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Isovector coupling channel and central properties of the charge density distribution in heavy spherical nuclei

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Abstract. The influence of the isovector coupling channel on the central depression parameter and the central value of the charge density distribution in heavy spherical nuclei was studied. The isovector coupling channel leads to about 50% increase of the central depression parameter, and weakens the dependency of both central depression parameter and central density on the asymmetry, impressively contributing to the semibubble form of the charge density distribution in heavy nuclei, and increasing the probability of larger nuclei with higher proton numbers and higher neutron-to-proton ratios stable.

Keywords. Isovector coupling channel; charge density; three-parameter Fermi distribution; central depression.

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1. Introduction

The periodic table of the elements is growing slowly but continuously. In 2004, physicists reported the synthesis of the element 115, in a co-work between the Joint Institute for Nuclear Research in Dubna, Russian Federation and the Lawrence Livermore National Laboratory in California, USA [1].

Heavy nuclei have large proton numbers and thus large Coulomb repulsive forces that push the protons to larger radii, leading to a depression in the central charge density. Though the Coulomb repulsive force is the leading factor producing a depression in the central charge density of heavy nuclei, the isovector coupling channel of the nucleon–nucleon interaction has also been found to have a significant effect on the central depression of the charge density distributions in lead isotopes [2]. The interest in the isospin dependence of nuclear forces is growing due to a new generation of radioactive beam facilities, like the Rare Isotope Accelerator planned in the United States of America, the SPIRAL2 at GANIL, France and the GSI Facility FAIR in Germany, which produce new data for neutron-rich nuclei.

The aim of this work is to extend the analysis given in ref. [2] to study the role played by the isovector coupling channel of the nuclear force in the systematics of the central charge density distribution in heavy spherically symmetric nuclei with known experimental masses around the ^{208}Pb nucleus [3]. This will help us understand how the isovector coupling channel affects the probability of the formation of new superheavy elements with larger number of protons and larger neutron-to-proton ratios.

Section 2 reviews the general theory of an effective nuclear interaction with density-dependent coupling parameters. Section 3 defines the central density distribution parameters, which are the central depression parameter w and the central density ρ_c. They are calculated in §4 for spherically symmetric nuclei with known experimental masses around the ^{208}Pb nucleus. The dependency of the central density distribution parameters w and ρ_c on the mass number A and the nuclear asymmetry $I = (N - Z)/A$ is studied, and the role played by the isovector coupling channel is analysed by comparing the results for both the isoscalar and the isovector coupling channels included in the nuclear force, with the results considering only the isoscalar channel and ignoring the isovector one. Section 5 summarizes the main conclusions.

2. Effective density-dependent interaction

The relativistic Brueckner–Hartree–Fock theory (RBHF) is generally accepted as one of the most reliable and feasible microscopic methods for describing the effective interactions in the nuclear medium [4,5]. Unfortunately, the application of the RBHF approach to finite nuclei is highly complicated, such that investigations using RBHF have to deal with the nuclear matter problem.

In order to overcome this problem, an attempt has been made to calculate finite nuclei using the relativistic Thomas–Fermi approach with effective density-dependent meson nucleon interactions (RDTF), where the density-dependent meson couplings are deduced from RBHF calculations by reproducing the nucleon self-energy resulting from RBHF nuclear matter calculations for different proton and neutron densities [6].

The effective nucleon–nucleon interaction is described in the RDTF by the electromagnetic field between protons and the exchange of four mesons: the isoscalar scalar meson σ, the isoscalar vector meson ω, the isovector scalar meson δ and the isovector vector meson ρ. Density-dependent coupling parameters for the isoscalar mesons are introduced by

$$\frac{g_i(\rho)}{g_i(\rho_0)} - 1 = a_i \left(\exp\left[b_i \left(1 - \left(\frac{\rho}{\rho_0} \right)^{1/3} \right) \right] - 1 \right), \quad i = \sigma, \omega, \tag{1}$$

where ρ_0 is the nuclear matter saturation density and a_i, b_i and $g_i(\rho_0)$ are the coefficients of the density-dependent function $g_i(\rho)$. Density-dependent coupling parameters for the isovector mesons are introduced by

$$g_i(\rho) = g_i(\rho_0) \exp\left[b_i \left(1 - \frac{\rho}{\rho_0} \right) \right], \quad i = \delta, \rho, \tag{2}$$

where b_i and $g_i(\rho_0)$ are the coefficients of the density-dependent function $g_i(\rho)$.

Table 1. The density-dependent parameter set. m_i is the mass of the i-meson. a_i, b_i and $g_i(\rho_0)$ are the coefficients of the parametrization of the density-dependent coupling parameters ($i = \sigma, \omega, \delta, \rho$). $m_N = 938.926$ MeV is the average nucleon mass used by ref. [5] and $\rho_0 = 0.185$ fm^{-3} is the saturation density resulting from the RBHF potential Bonn A [5].

Meson i	σ	ω	δ	ρ
m_i (MeV)	550	782.6	983	769
$g_i(\rho_0)$	9.297	11.269	4.701	2.370
a_i	0.2941	0.3451		
b_i	2.217	2.113	1.223	1.634

The coefficients a_i, b_i and $g_i(\rho_0)$ ($i = \sigma, \omega$) and b_i and $g_i(\rho_0)$ ($i = \delta, \rho$) are adjusted to the outcome of the RBHF calculations of the nucleon self-energy in nuclear matter of refs [7,8]. The coefficients of the resulting density-dependent parametrization of the RBHF potential Bonn A [5] are given in table 1. The masses m_N, m_σ, m_ω, m_δ and m_ρ and the saturation density ρ_0 are those of the Bonn A potential. For a detailed description of the RDTF, see refs [6,9].

The density-dependent parametrization given in table 1 was used in ref. [2] to study the central depression of the charge density distributions in lead isotopes.

3. Central density distribution parameters

The experimental determination of the density $\rho(r)$ at small radii r is very uncertain. The form factor

$$F(q) = \int \mathrm{d}^3 r\, \frac{\sin(qr)}{qr}\, \rho(r) \tag{3}$$

can be obtained experimentally only over a limited range of q-values, where q is the momentum transfer in the collision [10]. The spherical geometry reduces the contribution from near the centre, since

$$\mathrm{d}^3 r = 4\pi r^2\, \mathrm{d}r, \tag{4}$$

and contributions to the form factor $F(q)$ from near the centre, where r is small, are reduced due to the multiplication by r^2. Theoretical approaches are therefore essential in order to explore the systematics of central densities in nuclei.

The central depression parameter w of the charge density distribution is determined by fitting the central part of the distribution to a three-parameter Fermi distribution (3pF) [10]:

$$\rho_{3\mathrm{pF}}(r) = \left(1 + \frac{wr^2}{c^2}\right) \frac{\rho^0}{1 + \exp\left[(r - c)/a\right]}. \tag{5}$$

The three parameters of the 3pF are w, c and a, while ρ^0 is determined by

$$\int d^3r \ \rho_{3pF}(r) = Z. \tag{6}$$

The central density ρ_c is the density at the centre of the nucleus, and is determined by setting $r = 0$ in eq. (5), i.e.,

$$\rho_c = \rho_{3pF}(r = 0) = \frac{\rho^0}{1 + e^{-c/a}}. \tag{7}$$

4. Heavy spherical nuclei around ^{208}Pb

Charge density distributions of heavy spherically symmetric nuclei with known experimental masses around the ^{208}Pb nucleus are calculated using RDTF with the density-dependent parametrization given in table 1. The quantitative prediction of the central charge density in relativistic mean-field calculations is dominated by quantum shell effects, due to the filling of the various j-shell orbits [11]. Therefore, one has to provide the smooth part of the density according to the idea of the Strutinsky averaging method [12], and the semiclassical Thomas–Fermi (TF) calculation provides the smooth part of the density (see ref. [9], for derivation and details of the TF approximation).

Two remarks concerning the application of the TF approximation should be added. The first remark is that the Strutinsky averaging method goes beyond the TF approximation, but the evaluation of nuclear one- and two-body matrix elements shows that the semiclassical TF calculations provide the smoothly varying part of these matrix elements, dropping the shell effects in accordance with the Strutinsky averaging method [13]. The second remark is that extending the TF approximation leads to a better reproduction of the exponentially decreasing tail of the density distribution and the binding energy, but has a negligible effect on the central part of the density distribution, which is very well reproduced by the TF approximation [14].

The isoscalar coupling channel of the nuclear interaction is represented by the isoscalar scalar meson σ and the isoscalar vector meson ω, while the isovector coupling channel is represented by the isovector scalar meson δ and the isovector vector meson ρ. In order to analyse the role played by the isovector coupling channel, the results obtained using the parametrization of table 1 when all mesons ($\sigma\omega\delta\rho$) included are compared with the results when only isoscalar mesons ($\sigma\omega$) are included.

Figure 1 shows the charge density distributions of the $A = 214$ nuclei: Ra, Rn, Po, Pb when all mesons ($\sigma\omega\delta\rho$) are included, and figure 2 depicts the central part of these distributions. As explained earlier in this paper, experimental results for the density at small radii are very uncertain, and therefore no experimental curves are plotted. Table 2 lists the results obtained for the central depression parameter and the central density of the charge density distributions of heavy spherically symmetric nuclei with known experimental masses around the ^{208}Pb nucleus, and compares the results when all mesons ($\sigma\omega\delta\rho$) included with the results when only isoscalar mesons ($\sigma\omega$) are included. A factor

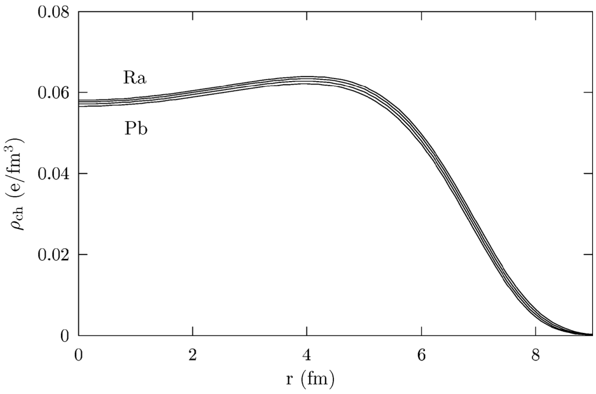

Figure 1. Charge density distributions of the $A = 214$ nuclei: Ra, Rn, Po, Pb (from top to bottom) when all mesons ($\sigma\omega\delta\rho$) are included.

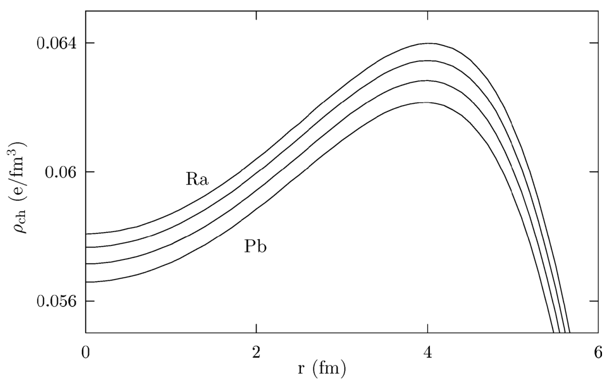

Figure 2. The central part of the charge density distributions of the $A = 214$ nuclei.

$$\frac{w(\sigma\omega\delta\rho)}{w(\sigma\omega)} \simeq 1.5 \quad (8)$$

can be deduced from the results given in table 2. The isovector coupling channel leads to about 50% increase of the central depression parameter of the charge density distribution in heavy nuclei, and therefore has an impressive contribution to the semibubble form with considerable reduction of central densities of the charge density distribution in heavy nuclei. The semibubble form for the charge density helps reducing the repulsive effect of the Coulomb force between protons, allowing nuclei with more protons to be more stable. Therefore, the isovector coupling channel increases the probability of nuclei with more protons being stable.

Figure 3 clarifies the dependency of the results given in table 2 for the central density on the mass number A and nuclear asymmetry I when all mesons ($\sigma\omega\delta\rho$) are included. It is found that the functional form

$$f(A, I) = c_1 + c_2 A^{1/3} + c_3 I^2 \quad (9)$$

Table 2. Central depression parameter w and central density ρ_c of the charge density distributions of heavy spherical nuclei. Table compares the results when all mesons ($\sigma\omega\delta\rho$) are included with the results when only isoscalar mesons ($\sigma\omega$) are included. $I = (N - Z)/A$ is the nuclear asymmetry.

		w		ρ_c (e/fm^3)	
Nucleus	I	($\sigma\omega\delta\rho$)	($\sigma\omega$)	($\sigma\omega\delta\rho$)	($\sigma\omega$)
^{182}Pb	0.0989	0.00794	0.00538	0.06344	0.06666
^{190}Pb	0.13684	0.00827	0.00564	0.06189	0.06457
^{200}Pb	0.18	0.00853	0.00555	0.05988	0.06207
^{202}Pb	0.18812	0.00863	0.00561	0.05946	0.06157
^{204}Pb	0.19608	0.00873	0.00568	0.05903	0.06107
^{204}Hg	0.21569	0.00862	0.00556	0.05848	0.06025
^{206}Po	0.18447	0.00873	0.00586	0.05909	0.06136
^{206}Pb	0.20388	0.00885	0.00574	0.05859	0.06058
^{206}Hg	0.2233	0.00873	0.00562	0.05803	0.05975
^{208}Po	0.19231	0.00884	0.00592	0.05868	0.06088
^{208}Pb	0.21154	0.00897	0.00580	0.05815	0.06008
^{210}Rn	0.18095	0.00907	0.00589	0.05870	0.06118
^{210}Po	0.2	0.00895	0.00577	0.05826	0.06042
^{210}Pb	0.21905	0.00883	0.00587	0.05773	0.05960
^{212}Rn	0.18868	0.00918	0.00595	0.05830	0.06070
^{212}Po	0.20755	0.00906	0.00583	0.05782	0.05993
^{212}Pb	0.22642	0.00895	0.00573	0.05727	0.05914
^{214}Ra	0.17757	0.00917	0.00611	0.05833	0.06097
^{214}Rn	0.19626	0.00928	0.00601	0.05789	0.06023
^{214}Po	0.21495	0.00918	0.00590	0.05739	0.05946
^{214}Pb	0.23364	0.00907	0.00579	0.05684	0.05866
^{216}Ra	0.18519	0.00927	0.00618	0.05793	0.06049
^{216}Rn	0.2037	0.00916	0.00607	0.05749	0.05976
^{216}Po	0.22222	0.00929	0.00596	0.05695	0.05898
^{218}Th	0.17431	0.00948	0.00614	0.05793	0.06077
^{218}Ra	0.19266	0.00936	0.00603	0.05754	0.06005
^{218}Rn	0.21101	0.00926	0.00592	0.05707	0.05931
^{218} Po	0.22936	0.00941	0.00602	0.05652	0.05851
^{220}Th	0.18182	0.00936	0.00619	0.05757	0.06031

describes accurately the dependency of both w and ρ_c on A and I. The coefficients c_1, c_2 and c_3 of the function $f(A, I)$ are determined by a best fit to the results given in table 2 for each w and ρ_c. A standard way of defining the best fit is to choose the parameters such that the sum of the squares of the deviations of the fit from the fitted values is minimized, and this is the so-called χ^2 minimization, i.e., c_1, c_2 and c_3 are determined by minimizing

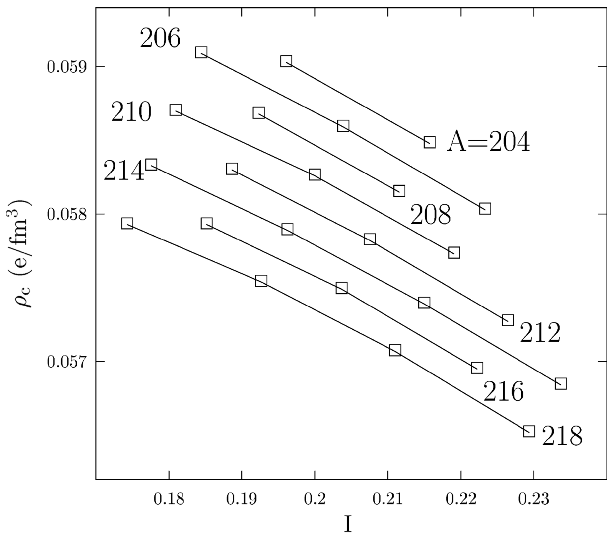

Figure 3. Dependency of the central value of the charge density distribution on the mass number A and nuclear asymmetry I when all mesons ($\sigma\omega\delta\rho$) are included.

$$\chi^2 = \sum_{i=1}^{29} [w(i) - w(A_i, I_i)]^2 \tag{10}$$

or

$$\chi^2 = \sum_{i=1}^{29} [\rho_c(i) - \rho_c(A_i, I_i)]^2 \ , \tag{11}$$

where $w(i)$ is the value of the central depression parameter of the charge density distribution of the nucleus i from the 29 nuclei given in table 2 and $w(A_i, I_i)$ is the value of the function (9) at mass number A_i and asymmetry I_i of the nucleus i. $\rho_c(i)$ and $\rho_c(A_i, I_i)$ have similar meanings.

The functional form (9) describes the first-order expansion of $f(A, I)$ with respect to the equivalent sharp radius of the nucleus, proportional to $A^{1/3}$, and analysis of the systematics of nuclear central densities, calculated from charge density parameters measured by elastic electron scattering and muonic atom spectroscopy, leads to the expectation that the central density decreases with the increase of the square of the nuclear asymmetry I^2 [15], i.e., one would expect c_3 to be negative in the case of ρ_c.

Table 3 lists the coefficients of the function (9) for w and ρ_c of the heavy spherical nuclei around ^{208}Pb, and compares the results when all mesons ($\sigma\omega\delta\rho$) are included with the results when only isoscalar mesons ($\sigma\omega$) are included. The comparison of the values of the coefficient c_3 between ($\sigma\omega\delta\rho$) and ($\sigma\omega$) leads to the result that the isovector coupling channel weakens the dependency of both w and ρ_c on the asymmetry, allowing nuclei with higher neutron-to-proton ratios, i.e., higher asymmetry values, to be more stable. Combining this result with the earlier result from

Table 3. Coefficients of the functions describing the dependency of the central depression parameter w and the central value of the charge density ρ_c on the mass number A and nuclear asymmetry I for heavy spherical nuclei around ^{208}Pb. Table compares the results when all mesons ($\sigma\omega\delta\rho$) are included with the results when only isoscalar mesons ($\sigma\omega$) are included. The resulting values of χ^2 for each of these functions are also given.

		c_1	c_2	c_3	χ^2
($\sigma\omega\delta\rho$)	w	−0.0179	4.56E-3	−4.88E-3	1.657E-7
	ρ_c (e/fm^3)	0.13091	−1.179E-2	−6.487E-2	8.9E-9
($\sigma\omega$)	w	−0.01104	2.92E-3	−1.134E-2	1.194E-7
	ρ_c (e/fm^3)	0.12492	−1.017E-2	−0.10039	2.812E-7

table 2 and eq. (8), it can be concluded that the inclusion of the isovector coupling channel increases the probability of larger nuclei with higher proton numbers and higher neutron-to-proton ratios being stable. For example, one would expect that a long-life isotope of the superheavy element 114 will be very neutron-rich, with the neutron-to-proton ratio beyond the value for the long-life isotope ^{244}Pu, which has a half-life of about 80 million years and a neutron-to-proton ratio of about 1.6, i.e., A for the superheavy element 114 will be about 300.

5. Summary

The central depression parameter and the central value of the charge density distribution of spherically symmetric nuclei with known experimental masses around the ^{208}Pb nucleus were determined by fitting the charge density distribution to a three-parameter Fermi distribution. The central depression parameter and the central density were parametrized as functions of the mass number A and the nuclear asymmetry I. In order to analyse the role played by the isovector coupling channel of the nucleon–nucleon interaction, results with all mesons included were compared with the results with only isoscalar mesons.

The isovector coupling channel leads to about 50% increase of the central depression parameter of the charge density distribution in heavy nuclei, impressively contributing to the semibubble form of the distribution, and the isovector coupling channel weakens the dependency of the parameters of the charge density distribution on the asymmetry, increasing the probability of larger nuclei with higher proton numbers and higher neutron-to-proton ratios being stable.

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