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Neutron skin effect of some Mo isotopes in pre-equilibrium reactions

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Abstract. The neutron skin effect has been investigated for even isotopes of molybdenum at 25.6 MeV $^{94-100}$Mo(p, xn) reaction using the geometry-dependent hybrid model of pre-equilibrium nuclear reactions. Here the initial neutron/proton exciton numbers were calculated from the neutron/proton densities obtained from an effective nucleon–nucleon interaction of the Skyrme type. Initial exciton numbers from different radii of even Mo isotopes were used to obtain the corresponding neutron emission spectra. In this investigation the calculated results are compared with the experimental data as also with each other. The results using central densities in the geometry-dependent hybrid model are in better agreement with the experimental data.

Keywords. Initial exciton number; neutron skin effect; geometry-dependent hybrid model.

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1. Introduction

Pre-equilibrium theory is widely used for analysing the spectra of particles emitted after the initial interaction of a nucleus with an incoming particle of energy E from 5 to 100 MeV. The exciton model proposed by Griffin [1] is very useful for this purpose [2]. It is generally recognized that the existence of a diffuse nuclear surface influences the emission spectra of particles at energies where pre-equilibrium (PEQ) reaction mechanism is dominant. The diffuse nuclear surface affects the PEQ emission in three ways: (1) the two-body interaction will be decreased in the surface region, (2) the hole energy will be limited by the shallower potential well at the surface and (3) the effective neutron skin will be formed in the neutron-rich targets. These effects were investigated for PEQ nucleon emission by some authors [3–5]. The exciton model and the geometry-dependent hybrid (GDH) model are well known for PEQ reactions. A hierarchy of configurations following one, two, three, etc., nucleon–nucleon scattering events is followed, each described by the exciton number $n = p + h$, where p and h are, respectively the numbers of excited particles above and below the Fermi energy. It has also been shown that with some freedom in the choice of parameters, these models for high energy process could give reasonable fit for the observed energy and angular distributions of the emitted particles [6–8]. The GDH model takes into account the density distribution of the nucleus and describes the nuclear density and potential by a two-parameter Fermi function and also the diffuse surface effects arising at the higher impact parameters are included in the model calculations [9,10].

There has been considerable interest in the thickness of neutron skin of the nuclei both theoretically and experimentally. The neutron skin thicknesses of nuclei provide us with a good testing ground for effective nucleon–nucleon interactions. In 2008, Tel *et al* suggested that the initial neutron and proton exciton numbers for nucleon-induced PEQ reactions could be calculated from the neutron and proton densities by using an effective nucleon–nucleon interaction with Skyrme force. This new method, taking into account single-particle wave functions, allows an increase or decrease in pre-compound emission spectra, with the simulation of effect, which are considered in the calculations, such as basic nucleon–nucleon potential interaction (Woods–Saxon, harmonic oscillator etc.) for nucleon-induced pre-compound reactions. It can help to investigate nuclear surface properties (and also neutron skin thickness effects) depending on the incident nucleon energy PEQ reactions and it gives more information about new nuclear reaction mechanism studies [11,12].

We study molybdenum in the present work because Mo and Mo containing alloys are important structural materials for fusion reactors, accelerator-driven systems, and in many other fields. It is also very useful as a refractory and corrosion resistant material in accelerator applications. Molybdenum is also used as a target material for the production of medically important radioisotopes [13]. To investigate the influence of a neutron skin in PEQ reactions induced by proton projectiles on neutron-rich even–even molybdenum targets, first the initial exciton numbers obtained from SKM* and SLy4 for proton-induced reaction on target nuclei were calculated. Then the new evaluated initial exciton numbers were introduced in the GDH model. The neutron skin effect was investigated using the GDH model of PEQ nuclear reactions for ^{94,96,98,100}Mo(p, xn) reaction at 25.6 MeV incident proton energies. The results obtained with different forces were compared with each other and with the experimental results.

2. Hartree–Fock calculations with Skyrme force

The Hartree–Fock (HF) method with the Skyrme force interaction is widely used for studying the properties of nuclei [13,14]. This method is successfully used for a wide range of nuclear characteristics, electromagnetic multipole moments, etc. [14–17]. This force consists of some two-body terms together with a three-body term,

$$V_{\mathrm{CS}} = \sum_{i<j} V_{ij}^{(2)} + \sum_{i<j<k} V_{ijk}^{(3)} \tag{1}$$

$$V_{ij}^{(2)} = t_0(1+x_0 p_\sigma)\delta(\vec{r}) + \frac{1}{2}t_1[\delta(\vec{r})\vec{k}^2 + \vec{k}'^2\delta(\vec{r})] + t_2\vec{k}'\cdot\delta(\vec{r})\vec{k} + iW_0(\vec{\sigma}_i - \vec{\sigma}_j)\cdot\vec{k}'\times\delta(\vec{r})\vec{k} \quad (2)$$

$$V_{ijk}^{(3)} = t_3\delta(\vec{r}_i - \vec{r}_j)\delta\vec{r}_j - \vec{r}_k. \quad (3)$$

The Skyrme forces with the three-body term replaced by a density-dependent two-body term are generalized and modified, which are unified in a single form by Ge *et al* [18] as an extended Skyrme force:

$$V_{\text{Skyrme}} = t_0(1+x_0 P_\sigma)\delta(\vec{r}) + \frac{1}{2}t_1(1+x_1 P_\sigma)\{\delta(\vec{r})\vec{k}^2 + \vec{k}'^2\delta(\vec{r}))\} + t_2(1+x_2 P_\sigma)\vec{k}'\cdot\delta(\vec{r})\vec{k} + \frac{1}{6}t_3(1+x_3 P_\sigma)\rho^\alpha(\vec{R})\delta(\vec{r}) + it_4(\vec{\sigma}_i+\vec{\sigma}_j)\cdot(\vec{k}'\times\delta(\vec{r})\vec{k}), \quad (4)$$

where $\vec{k}$ is the relative momentum, $\delta(\vec{r})$ is the delta function, P_σ is the space exchange operator, $\vec{\sigma}$ is the vector of Pauli spin matrices and $t_0, t_1, t_2, t_3, t_4, x_0, x_1, x_2, x_3, \alpha$ are Skyrme force parameters. The new Skyrme-like effective interactions (called SLy4) was proposed by Chabanat *et al* for neutron stars, supernovae and the neutron-rich nuclei [19]. These SLy4 parameters were adjusted to the properties of the symmetric infinite nuclear matter, with an additional constraint on the low- and high-density neutron equation of state. These parameter values and the other Skyrme force parameters can be found in [19–21]. In the interaction with Skyrme force, the neutron or proton densities are given by

$$\rho_q(\vec{r}) = \sum_{\beta\in q} w_\beta \psi_\beta(\vec{r})^+ \psi_\beta(\vec{r}), \qquad (q\text{: } n,\ \text{neutron or } p,\ \text{proton}), \quad (5)$$

where ψ_β is the single-particle wave function of the state β, the occupation probability of the state β is denoted by w_β.

3. New evaluated initial exciton number calculations using an effective nucleon–nucleon interaction for proton-induced PEQ reactions

The initial exciton numbers are very important in PEQ nuclear reactions. Nucleon-induced reactions are assumed in the hybrid and GDH models to begin with the excitation by the projectile of a two-particle–one-hole $(2p1h)$ doorway configuration. The incident nucleon may interact either with the same nucleon or with different isospin projection. But, in this energy range, the free scattering cross-sections or interactions between nucleons of differing isospin projections are approximately three times that of nucleons of the same isospin projections. So the σ_{np} free scattering n–p cross-sections are three times greater than σ_{nn} or σ_{pp} ($\sigma_{\text{np}} \approx 3\sigma_{\text{nn}}$ or σ_{pp}) over the energy range of interest for the precompound decay calculations under consideration [22].

The GDH model takes the initial exciton number as $n_0 = 3$ [9]. For proton-induced reactions, the initial proton exciton number X_p and the initial neutron exciton number X_n are given by Blann and Vonach [9] as

$$X_\mathrm{p} = \frac{2(3N + 2Z)}{3N + 2Z + 3N}, \quad X_\mathrm{n} = 2 - X_\mathrm{p}, \tag{6}$$

where N and Z are the neutron and proton numbers of the target nuclei, respectively.

The ALICE/ASH code is an advanced and modified version of the ALICE codes for PEQ calculations [23]. The initial exciton numbers in the ALICE/ASH code calculations can be expressed as

$$X_\mathrm{p} = 2\frac{(\sigma_\mathrm{pn}/\sigma_\mathrm{pp})N + 2Z}{2(\sigma_\mathrm{pn}/\sigma_\mathrm{pp})N + 2Z}, \quad X_\mathrm{n} = 2 - X_\mathrm{p}, \tag{7}$$

where σ_{xy} is the nucleon–nucleon interaction cross-section in the nucleus. The ratio of nucleon–nucleon cross-sections calculated by taking into account Pauli principle and the nucleon motion is parametrized as

$$\sigma_\mathrm{pn}/\sigma_\mathrm{pp} = \sigma_\mathrm{np}/\sigma_\mathrm{nn} = 1.375 \times 10^{-5}T^2 - 8.734 \times 10^{-3}T + 2.776, \tag{8}$$

where T is the kinetic energy of the projectile outside the nucleus. All the other code model parameters can be found in Broeders *et al* [23]. Considering neutron skin effect,

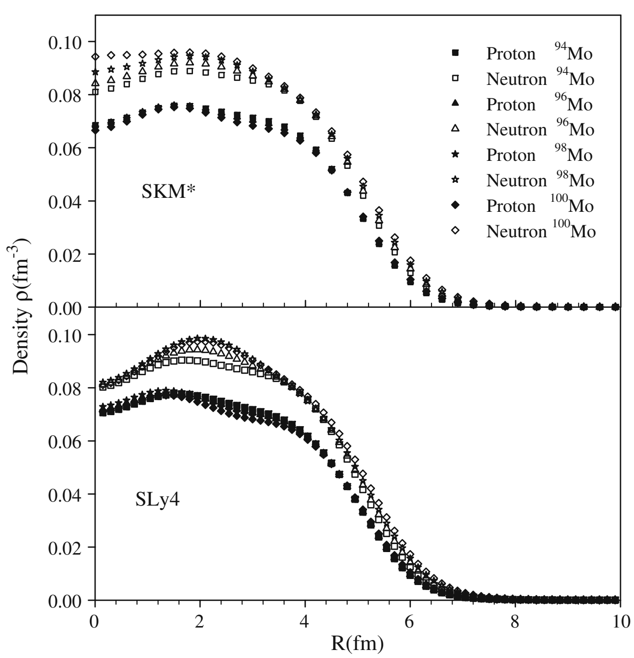

Figure 1. The comparison of calculated neutron and proton densities of Mo isotopes using SKM* and SLy4 parameters.

Castaneda *et al* [5] have suggested that the initial neutron (n) and proton (p) exciton numbers for each partial wave are calculated for a proton-induced reaction on a target. The initial neutron and proton exciton numbers for each partial wave can be given, respectively, as

$$X_{\mathrm{p}} \equiv \frac{2[3\rho_{\mathrm{n}}(R_l) + 2\rho_{\mathrm{p}}(R_l)]}{3\rho_{\mathrm{n}}(R_l) + 2\rho_{\mathrm{p}}(R_l) + 3\rho_{\mathrm{n}}(R_l)}, \quad X_{\mathrm{n}} = 2 - X_{\mathrm{p}}, \tag{9}$$

where l is the orbital angular momentum. The radius for the lth entrance channel partial wave has been defined by $R_l = \lambda(l + 1/2)$. In the GDH model, the Fermi energies and nuclear densities are defined to the impact parameter R_l [9]. Equation (9) preserves the relative np/nn scattering cross-section ratio of 3 as is used in the algorithm of eq. (6), but weighs collision probabilities by the nucleon densities, which have a radial dependence, rather than by the target neutron and proton numbers. Tel *et al* [12] have suggested that for nucleon-induced reaction cross-sections, the impact parameters $\rho_{\mathrm{n}}(R_l)$ and $\rho_{\mathrm{p}}(R_l)$ in eq. (9) can be replaced by the neutron density $\rho_{\mathrm{n}}(R)$ and the proton density $\rho_{\mathrm{p}}(R)$ from the values calculated by taking to single-particle wave functions with eq. (5). These $\rho_{\mathrm{n}}(R)$ and $\rho_{\mathrm{p}}(R)$ density values can be obtained using an effective interaction with Skyrme force. Therefore, for nucleon-induced pre-compound reactions the initial neutron and proton exciton numbers can be calculated from the neutron and proton densities using an effective nucleon–nucleon interaction with Skyrme force [11,12].

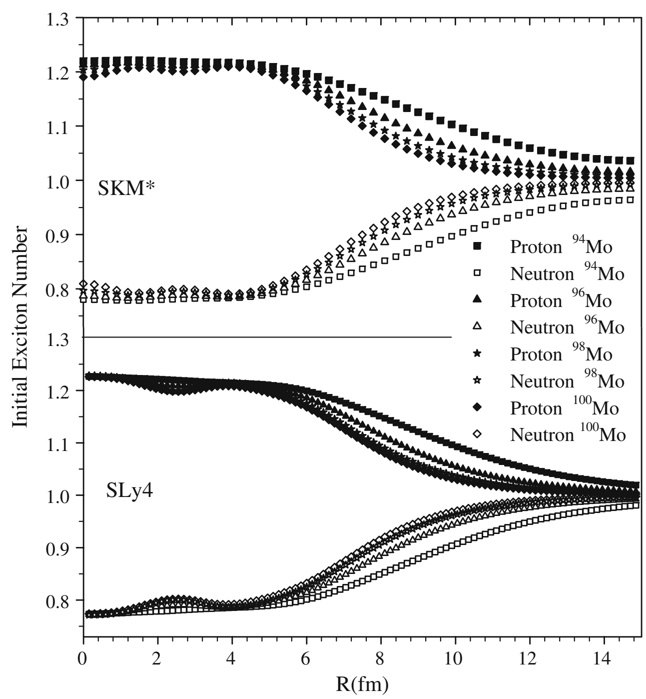

Figure 2. The calculated density-dependent initial exciton numbers using SKM* and SLy4 parameters.

Table 1. The calculated initial neutron X_n(EX1) and proton X_p(EX2) exciton numbers in the present work. σ_{pn}/σ_{pp} values are taken as 3.

Nucleus	R (fm)	SKM*				SLy4			
		$\rho_n(R)$	$\rho_p(R)$	X_n	X_p	$\rho_n(R)$	$\rho_p(R)$	X_n	X_p
^{94}Mo	0	0.081055	0.068484	0.78	1.22	0.080265	0.070599	0.77	1.23
	4.2	0.071667	0.059220	0.78	1.22	0.072042	0.058815	0.79	1.21
	8.4	0.000108	0.000053	0.86	1.14	0.000100	0.000048	0.86	1.14
^{96}Mo	0	0.084226	0.068362	0.79	1.21	0.081201	0.071719	0.77	1.23
	4.2	0.072068	0.058832	0.79	1.21	0.072001	0.058559	0.79	1.21
	8.4	0.000150	0.000053	0.90	1.10	0.000143	0.000048	0.90	1.10
^{98}Mo	0	0.088567	0.067804	0.80	1.20	0.082012	0.072859	0.77	1.23
	4.2	0.072549	0.058440	0.79	1.21	0.071923	0.058312	0.79	1.21
	8.4	0.000197	0.000053	0.92	1.08	0.000188	0.000048	0.92	1.08
^{100}Mo	0	0.094309	0.066560	0.81	1.19	0.081042	0.071089	0.77	1.23
	4.2	0.073314	0.058047	0.79	1.21	0.073894	0.057829	0.79	1.21
	8.4	0.000246	0.000053	0.93	1.07	0.000213	0.000047	0.93	1.07

4. Results and discussions

In the present paper, we have calculated the neutron and proton densities depending on radii using the HF method with an effective interaction including Skyrme force parameters (figure 1). The Skyrme force parameters of the SKM* and SLy4 can be found from [16–18]. The calculations using SKM* parameters with the single-particle wave functions of the harmonic oscillator potential were performed by HAFOMN code [24] and calculations using SLy4 with the single-particle wave functions of the Woods–Saxon potential were performed by Hartree–Fock program (modified for spin-orbit to accept SKI4) [25]. One can see that the neutron density increases with increasing mass number (in fact neutron number) up to 4 fm in figure 1.

Table 2. The initial neutron X_n(EX1) and proton X_p(EX2) exciton numbers obtained from the Blann and Vonach [9] and ALICE/ASH codes [23] at 25.6 MeV incident energy.

Nucleus	Blann and Vonach			ALICE/ASH		
	X_n	X_p	σ_{pn}/σ_{pp}	X_n	X_p	σ_{pn}/σ_{pp}
^{94}Mo	0.79	1.21	3	0.76	1.24	2.56
^{96}Mo	0.79	1.21	3	0.77	1.23	2.56
^{98}Mo	0.80	1.20	3	0.77	1.23	2.56
^{100}Mo	0.81	1.19	3	0.78	1.22	2.56

Table 3. The rms charge radii and neutron skin thickness of Mo isotopes (in fm).

Nucleus	r_c			SKM*			SLy4		
	Experimental	SKM*	SLy4	r_n	r_p	t	r_n	r_p	t
^{94}Mo	4.3515[b]	4.350	4.3574	4.346	4.277	0.069	4.3473	4.2842	0.0631
^{96}Mo	4.3825[a]	4.363	4.3714	4.396	4.293	0.103	4.3928	4.2984	0.0944
^{98}Mo	4.4075[a]	4.378	4.3851	4.445	4.311	0.134	4.4350	4.3123	0.1227
^{100}Mo	4.4428[a]	4.395	4.4095	4.494	4.330	0.164	4.4834	4.3371	0.1463

[a]Nadjakov [27].
[b]Angeli [28].

The relation between the calculated radii and the initial exciton numbers obtained from the densities with eq. (5) is shown in figure 2 and the obtained results are given in table 1. Above 10–12 fm, the initial proton/neutron exciton numbers (X_p and X_n, respectively) are closer to one. This means that in this region (outside the nucleus), proton is an unbound free particle and X_p is equal to one. The initial neutron X_n(EX1) and proton X_p(EX2) exciton numbers obtained from the Blann and Vonach [9] and ALICE/ASH codes [23] at $E_p = 25.6$ MeV are given in table 2. The rms charge radii of Mo isotopes using SKM*

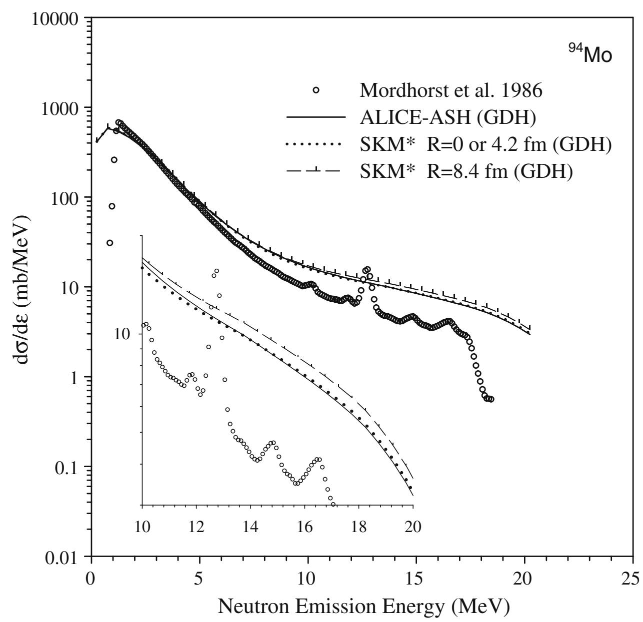

Figure 3. The comparison of neutron emission spectra of ^{94}Mo(p, xn) reaction with the values reported in literature at 25.6 MeV proton energy. The initial exciton numbers were calculated using neutron and proton densities from the centre to the surface ($R = 0$–8.4 fm) from SKM*. The experimental values were taken from ref. [26].

and SLy4 parameters are given in table 3. The obtained results with new Skyrme-like (SLy4) effective interactions agree well with the experimental values.

The neutron skin thickness t, a quantity of both theoretical and experimental interest, can be defined as the difference between the neutron rms radius r_n and the proton rms radius r_p,

$$t = r_\mathrm{n} - r_\mathrm{p}. \tag{10}$$

The neutron skin thickness calculated with 50 Skyrme interactions for the Mo isotopes are given in table 3. As can be seen from table 3, both the theoretical neutron skin thickness and the rms charge radii increase with increasing mass number.

The GDH calculations of the $^{94,96,98,100}\mathrm{Mo}(p, xn)$ reaction cross-sections using different initial exciton numbers (obtained from SKM* and SLy4 parameters) are compared with the experimental values at 25.6 MeV incident proton energy in figures 3–10. All calculations were performed in the framework of GDH models using ALICE/ASH computer code. The initial neutron exciton numbers and the initial proton exciton numbers were given with EX1 and EX2 (as in the same parameters of the ALICE/ASH code) in figures 3–10. In the ALICE/ASH (GDH) calculations, we have used the initial exciton number $n_0 = 3$ of 1 neutron, 1 proton and 1 hole (two-particle–one-hole). The initial exciton numbers were obtained from eqs (7), (8) for the theoretical values. By using eqs (7), (8), $\sigma_\mathrm{pn}/\sigma_\mathrm{pp}$ value was calculated as 2.56 at 25.6 MeV incident energy. All parameters of the ALICE/ASH code were taken as constant, but the initial exciton numbers of the

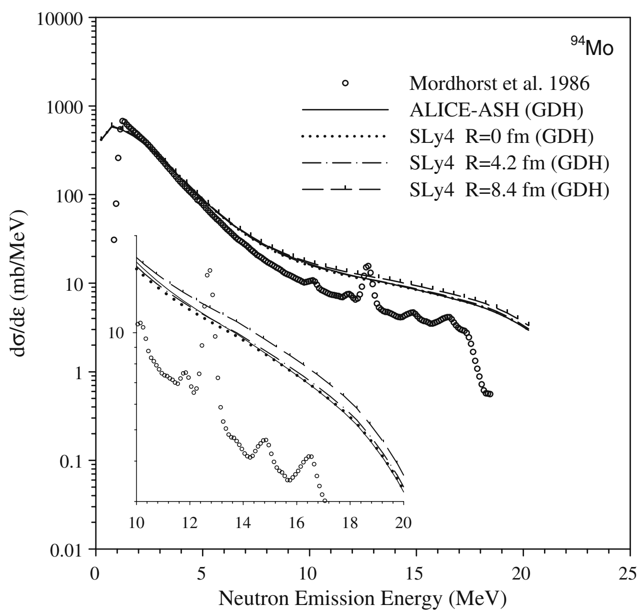

Figure 4. As figure 3 but calculations from SLy4.

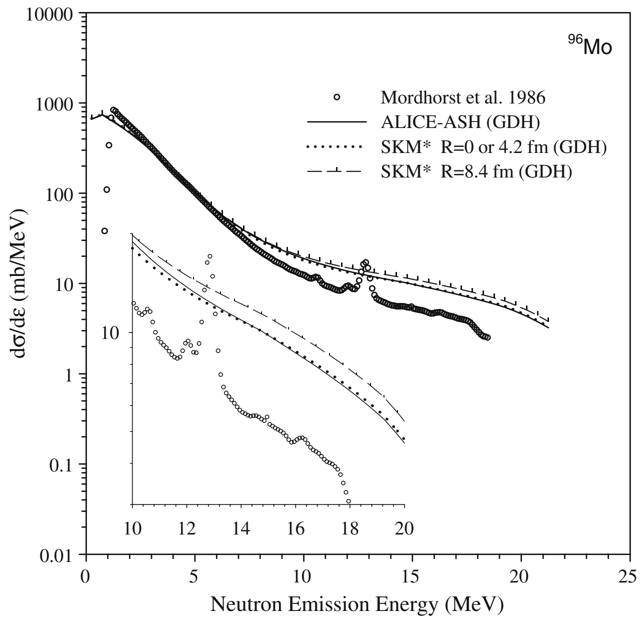

Figure 5. As figure 3 but calculations from SKM* for ^{96}Mo.

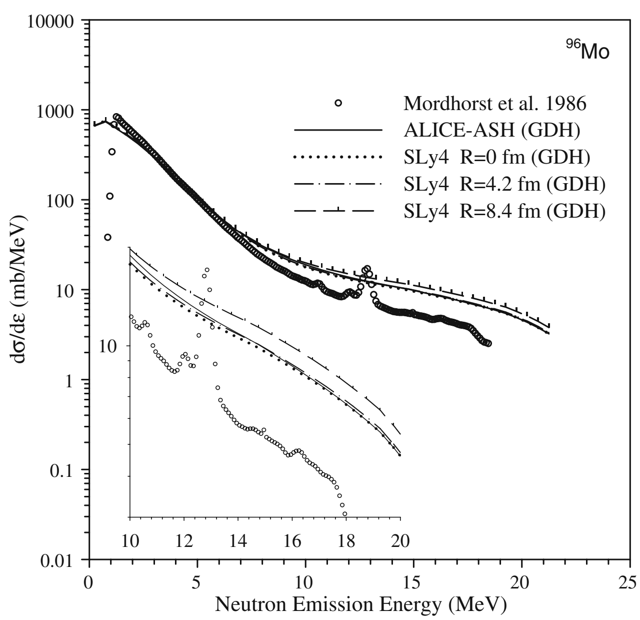

Figure 6. As figure 5 but calculations from SLy4.

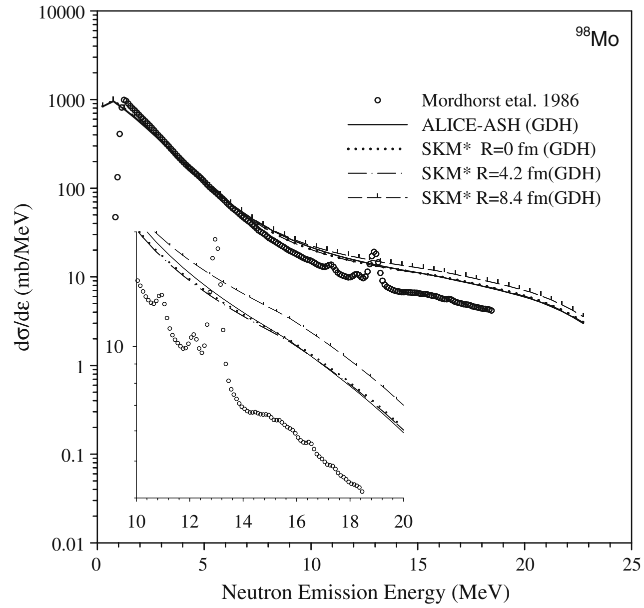

Figure 7. As figure 3 but calculations from SKM* for ^{98}Mo.

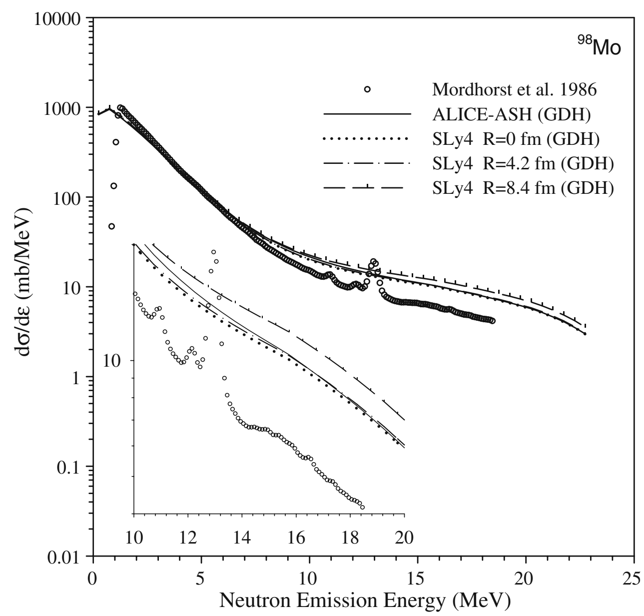

Figure 8. As figure 7 but calculations from SLy4.

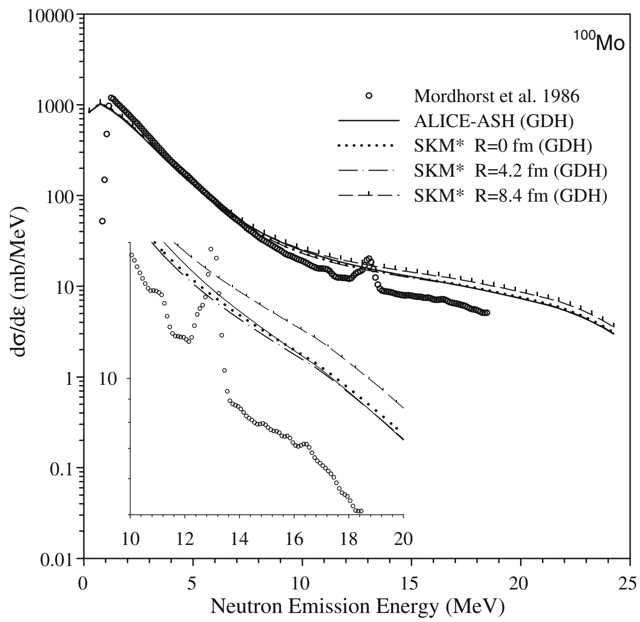

Figure 9. As figure 3 but calculations from SKM* for ^{100}Mo.

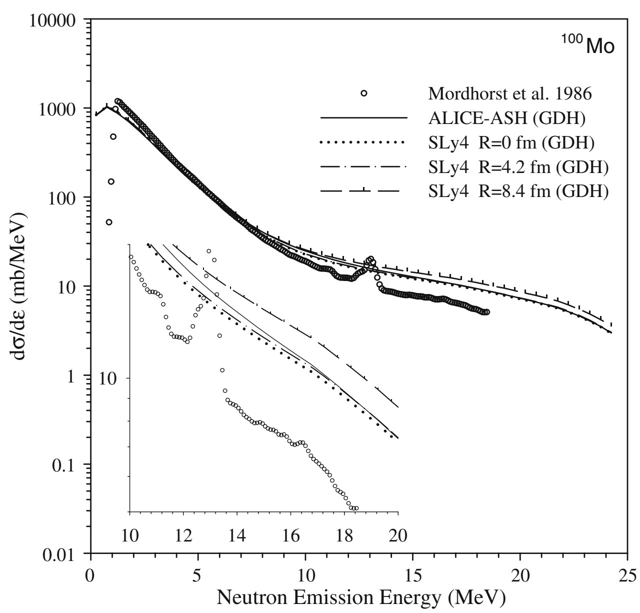

Figure 10. As figure 9 but calculations from SLy4.

calculated density values using SKM* and SLy4 parameters were varying. These densities, depending on the radius, were varying from $R = 0$ to 8.4 fm. Therefore, we have investigated the PEQ neutron emission spectra of the even–even Mo target nuclei for the initial exciton numbers depending on radii. The obtained results show that the GDH calculations at the centre ($R = 0$) agree well with the experimental values. But, the results do not agree with the experimental values with the increasing radius. The GDH model is made according to incoming orbital angular momentum l to account for the effects of the nuclear-density distribution. This leads to increased emission from the surface of the nucleus, and thus to increased emission of high-energetic particles.

Especially, the proton-induced nuclear reaction cross-section data can be used in technical applications such as the isotope production alternatives (for producing medical radioisotopes using cyclotrons), spallation reactions for producing neutrons in spallation neutron source and for other applications. So, when more experimental data for the proton scattering and emission differential cross-sections are obtained using new technologies, more reliable explanations for nuclear reaction mechanisms can be developed. These experimental cross-section data can be used for understanding the basic nucleon–nucleus interaction (and also nucleon–nucleon interaction), the binding energy systematic, nuclear structure and refined nuclear models.

Consequently, the calculated results using the initial exciton numbers with SKM* and SLy4 parameters at the centre ($R = 0$) are found to be in good agreement with the experimental values. The present GDH calculations for the PEQ reactions provide more information about the nuclear reaction mechanism and the effective nucleon–nucleon interaction as used in the Hartree–Fock calculations.

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