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Gamow–Teller 1^+ states in $^{112-124}$Sb isotopes

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Abstract. The violated supersymmetry property of the pairing interaction between nucleons were restored using the Pyatov method [Pyatov and Salamov, *Nucleonica* **22**, 127 (1977)]. The eigenvalues and eigenfunctions of the restored Hamiltonian with the separable residual Gamow–Teller effective interactions in the particle–hole and particle–particle channels were solved within the framework of proton–neutron quasirandom phase approximation (pnQRPA). The Gamow–Teller resonance energies for $^{112-124}$Sb isotopes and the differential cross-sections for Sn(^{3}He, t)Sb reactions at $E(^3\text{He}) = 200$ MeV occurring by the excitation of the Gamow–Teller resonance state were calculated. The calculated values were compared with other calculations and the corresponding experimental data.

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1. Introduction

The spin–isospin excitations in nuclei, especially the Gamow–Teller (GT) excitations, play an important role in the investigation of the nuclear structure. These collective excitations also provide information on the Gamow–Teller resonance (GTR) energy position and the GT strength distributions in nuclei. Reliable evidence on the GT strength distribution at relatively low energies is of great importance in understanding the basic astrophysical processes such as nucleosynthesis, stellar collapse and supernova formation [1]. There are both experimental and theoretical studies on the description of the GT strength distribution in nuclei. As far as experimental studies in the literature are concerned, different methods were attempted to extract this distribution. Charge exchange reactions can be efficiently used for this purpose. These reactions provide an important probe of the spin–isospin response of the nuclei. For incident energies above 100 MeV, the isovector spin-flip component of the effective interaction was shown to be significant [2,3], so that reaction cross-sections arose mainly from spin–isospin transitions. Because the momentum transfer at forward angles and low excitation energies is small, the reaction cross-section is dominated by GT transitions with $\Delta L = 0$, $\Delta S = 1$ and $\Delta J^{\pi} = 1^{+}$. The cross-section, extrapolated to zero-momentum transfer, is proportional to the GT strength between the same states. Out of these reactions, the (p, n) reaction is the most common, and for several years, this was used successfully in the investigation of the GT strength distribution [4–18]. As an alternative probe for the extraction of the GT strength distribution, the (^{3}He, t) [19–30] and (^{6}Li, ^{6}He) [31–38] reactions have also been proposed and used.

Theoretical descriptions of the GT strength distributions in medium and heavy mass nuclei are given in some studies [39–48]. Kuzmin and Soloviev [39] calculated the fragmentation of the GTR in heavy nuclei within the framework of the quasi-particle phonon model. Their results showed that the GTR spread only up to the excitation energies of ∼30 MeV. Colo *et al* [40] studied the spreading of the GTR in ^{208}Bi and calculated its particle decay width using the continuum Tamm–Dancoff approximation (TDA) and the Hartree–Fock (HF) formalism with several types of the Skyrme interaction. They found that the strength distribution of the GTR was located around an interval of 18–24 MeV. The energy of the main peak of the GTR is higher than the experimental value by 2–4 MeV. The spreading properties of the GTR in ^{208}Bi were studied by Dang *et al* [41] including two-particle, two-hole (2p2h) configurations. The coupling to 2p2h states spreads GTR energy at 16.6 MeV. Moukhai *et al* [42] considered the continuum RPA approach and used the Landau–Migdal parameter $g' = 0.76$ for calculating the GT strength distribution in ^{208}Bi. The mean excitation energy of the GT resonance was found to be around 19.2 MeV. These collective excitations in ^{208}Bi was also investigated by Suzuki and Sagawa [43] using the self-consistent HF and TDA. Their calculated value for the GT resonance energy was 0.4 MeV lower than the experimental value. Bender *et al* [44] investigated the effect of the spin–isospin channel of the Skyrme energy functional on the predictions of the GT distributions in ^{208}Pb nucleus. They concluded that some terms affected the strength and energy of the GTR in finite nuclei without altering the Landau parameter $g'_0 = 0.9$. Recently, we have studied the GT 1^{+} states in ^{208}Bi [45] and the GTR energy calculated from ground state of the ^{208}Bi was found to be 15.897 MeV. Drozdz *et al* [46] calculated the GT strength distributions for ^{48}Ca, ^{90}Zr and ^{208}Pb in local density approximation using the effective interactions based on a Brueckner G-matrix in nuclear matter. The conventional RPA theory was extended to include 1p1h as well as 2p2h excitations in a consistent way. They obtained a very reasonable description of the centroid energies as well as the spreading width of the resonances in all three nuclei. Bertsch and Hamamoto [47] studied the GT strength in ^{90}Zr using 2p2h configurations with the M3Y interaction including a tensor part. Recently, Babacan *et al* [48] studied the GT 1^{+} states in ^{90}Nb isotopes and it was found that the GT resonance energy was located at 7.61 MeV. Apart from the ^{90}Zr isotopes, the GT strength distributions in $^{112-124}$Sn isotopes was researched by Rodin and Urin [49] using the partially self-consistent proton neutron quasiparticle continuum random phase approximation (pnQCRPA). In ref. [50], a general review of the present knowledge of collective spin–isospin excitations in nuclei is given.

In the present study, GT 1^{+} states were investigated using the method developed by Pyatov and Salamov. According to this method, the broken supersymmetry property of the pairing interaction between nucleons was restored and the effect of restoration on the GTR for $^{112-124}$Sb isotopes was studied within the framework of the pnQRPA method with the separable residual GT effective interactions in the particle–hole (ph) and particle–particle (pp) channels. The same calculations were performed in the phenomenological shell model basis also (without restoration). The obtained results in both methods were compared with each other and with the corresponding experimental data.

2. Theoretical formalism

The total Hamiltonian for the atomic nuclei as many-body systems is specified by additive integrals of motion such as linear momentum $\vec{P}$, angular momentum $\vec{J}$ and particle number N. These conserved quantities are the consequence of the invariance of the nuclear Hamiltonian under symmetry transformations. But in constructing the nuclear model or in an approximate resolution of the problem, one often handles the Hamiltonians with broken symmetry. For example, the shell model Hamiltonian is not a translational invariant, i.e. in the shell model the total linear momentum is not conserved. Also, for deformed potentials the angular momentum is not conserved. On nuclei with $N \neq Z$, a shell potential contains isovector terms besides the Coulomb energy and the proton–neutron mass difference, breaking the isotopic invariance. On nuclei far off the closed shells, the static pairing (pairing field) breaks the gauge invariance. That is, the pairing potential is not commutative with the particle number operator and the GT operator. The restoration method of broken symmetries developed by Pyatov and Salamov was successful in the investigation of the electric dipole resonances (the excited 1^- states), the magnetic dipole resonances (the excited 1^+ states and scissor modes) in even–even nuclei and the isobar analogue resonances (IAR) (the excited 0^+ states and the isospin admixtures of the ground states) in odd–odd nuclei. It is important to make the corresponding restoration for each giant resonance mentioned above. For example, the broken translational invariance must be restored in the study of the electric dipole resonance because other violations have no influence on this resonance. Similarly, the restoration of the broken rotational invariance is important in the investigation of the magnetic dipole resonance. In the study of IAR states, the isospin invariance of the nuclear part of total Hamiltonian must be restored.

As this study deals with GT transitions, the restoration of the broken supersymmetry property of the static pairing interaction potential (the broken commutativity of the GT operator with the pairing interaction potential) is important. The other violations mentioned above have no influence on the GT transitions. In the present study, the supersymmetry property of the terms, including pairing interaction in the nuclear part of total Hamiltonian, was restored using the Pyatov method. There are two ways to make this restoration: First, the pairing interaction potential is added to the Hamiltonian after the restoration of its supersymmetry property. Secondly, the mentioned restoration is made in the quasiparticle space. In this study, the restoration was performed according to the latter method.

The schematic model (SM) Hamiltonian (the schematic model Hamiltonian does not include h_0 term) for GT excitations in the quasiparticle representation is usually accepted in the following form:

$$H_{\rm SM} = H_{\rm sqp} + h_{\rm ph} + h_{\rm pp}, \tag{1}$$

where $H_{\rm sqp}$ is the single quasiparticle (sqp) Hamiltonian, $h_{\rm ph}$ and $h_{\rm pp}$ are respectively the GT effective interactions in the ph and pp channels. As known, the effective interaction constants in the ph and pp channels are fixed from the experimental value of the GTR energy and the β decay $\log ft$ values between the low energy states of the parent and daughter nuclei (generally ground states). As mentioned earlier, the supersymmetry property of the pairing part in total Hamiltonian was restored according to the Pyatov method. In this respect, certain terms which naturally do not commute with the GT operator were excluded from total Hamiltonian and the broken commutativity of the remaining part due to the shell model mean-field approximation was restored by adding an effective interaction term h_0 as follows [51]:

$$[H_{\rm SM} - (h_{\rm ph} + h_{\rm pp}) - (V_1 + V_{\rm C} + V_{ls} + h_0, G^{\pm}_{1\mu}] = 0 \tag{2}$$

or

$$[H_{\rm sqp} - V_1 - V_{\rm C} - V_{ls} + h_0, G^{\pm}_{1\mu}] = 0 \tag{3}$$

where V_1, $V_{\rm C}$ and V_{ls} are the isovector, Coulomb and spin orbital term of the shell model potential, respectively. The restoration term h_0 in eq. (3) is included in a separable form:

$$\begin{aligned} h_0 = &\sum_{\rho=\pm} \frac{1}{4\gamma_\rho} \\ &\times \sum_{\mu=0,\pm1} [H_{\rm sqp} - V_1 - V_{\rm C} - V_{ls}), G^{\rho}_{1\mu}]^{\dagger} \\ &\times [H_{\rm sqp} - V_1 - V_{\rm C} - V_{ls}, G^{\rho}_{1\mu}]. \end{aligned} \tag{4}$$

The strength parameter γ_ρ of h_0 effective interaction is found from the commutation condition in eq. (3) and the following expression is obtained for this constant (for details, see ref. [52]).

$$\gamma_\rho = \frac{\rho(-1)^{\mu}}{2} \langle 0 | [[H_{\rm sqp} - V_1 - V_{\rm C} - V_{ls}, G^{\rho}_{1\mu}], G^{\rho}_{1\mu}] | 0 \rangle.$$

3. Hamiltonian

Let us consider a system of nucleons in a spherical symmetric average field with pairing forces. In this case, the corresponding quasiparticle Hamiltonian of the system is given by

$$H_{\rm sqp} = \sum_{\tau, j_m} \varepsilon_{j_\tau} \alpha^{\dagger}_{j_\tau m_\tau} \alpha_{j_\tau m_\tau}, \quad \tau = {\rm n, p}, \tag{5}$$

where ε_{j_τ} is the sqp energy of the nucleons with angular momentum j_τ, and $\alpha^\dagger_{j_\tau m_\tau}$ ($\alpha_{j_\tau m_\tau}$) is the quasiparticle creation (annihilation) operator. The GT operator in the quasiparticle space according to quasiboson approximation is given as follows:

$$G^-_{1\mu} = \sum_{np} [\bar{b}_{np} C^\dagger_{np}(\mu) + (-1)^{1+\mu} b_{np} C_{np}(-\mu)], \tag{6}$$

$$G^\dagger_{1\mu} = [G^-_{1\mu}]^\dagger,$$

where $C^\dagger_{np}(\mu)$ and $C_{np}(\mu)$ are the quasiboson creation and annihilation operators.

$$C^\dagger_{np}(\mu) = \sqrt{\frac{3}{2j_n+1}} \sum_{m_n,m_p} (-1)^{j_p-m_p} \langle j_p m_p 1\mu | j_n m_n \rangle \alpha^\dagger_{j_n m_n} \alpha^\dagger_{j_p -m_p}$$

and

$$C_{np}(\mu) = [C^\dagger_{np}(\mu)]^\dagger.$$

These operators satisfy the following commutation rules in the quasiboson approximation:

$$[C_{np}(\mu), C^\dagger_{n'p'}(\mu')] = \delta_{nn'}\delta_{pp'}\delta_{\mu\mu'}, \quad [C_{np}(\mu), C_{n'p'}(\mu')] = 0.$$

The h_0, $h_{\rm ph}$ and $h_{\rm pp}$ effective interactions in quasiboson approximation are described in the following forms:

$$h_0 = \sum_{npn'p'\mu\rho} \frac{1}{2\gamma_\rho} E^\rho_{np} E^\rho_{n'p'} [C_{np}(\mu) + \rho(-1)^{1+\mu} C^\dagger_{np}(-\mu)] \cdot [C^\dagger_{n'p'}(\mu') + \rho'(-1)^{1+\mu'} C_{n'p'}(-\mu')], \tag{7}$$

$$h_{\rm ph} = 2\chi_{\rm ph} \sum_{npn'p'\mu'} [\bar{b}_{np} C^\dagger_{np}(\mu) + (-1)^{1+\mu} b_{np} C_{np}(-\mu)] \cdot [\bar{b}_{n'p'} C_{n'p'}(\mu') + (-1)^{1+\mu'} b_{n'p'} C^\dagger_{n'p'}(-\mu')], \tag{8}$$

$$h_{\rm pp} = -2\chi_{\rm pp} \sum_{npn'p'\mu'} [d_{np} C^\dagger_{np}(\mu) - (-1)^{1+\mu} \bar{d}_{np} C_{np}(-\mu)] \cdot [d_{n'p'} C_{n'p'}(\mu') - (-1)^{1+\mu'} \bar{d}_{n'p'} C^\dagger_{n'p'}(-\mu')], \tag{9}$$

where E^ρ, $\bar{b}$, b, $\bar{d}$, d are the reduced matrix elements. The complete expressions of these matrix elements are given in refs [52,53]. Thus, the total Hamiltonian of the system according to PM is given as

$$H_{\rm PM} = H_{\rm sqp} + h_{\rm ph} + h_{\rm pp} + h_0. \tag{10}$$

4. Eigenvalues and eigenfunctions

The eigenvalues and eigenfunctions of the Hamiltonian given in eq. (10) were solved within the framework of the pnQRPA method. We shall consider the GT 1^+ excitations in odd–odd nuclei generated from the correlated ground state of the parent nucleus by the charge-exchange spin–spin forces and use the eigenstates of the single quasiparticle Hamiltonian H_{sqp} as a basis.

In pnQRPA, the ith excited GT 1^+ states in odd–odd nuclei are considered as the phonon excitations and described by:

$$|i\rangle = Q_i^\dagger(\mu)|0\rangle = \sum_{np}[\psi_{np}^i C_{np}^\dagger(\mu) - (-1)^{1+\mu}\varphi_{np}^i C_{np}(-\mu)]|0\rangle, \tag{11}$$

where $Q_i^\dagger(\mu)$ is the pnQRPA phonon creation operator, $|0\rangle$ is the phonon vacuum which corresponds to the ground state of an even–even nucleus and fulfills $Q_i(\mu)|0\rangle = 0$ for all i. The ψ_{np}^i and φ_{np}^i are quasiboson amplitudes.

Assuming that the phonon operators obey the commutation relations given below

$$\langle 0|[Q_i(\mu), Q_j^\dagger(\mu')]|0\rangle = \delta_{ij}\delta_{\mu\mu'},$$

we obtain the following orthonormalization condition for amplitudes ψ_{np}^i and φ_{np}^i:

$$\sum_{np}[\psi_{np}^i\psi_{np}^{i'} - \varphi_{np}^i\varphi_{np}^{i'}] = \delta_{ii'}. \tag{12}$$

The energies and wave functions of the GT 1^+ states are obtained from the pnQRPA equation of motion.

$$[H_{\text{PM}}, Q_i^\dagger(\mu)]|0\rangle = \omega_i Q_i^\dagger(\mu)|0\rangle, \tag{13}$$

where ω_i is the energy of the GT 1^+ states occurring in the neighbouring odd–odd nuclei. Let us note that these energies are calculated over the ground state of the parent nuclei. The following linear equations are obtained for the ψ_{np}^i and φ_{np}^i amplitudes:

$$\sum_{np}[(\rho_{npn'p'} - \omega_i\delta_{nn'}\delta_{pp'})\psi_{np}^i - (-1)^{1+\mu}\eta_{npn'p'}\varphi_{np}^i] = 0$$

$$\sum_{np}[\eta_{npn'p'}\psi_{np}^i - (-1)^{1+\mu}(\rho_{npn'p'} - \omega_i\delta_{nn'}\delta_{pp'})\varphi_{np}^i] = 0. \tag{14}$$

The following double commutators are solved to calculate ρ and η matrices.

$$\rho_{npn'p'} = [C_{n'p'}(\mu), [H_{\text{PM}}, C_{np}^\dagger(\mu)]], \tag{15}$$

$$\eta_{npn'p'} = [C_{n'p'}(\mu), [H_{\text{PM}}, C_{np}(-\mu)]]. \tag{16}$$

The following expressions are obtained for these matrices:

$$\rho_{npn'p'} = \varepsilon_{np}\delta_{nn'}\delta_{pp'} + \sum_{\rho} \frac{1}{2\gamma_\rho} E^{\rho}_{np} E^{\rho}_{n'p'} + 2\chi_{\rm ph}(\bar{b}_{n'p'}\bar{b}_{np} + b_{n'p'}b_{np}) - 2\chi_{\rm pp}(\bar{d}_{n'p'}\bar{d}_{np} + d_{n'p'}d_{np})$$

$$\eta_{npn'p'} = (-1)^{\mu}\left\{ \sum_{\rho} \frac{\rho}{2\gamma_\rho} E^{\rho}_{np} E^{\rho}_{n'p'} + 2\chi_{\rm ph}(\bar{b}_{n'p'}b_{np} + b_{n'p'}\bar{b}_{np}) - 2\chi_{\rm pp}(\bar{d}_{n'p'}d_{np} + d_{n'p'}\bar{d}_{np}) \right\}.$$

The energies ω_i are the roots of the secular equation:

$$\begin{vmatrix} \sum(\rho_{npn'p'} - \omega_i\delta_{nn'}\delta_{pp'}) & \sum \eta_{npn'p'} \\ \sum \eta_{npn'p'} & \sum(\rho_{npn'p'} - \omega_i\delta_{nn'}\delta_{pp'}) \end{vmatrix} = 0.$$

The ψ^i_{np} and φ^i_{np} amplitudes are found from eqs (12) and (14).

5. Matrix elements of $\beta^{\pm}$ transitions

One of the characteristic quantities for the GT 1^+ states occurring in the neighbouring odd–odd nuclei is the GT transition matrix elements. The $0^+ \to 1^+$ β^- and β^+ transition matrix elements are calculated using the following expressions:

$$M^i_{\beta^-}(0^+ \to 1^+_i) = \langle 1^+_i, \mu | G^-_{1\mu} | 0^+ \rangle = \langle 0 | [Q_i(\mu), G^-_{1\mu}] | 0 \rangle = -\sum_{np} (\psi^i_{np} b_{np} + \varphi^i_{np}\bar{b}_{np}) \tag{17}$$

$$M^i_{\beta^+}(0^+ \to 1^+_i) = \langle 1^+_i, \mu | G^+_{1\mu} | 0^+ \rangle = \langle 0 | [Q_i(\mu), G^+_{1\mu}] | 0 \rangle = \sum_{np} (\psi^i_{np} \bar{b}_{np} + \varphi^i_{np} b_{np}) \tag{18}$$

The $\beta^{\pm}$ reduced matrix elements are given by

$$B^{(\pm)}_{\rm GT}(\omega_i) = \sum_{\mu} |M^i_{\beta^\pm}(0^+ \to 1^+_i)|^2. \tag{19}$$

The $\beta^{\pm}$ transition strengths ($S^{\pm}$) must fulfill the Ikeda sum rule (ISR).

$$S^{\pm} = \sum_i B^{(\pm)}_{\rm GT}(\omega_i), \tag{20}$$

$$\mathrm{ISR} = S^{(-)} - S^{(+)} \cong 3(N - Z). \tag{21}$$

6. Results and discussions

In this section, the GTR energies in $^{112-124}$Sb isotopes and the differential cross-section for Sn(^{3}He, t)Sb reactions at $E(^3\text{He}) = 200$ MeV occurring by the excitation of the GTR state have been numerically calculated using PM. In the calculations, the Woods–Saxon potential with Chepurnov parametrization [54] was used and the pair correlation function was chosen as $C_p = C_n = 12/\sqrt{A}$ for the open shell nuclei. The basis used in our calculation contained all the neutron–proton transitions which change the radial quantum number n by $\Delta n = 0, 1, 2, 3$. The reliability of our basis was tested by calculating the Ikeda sum rule. The calculation results are presented in table 1. The second and third columns in the table represent the calculated values of sqp and PM, respectively. The fourth column shows the values of $3(N - Z)$.

As seen from table 1, the sqp and PM values are very close to each other and the average value of their differences from the ISR values are $\sim$1.65%. The calculated values of ISR in columns 2 and 3 of table 1 differ because some solutions of pnQRPA equations are lost. This loss however, has no important effect on the GTR properties and the low-lying β-decay states. Furthermore, the difference in the calculated values of ISR for nuclei having neutron excess can be attributed to a partial transmission of the GT transition strength to a higher energy region. To make an exact comment, the energy distributions of GT$^-$ transition strength should be investigated for various Sb isotopes.

The energy distributions of the GT β transition strengths in various Sb isotopes are given in figures 1–4. These distributions can be divided into three energy regions: low-energy region, GTR region and pigmy IVSMR region. The 1^+ excited states in the low-energy region are composed of proton–neutron quasiparticle transitions with $\Delta n = 0$, and these transitions are weakly collectivized. The 1^+ excited state with the biggest B_{GT} value in the spectrum is accepted as the GTR state. The calculations show a highly collectivized state. Our calculations also show the presence of the pigmy IVSMR region. The proton–neutron quasiparticle multiplets with $\Delta n \neq 0$ form the general 1^+ states in this region. As seen from figures 1–4, the GT transition strength has a more homogeneous distribution for heavier Sb isotopes. This shows that some part of the GTR strength has been transmitted to the pigmy IVSMR region. Furthermore, the GTR states calculated by [49], SM and PM are listed in table 2 in comparison with the corresponding experimental

Table 1. The Ikeda sum rule calculated by sqp and QRPA for various Sn isotopes.

A	sqp	QRPA	$3(N-Z)$
112	34.51	34.67	36
114	41.61	41.79	42
116	47.61	47.60	48
118	53.10	53.09	54
120	59.14	59.06	60
122	65.10	65.06	66
124	70.43	70.42	72

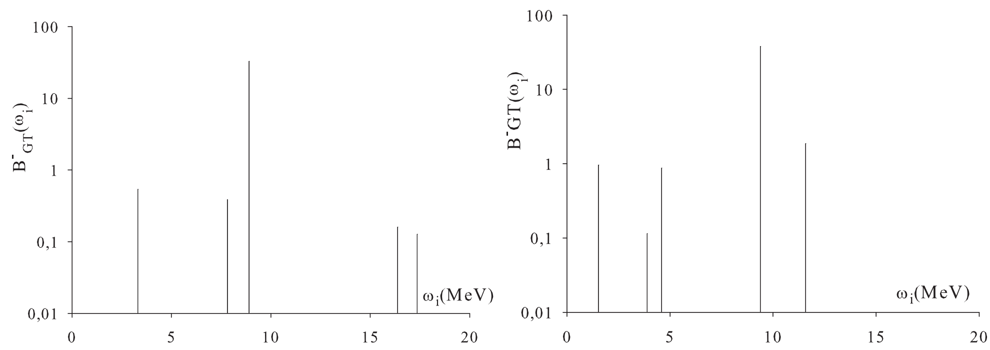

Figure 1. GT β transition strength distribution in ^{112}Sb (left) and ^{114}Sb (right).

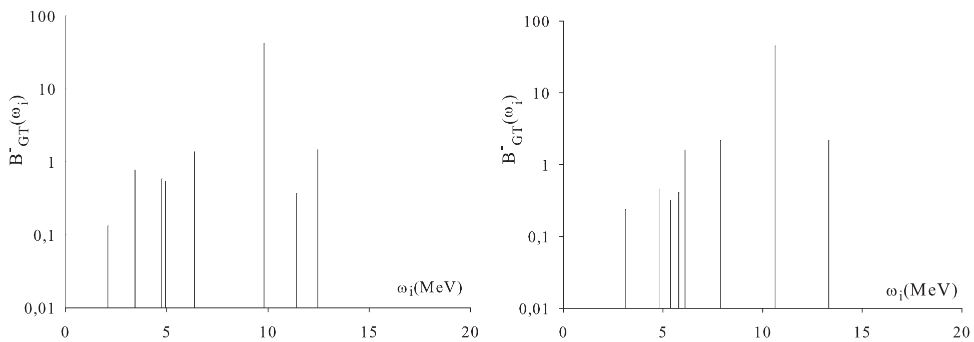

Figure 2. GT β transition strength distribution in ^{116}Sb (left) and ^{118}Sb (right).

Table 2. The 1^+ excited state with the biggest B_{GT} value in the spectrum of different models for $^{112-124}$Sb isotopes.

A	Exp. [55]	[49]	SM	PM
112	–	9.00	8.80	8.90
114	–	9.25	9.76	9.38
116	10.04	9.50	11.03	9.80
118	10.61	12.50	16.17	10.62
120	11.45	13.25	17.12	11.47
122	12.25	13.90	18.62	12.39
124	13.25	14.30	19.92	12.93

data. The calculated GTR relative strength in PM is found to be in a reasonable agreement with the respective experimental data [55].

The calculated GTR energy values of different models for $^{112-124}$Sb are given in figure 5. The energies were calculated from the ground state of the parent nucleus.

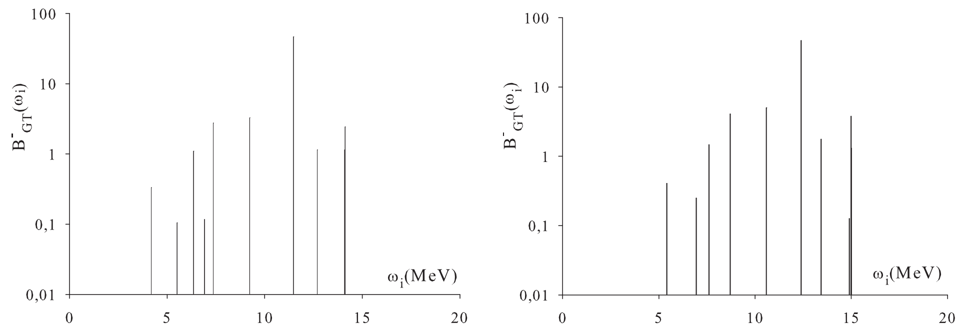

Figure 3. GT β transition strength distribution in ^{120}Sb (left) and ^{122}Sb (right).

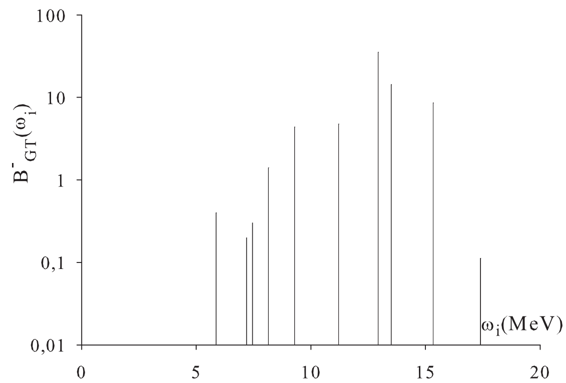

Figure 4. GT β transition strength distribution in ^{124}Sb.

The solid curve with rectangles corresponds to the calculated values by PM, the medium dash line with stars corresponds to the experimental values, the short dash line with circles corresponds to the calculated values by SM and the dotted curve with triangles corresponds to [49]. To investigate the influence of the restoration term (h_0) on ω_{GTR} values, the same χ_{ph} and χ_{pp} values were used in the calculations performed by PM and SM. It should be noted that the χ_{ph} and χ_{pp} values were chosen in such a way that the obtained ω_{GTR} values in PM were in good agreement with the corresponding experimental values. According to Rodin *et al* [49], the calculated ω_{GTR} values in PM are closer to the corresponding experimental data. As can be seen from figure 5, the energy values in PM are smaller than those in SM except for $A = 112$ isotope. This shows that the restoration term has an attractive character in $^{114-124}$Sb isotopes and a repulsive character in ^{112}Sb isotope. It can also be clearly seen that the h_0 term becomes more attractive as the $(N - Z)$ difference increases. In other words, the attractiveness of h_0 is more effective in heavier isotopes.

In figure 6, the energy differences between GTR and IAR calculated by PM are presented and compared with other theoretical calculations and the corresponding

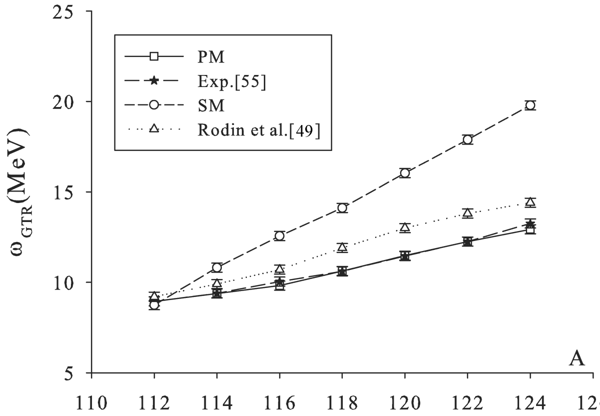

Figure 5. The GTR energies calculated by different models for various Sb isotopes.

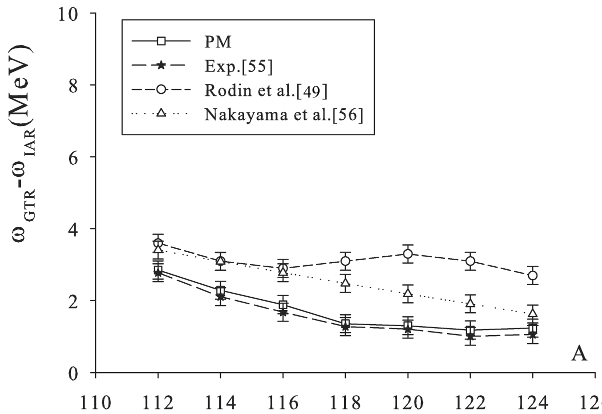

Figure 6. The energy difference between GTR and IAR for various Sn isotopes.

experimental data. The solid curve with rectangules corresponds to the calculated values by PM, the medium dash line with stars corresponds to the experimental values taken from ref. [55], the short dash line with circles corresponds to Rodin *et al* [49] and the dotted curve with triangles corresponds to the data calculated by the phenomenological formula ($\omega_{\rm GTR} - \omega_{\rm IAR} = 26A^{-1/3} - 18.5(N - Z)/A$ MeV) [56]. The $\omega_{\rm IAR}$ values in the PM calculations were taken from ref. [57]. It is clearly seen from figure 6 that the calculated energy differences in PM are closer to the corresponding experimental data in comparison with the other calculations.

To reliably predict the GTR properties in $^{112-124}$Sb isotopes, the differential cross-sections of zero degrees for the excitation of the GTR were calculated for $^{112-124}$Sn (^{3}He, t) $^{112-124}$Sb reactions at $E(^3{\rm He}) = 200$ MeV. The following formula was used in the calculations [55]:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{GT}}(q\approx 0,\theta=0^\circ)=\left(\frac{\mu}{\pi\hbar^2}\right)^2\left(\frac{k_f}{k_i}\right)N_{\mathrm{GT}}J_{\mathrm{GT}}^2B_{\mathrm{GT}}, \tag{22}$$

where J_{GT} is the volume integral of the central part in the effective spin–isospin interaction, N_{GT} is the distortion factor which may be approximated by the function $\exp(-xA^{1/3})$, the value of x is taken from ref. [13], μ, k and B_{GT} denote the reduced mass, the wave number in the centre of mass system and the square of the GT-reduced matrix elements, respectively. The J_{GT} volume integrals were taken from eq. (22) using the experimental cross-section values in ref. [55]. The J_{GT} integral was calculated for $B_{\mathrm{GT}}=3(N-Z)$ and $B_{\mathrm{GT}}=3(N-Z)/2$ values. The J_{GT} values found for both assumptions are given in figure 7. J_{GT} was found to be ≈100 MeV fm^3 in the first assumption and ≈140 MeV fm^3 in the second assumption. Although the J_{GT} value found in the first assumption is smaller than those found in ref. [13] ((172 ± 17) MeV fm^3) and [57] (168 MeV fm^3), the value found according to the second assumption is closer to the values in these studies. In figure 7, the solid curves with empty rectangles and empty circles correspond to the results of the first and second assumptions, respectively. As can be seen from figure 7, the J_{GT} value found from the first assumption ($B_{\mathrm{GT}}=3(N-Z)$) is ≈100 MeV fm^3. This value is less than the results obtained from ref. [13] ((172 ± 17) MeV fm^3) and [57] ((168 MeV fm^3)) by 45%. The J_{GT} value found from the second assumption ($B_{\mathrm{GT}}=3(N-Z)/2$) is ≈140 MeV fm^3 and closer to the results from ref. [13,57]. The differential cross-sections were calculated using J_{GT} values obtained in both assumptions. The calculation results are compared with the corresponding experimental values in figure 8. In figure 8, the solid curves with filled circle, empty rectangles and empty circles correspond to the experimental values, the volume integral with the first assumption and the volume integral with the second assumption, respectively. As can be seen from figure 8, the volume integral found from the second assumption is closer to the values in the experimental data. As a result, the calculated values of the volume integral of the central part in the effective spin–isospin interaction in $^{112-124}$Sb isotopes are is good agreement with

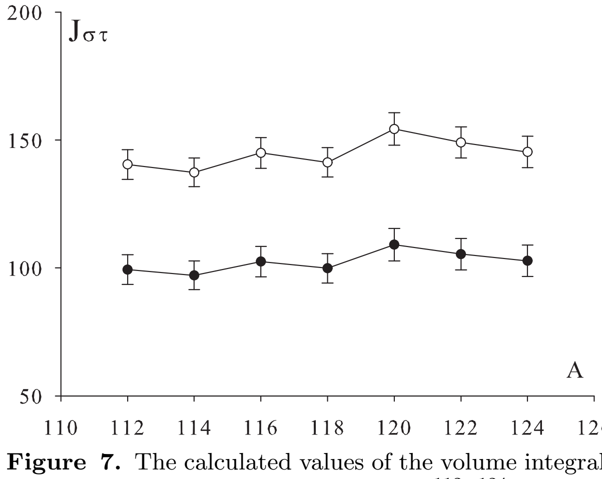

Figure 7. The calculated values of the volume integral of the central part in the effective spin–isospin interaction in $^{112-124}$Sb isotopes.

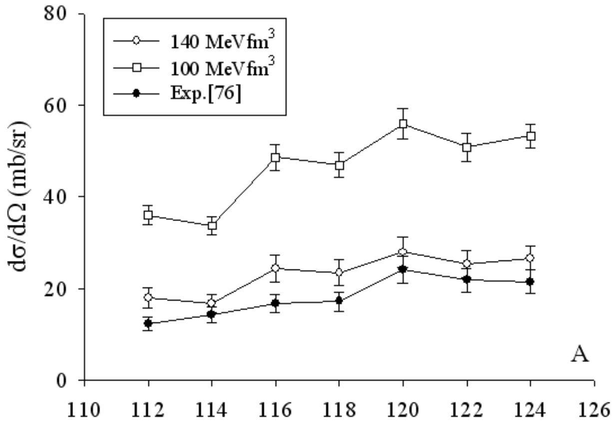

Figure 8. The differential cross-section values for Sn(^{3}He, t)Sb reactions at $E(^3\text{He})$ = 200 MeV occurring by the excitation of the GTR state.

the differential cross-section values for Sn(^{3}He, t)Sb reactions at $E(^3\text{He}) = 200$ MeV occurring by the excitation of the GTR state.

7. Conclusions

The GT 1^+ states in $^{112-124}$Sb isotopes were investigated for two different cases within the framework of the pnQRPA method. (1) The pnQRPA procedure was applied for the Hamiltonian which had the pairing part with supersymmetry property. (2) The same procedure was followed for SM Hamiltonian. Thus, it became possible to determine the influence of the restoration on the GTR for various Sb isotopes.

The GT^- strength distribution has a crucial role in understanding the GT 1^+ states and so the energy distributions of GT^- strength were calculated for various Sb isotopes. The spectrum of these 1^+ excited states consisted of low-energy region, the GTR region and the pigmy IVSMR region. The GT transition strength had a more homogeneous distribution with the increase of $(N - Z)$ difference. Thus, some part of the contribution of GTR to ISR was transmitted to the IVSMR states. This was also confirmed by the calculated values of the differential cross-sections for Sn(^{3}He, t)Sb reactions at 200 MeV and the GTR energies for $^{112-124}$Sb isotopes. The calculations of the differential cross-sections were done according to two different assumptions. The agreement of the calculated values in the second assumption ($B_{\text{GT}} = 3(N - Z)/2$) with the corresponding experimental values shows a partial transmission of the GTR strength to the IVSMR states. The increase in GTR energy with the increase of $(N - Z)$ difference means that the GTR and IVSMR states come closer to each other.

Furthermore, the GTR energy was calculated within SM to understand the effect of restoration on the GTR. The calculated values of GTR energies for all Sb isotopes under consideration show that the restoration term has an attractive character except for $A = 112$ isotope. The repulsiveness of the restoration term for this isotope can be associated with the strength of the attractive isovector interaction. As known, the isovector potential is proportional to the $(N - Z)$ difference. Therefore, the restoration term may be repulsive for some light isotopes. The attractiveness of the restoration term naturally affects the fixed values of the $\chi_{\rm ph}$ and $\chi_{\rm pp}$ constants. Thus, the fixed $\chi_{\rm ph}$ value in PM is greater than that in SM and the fixed $\chi_{\rm pp}$ value in PM is smaller than that in SM.

Both PM and SM give a good agreement with the corresponding experimental data depending on the choice of the $\chi_{\rm ph}$ and $\chi_{\rm ph}$ parameters. The restoration of supersymmetry property of the pairing interaction is however, physically important.

References

[1] W P Alford and B M Spicer, *Advances in nuclear physics* (Plenum, New York, 1998) Vol. 24
[2] T N Taddeucci *et al*, *Nucl. Phys.* **A469**, 125 (1987)
[3] W P Alford *et al*, *Phys. Lett.* **B179**, 20 (1986)
[4] Y Yoshida *et al*, *Nucl. Phys.* **A187**, 161 (1972)
[5] R R Doering *et al*, *Phys. Rev. Lett.* **35**, 1691 (1975)
[6] A Galonsky *et al*, *Phys. Rev.* **C14**, 748 (1976)
[7] C D Goodman *et al*, *Phys. Rev. Lett.* **44**, 1755 (1980)
[8] D E Bainum *et al*, *Phys. Rev. Lett.* **44**, 1751 (1980)
[9] C Gaarde *et al*, *Nucl. Phys.* **A369**, 258 (1981)
[10] C Gaarde, *Nucl. Phys.* **A396**, 127c (1983)
[11] B S Flanders *et al*, *Phys. Rev.* **C40**, 1985 (1989)
[12] C Gaarde, *Nucl. Phys.* **A507**, 79c (1990)
[13] M Fujiwara *et al*, *Nucl. Phys.* **A599**, 223c (1996)
[14] Y Fujita *et al*, *Phys. Lett.* **B365**, 29 (1996)
[15] K J Raywood *et al*, *Nucl. Phys.* **A625**, 675 (1997)
[16] D L Prout *et al*, *Phys. Rev.* **C63**, 014603-1 (2000)
[17] T Wakasa *et al*, *Phys. Rev.* **C55**, 2909 (1997)
[18] C D Goodman, *Nucl. Instrum. Methods in Phys. Res.* **A462**, 545 (2001)
[19] S I Hayakawa *et al*, *Nucl. Phys.* **A139**, 465 (1969)
[20] C Ellegaard *et al*, *Phys. Rev. Lett.* **50**, 1745 (1983)
[21] G Mairle *et al*, *Nucl. Phys.* **A490**, 371 (1988)
[22] A Brockstedt *et al*, *Nucl. Phys.* **A530**, 571 (1991)
[23] J Janecke *et al*, *Nucl. Phys.* **A526**, 1 (1991)
[24] J Janecke *et al*, *Phys. Rev.* **C48**, 2828 (1993)
[25] M Fujiwara *et al*, *Nucl. Phys.* **A577**, 43c (1994)
[26] H Akimune *et al*, *Phys. Lett.* **B323**, 107 (1994)
K Pham *et al*, *Phys. Rev.* **C51**, 526 (1994)
[27] H Akimune *et al*, *Phys. Rev.* **C52**, 604 (1995)
[28] I Daito *et al*, *Phys. Lett.* **B418**, 27 (1998)
[29] J Janecke *et al*, *Nucl. Phys.* **A687**, 270c (2001)
[30] A Krazsnahorkay *et al*, *Phys. Rev.* **C64**, 067302-1 (2001)
[31] R G T Zegers *et al*, *Nucl. Phys.* **A731**, 121 (2004)
[32] C Gaarde and T Kammuri, *Nucl. Phys.* **A215**, 314 (1973)
[33] W R Wharton and P T Debevec, *Phys. Rev.* **C11**, 1963 (1975)
[34] N Anantaraman *et al*, *Phys. Rev. Lett.* **57**, 2375 (1986)

[35] M Moosburger *et al*, *Phys. Rev.* **C41**, 2925 (1990)
[36] H Wirth *et al*, *Phys. Rev.* **C41**, 2698 (1990)
[37] E Aschenauer *et al*, *Phys. Rev.* **C44**, 2771 (1991)
[38] H Laurent *et al*, *Nucl. Phys.* **A569**, 297c (1994)
[39] V A Kuzmin and V G Soloviev, *J. Phys. G : Nucl. Part. Phys.* **10**, 1507 (1984)
[40] G Colo *et al*, *Phys. Rev.* **C50**, 1496 (1994)
[41] N D Dang *et al*, *Phys. Rev. Lett.* **79**, 1638 (1997)
N D Dang *et al*, *Nucl. Phys.* **A621**, 719 (1997)
[42] E A Moukhai, V A Rodin and M H Urin, *Phys. Lett.* **B447**, 8 (1999)
[43] T Suzuki and H Sagawa, *Eur. Phys. J.* **A9**, 49 (2000)
[44] M Bender *et al*, *Phys. Rev.* **C65**, 054322 (2002)
[45] T Babacan, D I Salamov and A Kucukbursa, *Phys. Rev.* **C71**, 037303 (2005)
[46] S Drozdz *et al*, *Phys. Lett.* **B166**, 1323 (1986)
[47] B F Bertsch and I Hamamoto, *Phys. Rev.* **C26**, 1323 (1982)
[48] T Babacan, D I Salamov and A Kucukbursa, *Math. Comp. Appl.* **10/3**, 359 (2005)
[49] V A Rodin and M H Urin, *Nucl. Phys.* **A687**, 276 (2001); *Phys. Atom. Nucl.* **66**, 2128 (2003)
[50] Franz Osterfeld, *Rev. Mod. Phys.* **64**, 491 (1992)
[51] N I Pyatov and D I Salamov, *Nucleonica* **22**, 127 (1977)
[52] D I Salamov *et al*, *Proceedings of 5th Conf. on Nucl. and Part. Phys. (NUPPAC 05)* (Cairo, 2006) p. 361
[53] Necla Cakmak, *The study of charge-exchange collective excitations in odd mass nuclei by Pyatov method*, Ph.D. Thesis (Anadolu University, Turkey, 2008)
[54] V G Soloviev, *Theory of complex nuclei* (Pergamon, New York, 1976)
[55] K Pham *et al*, *Phys. Rev.* **C51**, 526 (1995)
[56] K Nakayama *et al*, *Phys. Lett.* **B214**, 217 (1982)
[57] T Babacan *et al*, *J. Phys.* **G30**, 759 (2004)