrt{\frac{2}{\pi}}t\right)\right], \tag{24}$$

where $C(t)$ is the Fresnel function. Integrating the above equation over the electron transit time through the wiggler interaction length yields the average change in γ per electron, i.e.,

$$\begin{aligned}\langle\Delta\gamma\rangle_\phi &= \int_0^T\langle\dot{\gamma}\rangle_\phi{\rm d}t\\ &= \frac{c^3k_{\rm w}^2}{2}\gamma\tilde{E}_{\rm r}\beta_{\rm w}(k_{\rm r}+k_{\rm w})\left[\frac{\Lambda}{\Omega^2}(-2+2\cos\Omega T+\Omega T\sin\Omega T)\right.\\ &\quad\left.-\frac{F}{2}\left(\sqrt{2\pi}TC\left(\sqrt{\frac{2}{\pi}}T\right)-\sin T^2\right)\right],\end{aligned} \tag{25}$$

where $L(=c\beta_\parallel T)$ is the FEL interaction length. The small-signal gain is defined by $G = \Delta P_{\rm r}/P_{\rm r}$, in which $P_{\rm r} = (c\pi r_{\rm b}^2E_{\rm r}^2)/4\pi$ is the electromagnetic power and $\Delta P_{\rm r}$ is the change in electromagnetic power in one transit, i.e., $\Delta P_{\rm r} = -mc^3 n_{\rm b}\pi r_{\rm b}^2 \beta_\parallel \langle \Delta\gamma\rangle_\phi$. Considering the definitions of $P_{\rm r}$, $\Delta P_{\rm r}$ and eq. (25), G can be expressed as

$$G = \frac{\omega_{\rm b}^2 L^3 k_{\rm w}^2 (k_{\rm r}+k_{\rm w})\beta_{\rm w}^2}{\gamma^2\beta_\parallel^2}\left[\left(1-\frac{\gamma^2(\beta_\parallel+\beta_{\rm p})^2\beta_{\rm w}^2}{\sigma}\right)g(\Omega T) + \frac{\kappa_0\Omega_0\gamma^2\beta_\parallel(\beta_\parallel+\beta_{\rm p})\beta_{\rm w}}{2\sigma}\vartheta(T)\right], \tag{26}$$

where

$$g(\Omega T) = \frac{1}{\Omega^3 T^3}[2 - 2\cos\Omega T - \Omega T\sin\Omega T],$$
$$\vartheta(T) = \frac{1}{T^3}\left[\sqrt{2\pi}\,T C\left(\sqrt{\frac{2}{\pi}}T\right) - \sin T^2\right],$$
$$\sigma = (\Omega_0 + \Omega_0\kappa_0\beta_\parallel)(\beta_\parallel+\beta_{\rm p}) - (1-\beta_\parallel^2)\omega_{\rm b}^2 - (1+2\omega_{\rm b}^2)(\beta_\parallel+\beta_{\rm p})^2. \tag{27}$$

The extrema of gain occur for $\Omega T \simeq \pm 2.6$ at which $g(\Omega T) \simeq \pm 0.135$, and $\vartheta(T) \simeq \pm 2.8\times 10^{-5}$ corresponding to wave numbers,

$$k_{\rm r} = \frac{k_{\rm w}}{1-\beta_\parallel}\left[1+\beta_{\rm p} \mp \frac{2.6\beta_\parallel}{k_{\rm w}L}\right]. \tag{28}$$

Thus the gain extrema is given by

$$G_{\max} \simeq \frac{\omega_{\rm b}^2 L^3 k_{\rm w}^3 \beta_{\rm w}^2}{\gamma^2\beta_\parallel^2(1-\beta_\parallel)}\left[1+\beta_{\rm p} \mp \frac{2.6}{k_{\rm w}L}\beta_\parallel\right] \times\left[\pm 0.135\left(1-\frac{\gamma^2(\beta_\parallel+\beta_{\rm p})^2\beta_{\rm w}^2}{\sigma}\right) \pm 2.8\times 10^{-5}\frac{\kappa_0\Omega_0\gamma^2\beta_\parallel(\beta_\parallel+\beta_{\rm p})\beta_{\rm w}}{2\sigma}\right], \tag{29}$$

where $\pm$ signs must be chosen to give a positive gain in eq. (29). Eliminating self-fields, the maximum gain reduces to

$$G_{0\max} \simeq \frac{\omega_{\rm b}^2 L^3 k_{\rm w}^3 \Omega_{\rm w}^2(\beta_\parallel+\beta_{\rm p})^2}{\gamma^2\beta_\parallel^2(1-\beta_\parallel)(\Omega_0+\Omega_0\kappa_0\beta_\parallel-\beta_\parallel-\beta_{\rm p})^2}\left[1+\beta_{\rm p}\mp\frac{2.6}{k_{\rm w}L}\beta_\parallel\right]$$
$$\times\left[\pm 0.135\left(1-\frac{\gamma^2\Omega_{\rm w}^2(\beta_\parallel+\beta_{\rm p})^4}{(\Omega_0+\Omega_0\kappa_0\beta_\parallel-\beta_\parallel-\beta_{\rm p})^2((\Omega_0+\Omega_0\kappa_0\beta_\parallel)(\beta_\parallel+\beta_{\rm p})-(\beta_\parallel+\beta_{\rm p})^2)}\right)\right.$$
$$\left.\pm 1.4\times 10^{-5}\frac{\kappa_0\Omega_0\gamma^2\beta_\parallel\Omega_{\rm w}(\beta_\parallel+\beta_{\rm p})^2}{(\Omega_0+\Omega_0\kappa_0\beta_\parallel-\beta_\parallel-\beta_{\rm p})((\Omega_0+\Omega_0\kappa_0\beta_\parallel)(\beta_\parallel+\beta_{\rm p})-(\beta_\parallel+\beta_{\rm p})^2)}\right]. \tag{30}$$

If we set the normalized axial magnetic field equal to zero in eq. (29) in the absence of self-fields, we get

$$G_0 \simeq \pm 0.135 \frac{\omega_b^2 L^3 k_w^3 \Omega_w^2}{\gamma^2 \beta_0^2 (1-\beta_0)} (1+\gamma^2 \Omega_w^2) \left[1 + \beta_p \mp \frac{2.6}{k_w L} \beta_\| \right], \tag{31}$$

where $\beta_{p0} = (1 + 2\omega_b^2)^{1/2}$ and $\beta_0 = (1 - \Omega_w^2 - \gamma^{-2})^{1/2}$. Our results will be the same as that of Freund, Kehs and Granatstein [16], when we eliminate self-field effects and $\kappa_0 \to 0$.

4. Numerical studies and discussion

A numerical study of the diamagnetic and paramagnetic effects on gain in electromagnetically pumped free-electron laser with nonuniform guide magnetic field, by taking into account the self-electric and magnetic fields has been made. The normalized wiggler magnetic field frequency Ω_w is taken as 0.05, the Lorentz relativistic factor γ is chosen as 25, the normalized electron beam frequency ω_b is taken as 0.08 and the normalized inverse scale length for variation of the axial guide magnetic field is taken as 0.3. In this configuration, $\delta B_0 (= B_0 k_0 z)$ satisfies the condition $|\delta B_0 / B_0| \ll 1$.

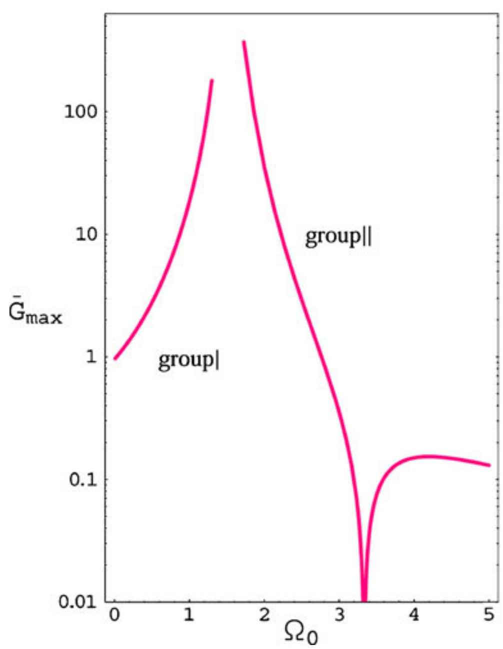

Figure 1. Graph of the normalized gain, $\bar{G}_{max}$, for electromagnetic wave wiggler vs. the normalized axial guide magnetic field in the logarithmic scale for $\Omega_w = 0.05$, $\omega_b = 0.08$, $\kappa_0 = 0.3$ and $\gamma = 25$.

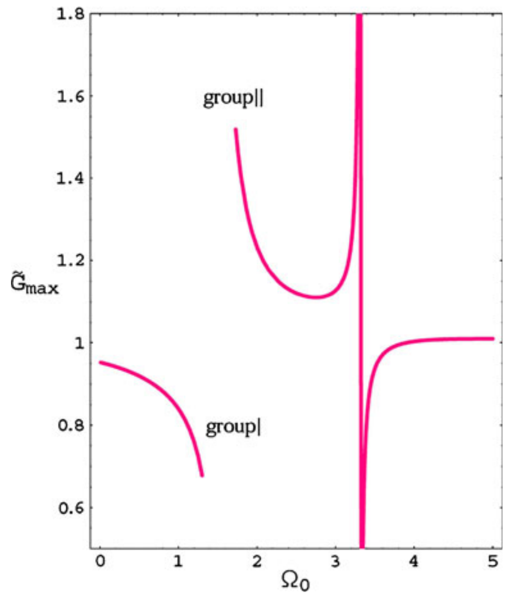

Figure 2. Graph of the gain ratio, $\tilde{G}_{\max}$, for electromagnetic wave wiggler vs. the normalized axial guide magnetic field for $\Omega_{\rm w} = 0.05$, $\omega_{\rm b} = 0.08$, $\kappa_0 = 0.3$ and $\gamma = 25$.

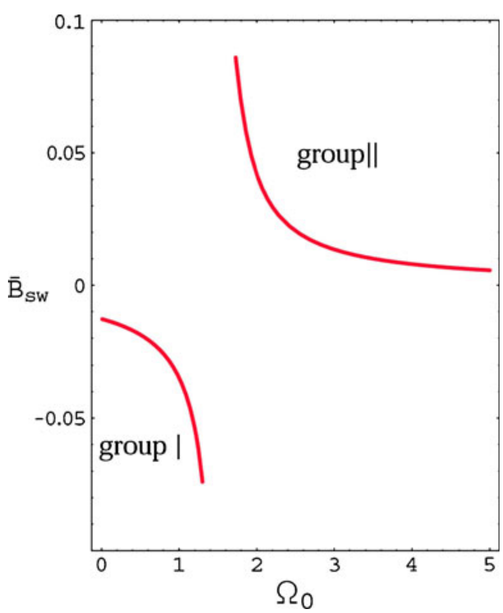

Figure 3. Graph of the transverse magnetic field, $\bar{B}_{\rm sw}$, vs. the normalized axial guide magnetic field for $\Omega_{\rm w} = 0.05$, $\omega_{\rm b} = 0.08$, $\kappa_0 = 0.3$ and $\gamma = 25$.

The graph of normalized gain, $\bar{G}_{\max} \equiv G_{\max}/G_0$ (i.e., the ratio of the maximum gain in the presence of the self-fields to maximum gain in the absence of the self-fields and axial guide magnetic field) as a function of the normalized axial magnetic field is depicted in figure 1. Here the FEL interaction length L is taken as $200\pi/k_{\mathrm{w}}$ and the normalized axial velocity $\beta_{\|}$ is taken as ~ 1 (i.e., $\beta_{\|} > 0.95$). It is shown that the normalized gain for group I orbits increases monotonically from 1 for $\Omega_0 = 0$ and goes to its peak (178.96) at $\Omega_0 = 1.3$. For group II orbits, the normalized gain starts from ($\Omega_0 = 1.72$, $\bar{G}_{\max} = 370.13$) then goes down to zero at $\Omega_0 = 3.27$, again goes up until it reaches the maximum value of around 0.2. Figure 2 shows the gain ratio $\tilde{G}_{\max} \equiv G_{\max}/G_{\max 0}$ (i.e., the ratio of the maximum gain in the presence of the self-fields to the maximum gain in the absence of the self-fields) as a function of the normalized axial magnetic field. As shown in figure 2, the gain ratios for group I orbits are less than 1, and so the effect of self-fields causes the gain decrement, while for group II orbits, the gain enhancement starts around ($\Omega_0 = 1.72$, $\tilde{G}_{\max} = 1.51$) then decreases to reach its minimum (1.11) at $\Omega_0 \simeq 2.75$. Thereafter, by increasing Ω_0 it goes up to its critical point at $\Omega_{0\mathrm{cr}} \simeq 3.34$. Figure 3 illustrates the normalized wiggler-induced self-magnetic field, $\bar{B}_{\mathrm{sw}} \equiv \mathbf{B}_{\mathrm{sw}}/\mathbf{B}_{\mathrm{w}}$, against Ω_0 for nonuniform guide field. As shown in this figure, for group I orbits $\bar{B}_{\mathrm{sw}}$ is negative, hence the wiggler-induced self-magnetic field is diamagnetic, and that is why the self-magnetic field decreases the gain for group I orbits. On the other hand, for group II orbits $\bar{B}_{\mathrm{sw}}$ is positive, which means that self-magnetic field is paramagnetic and causes the gain

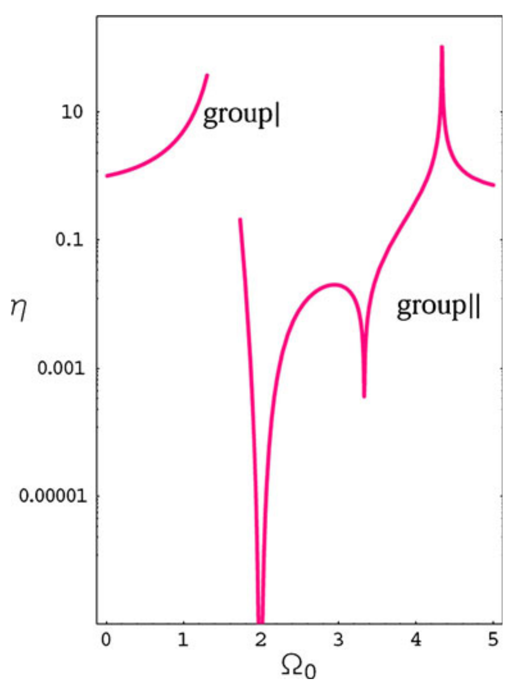

Figure 4. Graph of the normalized gain ratio, η, vs. the normalized axial guide magnetic field for $\Omega_{\mathrm{w}} = 0.05$, $\omega_{\mathrm{b}} = 0.08$, $\kappa_0 = 0.3$ and $\gamma = 25$.

to increase. The plot of normalized gain ratio, $\eta = G_{\max-t}/G_{\max-u}$, i.e., the ratio of maximum gain in the presence of nonuniform field to maximum gain in the uniform axial guide field, vs. Ω_0 for group I and II orbits, is depicted in figure 4. This figure shows that η for group I orbits is >1 and increases to reach its maximum (37.59) at $\Omega_0 = 1.3$, near its transverse velocity resonance frequency. For group II orbits, η is less than 1 when $1.72 < \Omega_0 < 4.16$, for $\Omega_0 \simeq 4.16$ it becomes 1 and then goes up sharply to its maximum (44.53) at $\Omega_{0\text{cr}} = 4.33$.

5. Summary and conclusions

In the conventional FEL, a relativistic beam of electrons passes through a magnetic undulator to amplify co-propagating light pulse. During the interaction, about half of the electrons in the beam lose energy to the light pulse, whereas the others take energy away from the light pulse. This process induces an energy spread in the electron beam that eventually leads to saturation and gain reduction in strong optical fields [20]. The design of the undulator or magnetic guide field can be modified to alter the light pulse/electron interaction and the final electron beam distribution. The tapered FEL decreases the undulator wavelength or the undulator field strength along the undulator length in order to maintain resonance with electrons that lose energy to the light pulse. The tapered FEL increases gain and efficiency in strong optical fields and extends the usual saturation limit to stronger optical fields. The enhancement of the efficiency in a tapered wiggler/guide magnetic field is accomplished by reducing the resonant energy for the interaction as the electron beam loses energy to the wave [14,20].

In this paper we have presented an analytical model for self-fields in a FEL with electromagnetic wave wiggler and tapered magnetic field. Mutual influence of the electron velocity and self-magnetic field has been considered to account for the total self-magnetic field. The wiggler-induced self-magnetic field has been developed as an infinite series by a self-consistent method. Our calculations show that the wiggler-induced self-magnetic field decreases the effective wiggler magnetic field for group I orbits, whereas for group II orbits it increases the effective wiggler magnetic field. Therefore, the wiggler-induced self-magnetic field has a diamagnetic effect for group I orbits and a paramagnetic effect for group II orbits [21]. The low-gain regime is relevant to short wavelength FELs driven by high energy but low current electron beams. In this regime, the gain of an electromagnetic wiggler is proportional to B_{w}^2 where B_{w} is the magnitude of the wiggler magnetic field. Therefore, we expect that in the presence of self-fields the gain decreases due to the diamagnetic effect for group I orbits and it increases due to the paramagnetic effect for group II orbits.

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