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Accretion, primordial black holes and standard cosmology

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Abstract. Primordial black holes evaporate due to Hawking radiation. We find that the evaporation times of primordial black holes increase when accretion of radiation is included. Thus, depending on accretion efficiency, more primordial black holes are existing today, which strengthens the conjecture that the primordial black holes are the proper candidates for dark matter.

Keywords. Primordial black hole; accretion; accretion efficiency.

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1. Introduction

Black holes formed in the early Universe are known as primordial black holes (PBHs). A comparison of the cosmological density of the Universe at any time after the Big Bang with the density associated with a black hole shows that PBHs would have of the order particle horizon mass. PBHs can thus span enormous mass range starting from 10^{-5} g to more than 10^{15} g. These black holes are formed as a result of initial inhomogeneities [1,2], inflation [3,4], phase transitions [5], bubble collisions [6,7] or the decay of cosmic loops [8]. In 1974, Hawking discovered that the black holes emit thermal radiation due to quantum effects [9]. So the black holes get evaporated depending upon their masses. Smaller the masses of the PBHs, quicker they evaporate. But the density of a black hole varies inversely with its mass. So high density is needed for forming lighter black holes, and such high densities are available only in the early Universe. Thus, primordial black holes are the only black holes whose masses could be small enough to have evaporated by now. Further, PBHs can act as seeds for structure formation [10] and can also form a significant component of dark matter [11–13].

As the cosmological environment is very hot and dense in the radiation-dominated era, it is expected that appreciable absorption of the energy-matter from the surroundings can take place. Calculation of such PBH accretion in standard cosmology have a long history but are plagued with significant uncertainties. The early work by Zel'dovich and Novikov [1] speculated that PBHs might even be able to grow as fast as the horizon. Subsequent works, especially by Carr and Hawking [2,14], made a convincing case that such growth could not occur and moreover, once the PBH became significantly smaller than the horizon, accretion would become very inefficient. But it has been noticed that such accretion is most effective in altered gravity scenarios. This accretion is responsible for the prolongation of the lifetime of PBHs in brane-world models [15] as well as in scalar–tensor models [16,17].

Using standard cosmology, Barrow and Carr [18] have studied the evaporation of PBHs. They have, however, not included the effect of accretion of radiation which seems to play an important role in scalar–tensor models. Majumdar *et al* [19] have provided a viable solution of the baryon asymmetry problem including accretion. In the present work, we include accretion of radiation while studying the evaporation of PBHs and have shown how evaporation times of PBHs change with accretion efficiency.

2. PBH evaporation in standard cosmology

For a spatially flat ($k = 0$) FRW Universe with scale factor a, the Einstein equation is [20]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \tag{1}$$

where ρ is the density of the Universe.

The energy conservation equation is

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)(1+\gamma)\rho = 0 \tag{2}$$

on assuming that the Universe is filled with perfect fluid described by equation of state $p = \gamma\rho$. The parameter γ is $\frac{1}{3}$ for radiation-dominated era $(t < t_1)$ and is 0 for matter-dominated era $(t > t_1)$, where time t_1 marks the end of the radiation-dominated era $\approx 10^{11}$ s.

Now eq. (2) gives

$$\rho \propto \begin{cases} a^{-4}, \; t < t_1 \\ a^{-3}, \; t > t_1 \end{cases}.$$

Using this solution in eq. (1), one gets the well-known temporal behaviour of the scale factor $a(t)$ as

$$a(t) \propto \begin{cases} t^{1/2}, \; t < t_1 \\ t^{2/3}, \; t > t_1 \end{cases}. \tag{3}$$

Due to Hawking evaporation, the rate at which the PBH mass (M) decreases is given by

$$\dot{M}_{\rm evap} = -4\pi r_{\rm BH}^2 a_{\rm H} T_{\rm BH}^4, \tag{4}$$

where $r_{\rm BH}$ is the black hole radius $= 2GM$ with G as Newton's gravitational constant, $a_{\rm H}$ is the black body constant and $T_{\rm BH}$ is the Hawking temperature $= 1/8\pi GM$.

Now eq. (4) becomes

$$\dot{M}_{\rm evap} = -\frac{a_{\rm H}}{256\pi^3}\frac{1}{G^2M^2}. \tag{5}$$

Integrating eq. (5), we get

$$M = \left[M_i^3 + 3\alpha(t_i - t)\right]^{1/3}, \tag{6}$$

where $\alpha = \frac{a_{\rm H}}{256\pi^3}\frac{1}{G^2}$ and M_i is the black hole mass at its formation time t_i. It is worthwhile to remark that we assume M_i to be the same as the horizon mass as conjectured in [21]. We shall, however, demonstrate that two masses will have different temporal growth.

3. Accretion

When a PBH passes through radiation-dominated era, the accretion of radiation leads to increase of its mass with the rate given by

$$\dot{M}_{\rm acc} = 4\pi f r_{\rm BH}^2 \rho_{\rm r}, \tag{7}$$

where $\rho_{\rm r}$ is the radiation energy density of the surrounding of the black hole $= \frac{3}{8\pi G}\left(\dot{a}/a\right)^2$ and f is the accretion efficiency. The accretion efficiency f depends upon complex physical processes such as the mean free paths of the particles comprising the radiation surrounding the PBHs. Any peculiar velocity of the PBH with respect to the cosmic frame can increase the value of f [19,22]. As the precise value of f is unknown, it is customary [23] to take the accretion rate to be proportional to the product of the surface area of the PBH and the energy density of radiation with $f \sim O(1)$.

After substituting the expressions for $r_{\rm BH}$ and $\rho_{\rm R}$, eq. (7) becomes

$$\dot{M}_{\rm acc} = 6fG\left(\frac{\dot{a}}{a}\right)^2 M^2. \tag{8}$$

Using eq. (3), we get

$$\dot{M}_{\rm acc} = \frac{3}{2} fG \frac{M^2}{t^2}. \tag{9}$$

On integration, eq. (9) gives

$$M(t) = \left[M_i^{-1} + \frac{3}{2} fG\left(\frac{1}{t} - \frac{1}{t_i}\right)\right]^{-1}. \tag{10}$$

Using horizon mass which varies with time as $M_{\rm H}(t) = G^{-1}t$, as initial mass of PBH, we get

$$M(t) = M_i\left[1 + \frac{3}{2} f\left(\frac{t_i}{t} - 1\right)\right]^{-1}. \tag{11}$$

We draw two important conclusions from eq. (11). First we obtain the variation of accreting mass with time for different f as shown in figure 1. The figure clearly indicates that the mass of the PBH increases with accretion efficiency.

For large t, $M_{\rm BH}$ of eq. (11) asymptotes to its maximum value as

$$M_{\rm max} = \frac{M_i}{1 - \frac{3}{2}f} \tag{12}$$

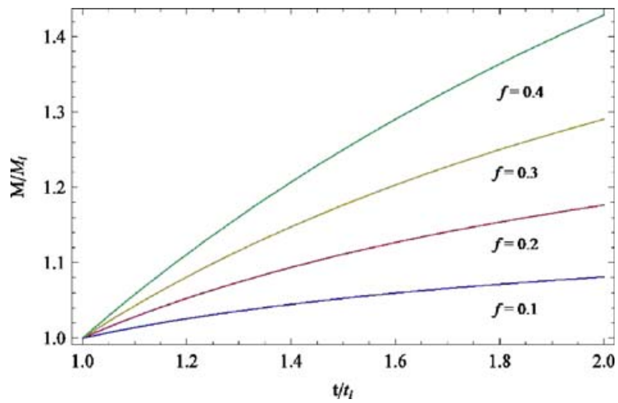

Figure 1. Variation of accreting mass for $f = 0.1, 0.2, 0.3, 0.4$.

which leads to an upper bound,

$$f < \frac{2}{3}. \tag{13}$$

The second conclusion is with regard to the variation of PBH mass *vis à vis* that of the horizon with time.

As the horizon mass grows as $M_{\mathrm{H}}(t) \sim G^{-1}t$, from eq. (11) one finds that M_{H} grows faster than the black hole mass M_{BH}. This is graphically shown in figure 2. Thus enough radiation density is available within the cosmological horizon for a PBH to accrete causally, making accretion effective in this scenario.

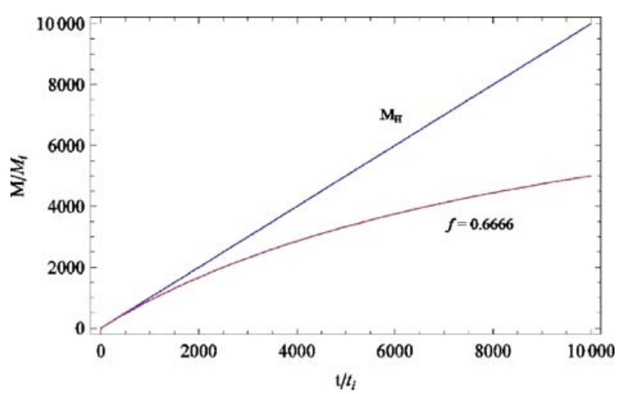

Figure 2. Variation of M_{PBH} having $f = 0.6666$ and M_{H} with t.

4. PBH dynamics in different era

Primordial black holes, as discussed before, are only formed in radiation-dominated era. So depending on their evaporation, we can divide PBHs into two categories. (i) PBHs evaporating in radiation-dominated era $(t < t_1)$ and (ii) PBHs evaporating in matter-dominated era $(t > t_1)$.

Case I $(t < t_1)$
Black hole evaporation eq. (6) implies

$$M = M_i \left[1 + \frac{3\alpha}{M_i^3}(t_i - t)\right]^{1/3}. \tag{14}$$

If we consider both evaporation and accretion simultaneously, then the rate at which primordial black hole mass changes is given by

$$\dot{M}_{\mathrm{PBH}} = \frac{3}{2} f G \frac{M^2}{t^2} - \alpha \frac{1}{M^2}. \tag{15}$$

This equation cannot be solved analytically. So we have solved it by using numerical methods.

For PBHs with formation mass $M_i^2 > a_{\mathrm{H}}/384 f G$, the magnitude of the first term (accretion) exceeds that of the second term (evaporation). In the radiation-dominated era for a PBH whose formation mass satisfies the above relation, accretion is dominant upto a value of t, say t_{c}, at which accretion rate equals evaporation rate (the PBH mass rises to a maximum value $M_{\max}$ at this stage), and after that evaporation dominates over accretion. For our calculation purpose, we have used $\alpha \approx G^{-2} = 10^{28}\ \mathrm{g}^3/\mathrm{s}$ and $G = 10^{-38}\ \mathrm{s/g}$.

For a given M_i, the solution as given by eq. (14) and the solution of eq. (15) are shown in figure 3. The figure clearly shows that the evaporation time of PBH increases with accretion efficiency.

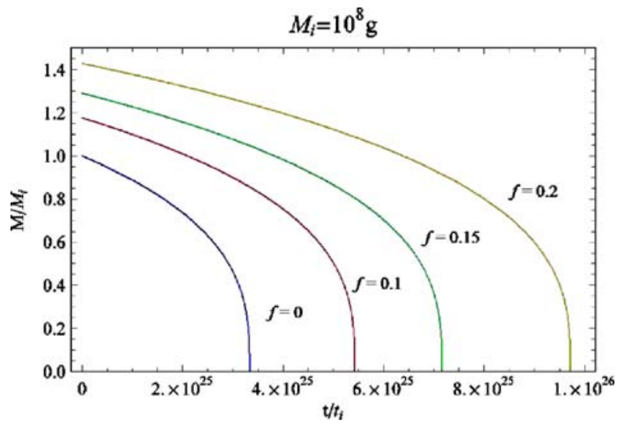

Figure 3. Variation of PBH mass for $f = 0, 0.1, 0.15, 0.2$.

Table 1. The formation times and initial masses of the PBHs which are evaporating now are displayed for several accretion efficiencies.

$t_{\text{evap}} = t_0 = 4.42 \times 10^{17}\text{s}$		
f	t_i (s)	M_i (g)
0	2.3669×10^{-23}	2.3669×10^{15}
0.1	2.0119×10^{-23}	2.0119×10^{15}
0.2	1.6568×10^{-23}	1.6568×10^{15}
0.3	1.3018×10^{-23}	1.3018×10^{15}
0.4	0.9467×10^{-23}	0.9467×10^{15}
0.5	0.5916×10^{-23}	0.5916×10^{15}
0.6	0.23669×10^{-23}	0.23669×10^{15}

Case II $(t > t_1)$
As there is no accretion in matter-dominated era, the first term in the combined eq. (15) for variation of M_{PBH} with time needs to be integrated only up to t_1 .

Based on numerical solutions with the above provision, we construct table 1 for the PBHs which are evaporating now.

It is clear from the table that accretion makes it possible for the PBHs evaporating now to be formed at earlier times with smaller initial masses.

5. Constraints on PBH

The fraction of the Universes' mass going into PBHs at time t is [2]

$$\beta(t) = \left[\frac{\Omega_{\text{PBH}}(t)}{\Omega_{\text{R}}}\right](1+z)^{-1}, \tag{16}$$

where $\Omega_{\text{PBH}}(t)$ is the density parameter associated with PBHs formed at time t, z is the red-shift associated with time t. Ω_{R} is the microwave background density having a value of 10^{-4}.

For $t < t_1$, red-shift definition implies, $(1+z)^{-1} = (t/t_1)^{1/2}\,(t_1/t_0)^{2/3}$. So

$$\beta(t) = \left(\frac{t}{t_1}\right)^{1/2}\left(\frac{t_1}{t_0}\right)^{2/3}\Omega_{\text{PBH}}(t) \times 10^4. \tag{17}$$

Using $M = G^{-1}t$, we can transcribe eq. (17) to write the fraction of the Universe going into PBHs' as a function of mass *M* as

$$\beta(M) = \left(\frac{M}{M_1}\right)^{1/2}\left(\frac{t_1}{t_0}\right)^{2/3}\Omega_{\text{PBH}}(M) \times 10^4. \tag{18}$$

Observations of the cosmological deceleration parameter imply $\Omega_{\text{PBH}}(M) < 1$ over all mass ranges for which PBHs have not evaporated yet. But presently evaporating PBHs(M_*) generate a γ-ray background where most of the energy is appearing at around 100 MeV

Table 2. Upper bounds on the initial mass fraction of PBHs that are evaporating for various accretion efficiencies f.

$t_{\rm evap} = t_0$		
f	M_* (g)	$\beta(M_*) <$
0	2.3669×10^{15}	5.71227×10^{-26}
0.2	1.6568×10^{15}	4.77918×10^{-26}
0.4	9.467×10^{14}	3.61264×10^{-26}
0.6	2.3669×10^{14}	1.80638×10^{-26}
0.666	2.36689×10^{12}	1.80637×10^{-27}
0.6666	2.36687×10^{11}	5.71233×10^{-28}

[24]. If the fraction of the emitted energy which goes into photons is ϵ_γ, then the density of the radiation at this energy is expected to be $\Omega_\gamma = \epsilon_\gamma \Omega_{\rm PBH}(M_*)$. Since $\epsilon_\gamma \sim 0.1$ and the observed γ-ray background density around 100 MeV is $\Omega_\gamma \sim 10^{-9}$, one gets $\Omega_{\rm PBH} < 10^{-8}$.

Equation (18), therefore, becomes

$$\beta(M_*) < \left(\frac{M_*}{M_1}\right)^{1/2} \times \left(\frac{t_1}{t_0}\right)^{2/3} \times 10^{-4}. \tag{19}$$

The variation of $\beta(M_*)$ with f drawn from variation of M_* with f is shown in table 2. The bound on $\beta(M_*)$ is strengthened as f approaches $2/3$, its maximum value.

6. Discussion and conclusion

Consideration of evaporation alone makes the primordial black holes which are created on or before 2.3669×10^{-23} s completely evaporate by the present time. But, we found that if we include accretion, then the primordial black holes which are created at the same instant of time will live longer depending on their accretion efficiency. Our analysis also leads to an upper bound on the accretion efficiency as $f < \frac{2}{3}$. Further, the constraint on the fraction of the Universes' mass going into PBHs' obtained by us is consistent with previous results [25,26] that $\beta(M_*) < 10^{-25}$.

Thus, accretion increases the number of existing PBHs depending on accretion efficiency, which lends support to the proposal of considering PBHs as the viable candidate for dark matter. We, thus, provide within standard cosmology a possible realization of the speculation advanced earlier [11–13].

In the present context, one may consider back reaction of the primordial black hole evaporation which can lead to non-trivial consequences [27]. Back reaction modifies the radius and temperature of PBH [28] which ultimately affects the accretion and evaporation rates. Thus it might be interesting to see in what way the resulting modification could in turn impact the evolution of black holes. Such effects, it is argued [29], may make the Hawking process terminate while the PBH still has macroscopic mass. There are also competing speculations that black holes completely evaporate leaving no remnants [30] or that black holes cease to evaporate as they approach Planck mass [31]. Whatever may be the cause of the stability of final remnant of radiating PBHs, the finite mass relics would provide a possible cold dark matter candidate [32].

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