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Sub-natural linewidth resonances in coherently-driven double Λ system

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Abstract. We investigate theoretically the pump-probe spectroscopy of coherently-driven four-level Λ system with two closely spaced excited common levels, thereby forming a double Λ system. Using the master equation approach, analytical results are obtained for the absorption spectrum of a weak probe in the presence of a strong pump. The model is applied to the double Λ system $5^2S_{1/2}F = 1,2 \rightarrow 5^2P_{3/2}F' = 1,2$ of ^{87}Rb atom. It is shown that the absorption spectrum consists of a triplet, of which one resonance is of sub-natural linewidth depending on the atom–field interaction parameters. The effect of Doppler broadening on the absorption spectrum is also investigated.

Keywords. Atomic coherence and interference; subnatural linewidth; electromagnetically-induced transparency.

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1. Introduction

Laser-induced coherence of atomic states and its use to control the optical response of a gas phase atomic medium have received much theoretical and experimental attention [1–8]. One interesting aspect of these studies is the possibility of producing atomic resonances with sub-natural linewidths [2–5]. Interest in these ultra-narrow sub-natural resonances has intensified in recent years owing to their applications in time and frequency standards, precision atomic clocks and ultra-sensitive magnetometry [4,5]. A prototype system that is investigated in this context is a three-level atomic system in Λ, V or double resonance configurations [1–5], where a strong pump field induces coherence between levels to which it is coupled and a weak probe field is used on the other transition to investigate this atomic coherence and its manifestation. Such three-level configurations are readily constructed from the hyperfine manifolds of D_1 or D_2 transitions of alkali atoms [9].

In this paper we investigate theoretically the sub-natural resonances in a coherently-driven Λ system, but unlike earlier reported works, we consider here

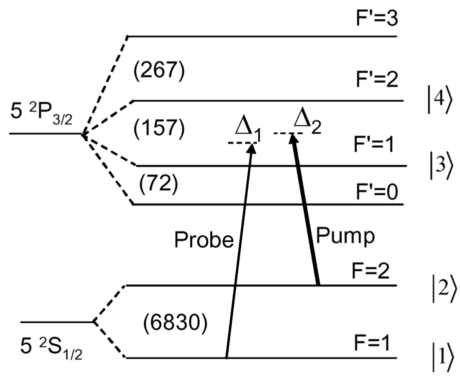

Figure 1. Schematic representation of energy levels in the D_2 transition of ^{87}Rb atom. Relevant hyperfine levels, which form the double Λ system, are $|1\rangle, |2\rangle, |3\rangle$ and $|4\rangle$. The detunings Δ_1 and Δ_2 are measured with respect to $|4\rangle$. The bracketed entries represent the separation between adjacent hyperfine levels in MHz.

a four-level system with closely spaced two excited common levels. The excitation scheme thus consists of two simultaneous Λ resonances, i.e., a double Λ resonance [6–8], coupled by a strong pump and a weak probe. Such a study has a very direct bearing on pump-probe coherent spectroscopy experiments in alkali atoms, where the excited hyperfine levels are relatively closely spaced. Further, the level configuration studied here is the simplest of the four-level systems and provides useful insight into the coherent dynamics of complex systems.

2. Theoretical formulation

We consider a typical experimental situation in the D_2 transition of ^{87}Rb atom as shown in figure 1. Here a strong pump laser with frequency Ω_2 and amplitude E_2 is used to dress the hyperfine transitions $5^2S_{1/2}F = 2 \rightarrow 5^2P_{3/2}F'$ and the resulting dressed states are probed by scanning a probe laser of frequency Ω_1 and amplitude E_1 across $5^2S_{1/2}F = 1 \rightarrow 5^2P_{3/2}F'$. The four-level subset forming the double Λ-system that is relevant for the discussion here consists of $|1\rangle, |2\rangle, |3\rangle$ and $|4\rangle$ levels such that the only non-vanishing dipole matrix elements d_{ij} are d_{13}, d_{14}, d_{23} and d_{24}. The separation between the excited levels is $S = \omega_{43} = \omega_4 - \omega_3$, which for ^{87}Rb is ~157 MHz [9].

The master equation for this system is

$$\begin{aligned}\frac{\partial\rho}{\partial t} &= -i[H_0,\rho] - \gamma_{31}\{A_{33}\rho - 2A_{13}\rho A_{31} + \rho A_{33}\}\\ &\quad -\gamma_{41}\{A_{44}\rho - 2A_{14}\rho A_{41} + \rho A_{44}\}\\ &\quad -\gamma_{32}\{A_{33}\rho - 2A_{23}\rho A_{32} + \rho A_{33}\}\\ &\quad -\gamma_{42}\{A_{44}\rho - 2A_{24}\rho A_{42} + \rho A_{44}\}. \end{aligned} \tag{1}$$

Here ρ is the reduced atomic density operator and H_0 is the semi-classical Hamiltonian that describes the atom-field system in a frame rotating at the fast optical frequencies,

$$\begin{aligned} H_0 = {} & \alpha_1(A_{14}+A_{41}) + \alpha'_1(A_{13}+A_{31}) \\ & +\alpha_2(A_{24}+A_{42}) + \alpha'_2(A_{23}+A_{32}) \\ & +(\Delta_1-\Delta_2)A_{22} + \Delta'_1 A_{33} + \Delta_1 A_{44} \end{aligned} \tag{2}$$

where the operator $A_{mn} = |m\rangle\langle n|$ with $|m\rangle$ representing the mth state of the atom $(m = 1, 2, 3, 4)$. In eq. (2) $2\gamma_{31}$, $2\gamma_{32}$, $2\gamma_{41}$ and $2\gamma_{42}$ are the Einstein A-coefficients, $\Delta_1 = \omega_{41} - \Omega_1$, $\Delta_2 = \omega_{42} - \Omega_2$, $\Delta'_1 = \omega_{31} - \Omega_2$ are the relevant detunings, α_1, α_2, α'_1 and α'_2 are the Rabi frequencies of the fields given by $\alpha_1 = \vec{E}_1 \cdot \vec{d}_{14}/2\hbar$, $\alpha_2 = \vec{E}_2 \cdot \vec{d}_{24}/2\hbar$, $\alpha'_1 = \vec{E}_1 \cdot \vec{d}_{13}/2\hbar$ and $\alpha'_2 = \vec{E}_2 \cdot \vec{d}_{23}/2\hbar$.

For a weak probe laser, steady-state solutions for ρ_{ij} can be obtained perturbatively up to the first order in α_1 and α'_1 respectively. We thus have

$$\left.\begin{aligned} &\rho_{11}^{(0)} = 1, \quad \rho_{ij}^{(0)} = 0 \ (i = j \neq 1) \\ &\rho_{ii}^{(1)} = \delta_{i1}, \quad \rho_{12}^{(1)} = -(\alpha_1\alpha_2 a_2 + \alpha'_1\alpha'_2 a_3)/B \\ &\rho_{13}^{(1)} = i[\alpha'_1 a_1 a_3 - \alpha_2(\alpha_1\alpha'_2 - \alpha'_1\alpha_2)]/B, \\ &\rho_{14}^{(1)} = i[\alpha_1 a_1 a_2 - \alpha'_2(\alpha_1\alpha'_2 - \alpha'_1\alpha_2)]/B \end{aligned}\right\}, \tag{3}$$

where

$$\begin{aligned} &a_1 = i(\Delta_2 - \Delta_1), \quad a_2 = \gamma_{31} + \gamma_{32} - i\Delta'_1, \quad a_3 = \gamma_{41} + \gamma_{42} - i\Delta_1, \\ &B = a_1 a_2 a_3 + \alpha_2^2 a_2 + \alpha_2'^2 a_3. \end{aligned} \tag{4}$$

The absorption A of the weak probe is then given by

$$A = -\mathrm{Im}\left(\frac{\rho_{13}\gamma_{31}}{\alpha'_1} + \frac{\rho_{14}\gamma_{41}}{\alpha_1}\right), \tag{5}$$

where ρ_{13} (ρ_{14}) is the induced polarization on the $|1\rangle \rightarrow |3\rangle$ ($|1\rangle \rightarrow |4\rangle$) transition. Substituting eqs (3) and (4) in (5), we obtain A for a stationary atom. In order to take into account the thermal velocity distribution of atoms, we average A over Maxwell–Boltzmann probability distribution

$$P(\omega) = \frac{1}{\sqrt{2\pi D^2}} \exp[-(\omega-\omega_0)^2/2D^2], \tag{6}$$

where $D = \Delta W_\mathrm{D}/2\sqrt{2\ln 2}$ and ΔW_D is the FWHM of the Doppler profile.

3. Results and discussion

Using the theoretical formulation of §2, we obtain probe absorption spectrum for ^{87}Rb atom in a double Λ configuration of figure 1. From the atomic data of ^{87}Rb

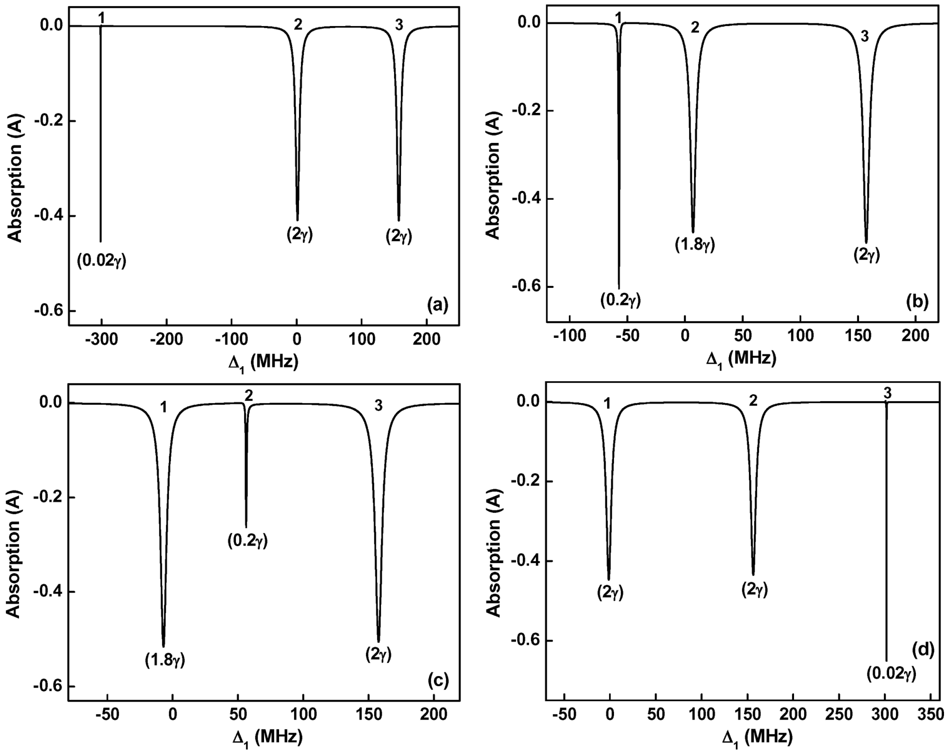

Figure 2. Probe absorption vs. detuning of the probe laser (Δ_1) calculated for $\alpha_2 = 20$ MHz. (**a**)–(**d**) correspond to $\Delta_2 = -300, -50, 50, 300$ MHz respectively. The bracketed entries give linewidth with respect to the natural linewidth (2γ).

[9], we have $\alpha_1' = \alpha_1$ and $\alpha_2' = \alpha_2/\sqrt{5}$. Also all detunings are not independent and in fact $\Delta_1' = \Delta_1 - S$ and $\Delta_2' = \Delta_2 - S$. Consequently, the probe absorption spectrum, $A(\Delta_1)$, is a function of α_2 and Δ_2, which are respectively the pump Rabi frequency and detuning for Λ configuration formed by $|1\rangle$, $|2\rangle$ and $|4\rangle$ levels. We further take $2(\gamma_{31} + \gamma_{32}) = 2(\gamma_{41} + \gamma_{42}) = 6.1$ MHz, which is the natural linewidth (2γ) of D_2 transition of ^{87}Rb.

In the absence of the inhomogenous broadening ($D = 0$) the calculated absorption spectra for fixed α_2 and a few representative values of Δ_2 are shown in figure 2. Note here that the absorption spectrum exhibits three resonances corresponding to the transitions $|1\rangle \rightarrow |d_j\rangle$ where $|d_j\rangle$, $j = 1, 2, 3$, are the dressed states generated by the coherent coupling of $|2\rangle \rightarrow |3\rangle, |4\rangle$ transitions by the pump laser. We observe that in general the widths of the resonances are different and interestingly, the resonance at $\Delta_1 \sim \Delta_2$ has sub-natural linewidth, which can be made as narrow as possible by a proper choice of Δ_2. We may also note here that the sum of the widths of the triplet spectrum is 4γ.

Figure 3 shows the effects of Doppler broadening on the probe absorption spectra for a few values of D. These calculations are performed for an experimental

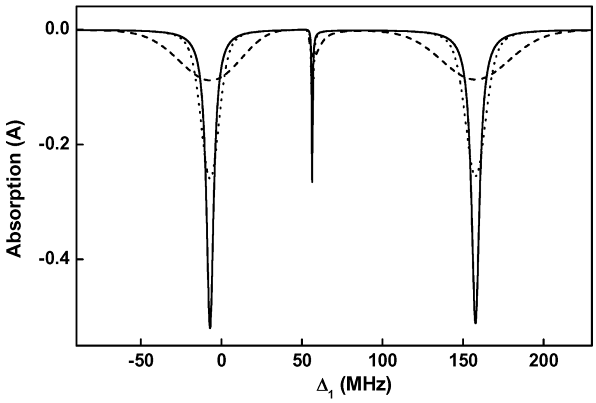

Figure 3. Effect of Doppler broadening on probe absorption spectra for $D = 0.0$ MHz (solid line), 5.0 MHz (dotted line) and 20 MHz (dashed line). Here $(\alpha_1, \alpha_2, \Delta_2) = (0.1, 20, 50)$ MHz.

situation of co-propagating pump and probe beams. It is seen from figure 3 that with increase in D, the resonances get broadened. Interestingly, the minimum at $\Delta_1 \sim \Delta_2$ survives in spite of Doppler averaging, which corresponds to the position of EIT resonance.

4. Conclusion

In this paper we have theoretically investigated the sub-natural resonances in a double Λ configuration interacting with a coherent bichromatic field consisting of a strong pump and a weak probe. Results are presented for an appropriate level scheme of ^{87}Rb. The probe absorption spectrum in such a configuration exhibits three resonances and one of these can be made as narrow as possible for appropriate choice of laser–atom interaction parameters. As Doppler averaging broadens the resonances, a Doppler-free spectroscopic technique is essential to observe such resonances experimentally.

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