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Progress in all-order breakup reaction theories

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Abstract. Progress in breakup reaction theories, like the distorted wave Born approximation, the continuum discretized coupled channels method and the dynamical eikonal approximation, is brought into focus. The need to calculate exclusive reaction observables and the utility of benchmark tests as arbitrators of theoretical models are discussed.

Keywords. Breakup theories; post and prior forms; distorted wave Born approximation; continuum discretized coupled channels; dynamical eikonal.

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1. Introduction

Breakup reactions are one of the most sought-after methods to study the structure of nuclei right from stable to those near the drip lines. For the latter case it is perhaps the only technique which is reasonably successful in dealing with exotic species – like halo nuclei. It is thus challenging to construct a theoretical model for these reactions which would be applicable over the entire span of nuclei having binding energies ranging from the MeV to the keV scale.

Theoretical approaches to breakup reactions trace their origin to the construction of the exact transition matrix (the T-matrix) which are of the post and the prior form types depending on whether one uses the initial (prior) or the final (post) channel form of the asymptotic Hamiltonian [1]. Although these two forms are equal, the prior form has been further subdivided into a class called the alternate prior form, which however is not equal to the post form.

Theories of breakup reactions like the distorted wave Born approximation (DWBA) and the continuum discretized coupled channels (CDCC) can be constructed by the suitable choice of the exact scattering wave function. The CDCC calculation owes its allegiance to the alternate prior form of the T-matrix, while the DWBA can be done on both the post and prior forms. In the latter case, it was found that the post form of the theory appears to corroborate experimental data in a better way than the prior or the alternate prior form. What is interesting however, is that many of the recent data emanating from different laboratories across the world are reasonably well explained by these methods as also by elaborate models like the dynamical eikonal approximation and the time-dependent

Schrödinger equation method. This is surprising because these models are based on different approximations. An obvious explanation, though, is that most of the reaction observables calculated – like the momentum distributions, relative energy spectra, angular distributions – are of an inclusive nature and involve lots of summations or integrations, which suppress contribution of many partial waves. There is thus a strong case to calculate more exclusive observables, like double- and triple-differential cross-sections, where the differences within the theories could be more apparent.

This paper is organized in the following way. The exact transition amplitude and different approximations to the total wave function leading to various forms of the DWBA, CDCC and dynamical eikonal approximation are discussed in §2. The applications, benchmark tests and future directions are also discussed therein.

2. The exact transition matrix and various approximations

Let us consider the reaction $a + t \rightarrow b + c + t$, in a three-body model, where the projectile a, incident with momentum $\mathbf{q}_a$, breaks up into fragments b and c with momenta $\mathbf{q}_b$ and $\mathbf{q}_c$, respectively in the Coulomb and nuclear fields of a target t.

The Hamiltonian of the system is written as

$$H = T_b + T_c + T_t + V_{bc} + V_{bt} + V_{ct}, \tag{1}$$

where T_i is the kinetic energy of particle i and V_{ij} is the two-body interaction between i and j; their separation will be denoted by $\mathbf{r}_{ij}$ in the following.

To find the interaction in the initial and final channels, we note that the asymptotic Hamiltonians in the initial (prior) and final (post) channels are

$$H_{\rm i} = T_b + T_c + T_t + V_{bc} \tag{2}$$

and

$$H_{\rm f} = T_b + T_c + T_t \tag{3}$$

respectively. Hence the initial (prior) channel interaction

$$V_{\rm i} = H - H_{\rm i} = V_{bt} + V_{ct} \tag{4}$$

and the final (post) channel interaction

$$V_{\rm f} = H - H_{\rm f} = V_{bc} + V_{bt} + V_{ct}. \tag{5}$$

There are two exact T-matrices [6]

$$T_{fi}^{(+)[\rm post]} = \langle {\rm e}^{i\mathbf{q}_c\cdot\mathbf{r}_{ct}} {\rm e}^{i\mathbf{q}_b\cdot\mathbf{r}_{bt}} | V_{bc} + V_{bt} + V_{ct} | \Psi_i^{(+)} \rangle \tag{6}$$

and

$$T_{fi}^{(-)[\rm prior]} = \langle \Psi_i^{(-)} | V_{bt} + V_{ct} | {\rm e}^{i\mathbf{q}_a\cdot\mathbf{r}_{at}} \phi_a(\mathbf{r}_{bc}) \rangle, \tag{7}$$

which are the starting points for a discussion on the theory of breakup processes. The ground state wave function of the projectile, $\phi_a(\mathbf{r}_{bc})$, satisfies

$$(T_b + T_c + V_{bc})\phi_a(\mathbf{r}_{bc}) = -\epsilon_a \phi_a(\mathbf{r}_{bc}), \tag{8}$$

where ϵ_a is the separation energy between fragments b and c in the ground state of the projectile. $\Psi_i^{(+)}$ is the exact scattering wave function with outgoing wave boundary condition (denoted by (+) sign in the superscript) and $\Psi_f^{(-)}$ is the exact scattering wave function with ingoing wave boundary condition (denoted by (–) sign in the superscript). They are the exact eigenfunctions of the three-body Hamiltonian eq. (1). Thus they satisfy

$$H\Psi_i^{(+)} = E\Psi_i^{(+)} \tag{9}$$

and

$$H\Psi_f^{(-)} = E\Psi_f^{(-)}, \tag{10}$$

where E is the total energy of the system. We now use the Gell-Mann–Goldberger two-potential formula [2] to rewrite eqs (6) and (7) as

$$T_{fi}^{(+)[\text{post}]} = \langle \chi_{q_c}^{(-)}(\mathbf{r}_{ct})\chi_{q_b}^{(-)}(\mathbf{r}_{bt})|V_{bc} + V_{bt} + V_{ct} - U_{bt} - U_{ct}|\Psi_i^{(+)}\rangle \tag{11}$$

and

$$T_{fi}^{(-)[\text{prior}]} = \langle \Psi_f^{(-)}|V_{ct} + V_{bt} - U_{at}|\chi_{q_a}^{(+)}(\mathbf{r}_{at})\phi_a(\mathbf{r}_{bc})\rangle. \tag{12}$$

In eqs (11) and (12), wave functions $\chi_{q_c}^{(-)}(\mathbf{r}_{ct})\chi_{q_b}^{(-)}(\mathbf{r}_{bt})$ and $\chi_{q_a}^{(+)}(\mathbf{r}_{at})$ satisfy the Schrödinger equations

$$[T_{r_{ct}} + T_{r_{bt}} + U_{bt} + U_{ct}]\chi_{q_c}^{(-)}(\mathbf{r}_{ct})\chi_{q_b}^{(-)}(\mathbf{r}_{bt}) = E\chi_{q_c}^{(-)}(\mathbf{r}_{ct})\chi_{q_b}^{(-)}(\mathbf{r}_{bt}) \tag{13}$$

and

$$[T_{r_{at}} + U_{at}]\chi_{q_a}^{(+)}(\mathbf{r}_{at}) = (E + \epsilon_a)\chi_{q_a}^{(+)}(\mathbf{r}_{at}), \tag{14}$$

respectively. In eqs (13) and (14), U_{it} are auxiliary potentials acting between particle i and the target.

Assuming $V_{bt} = U_{bt}$ and $V_{ct} = U_{ct}$ in eq. (11), we have [3,4]

$$T_{f\text{i}}^{(+)[\text{post}]} = \langle \chi_{q_c}^{(-)}(\mathbf{r}_{ct})\chi_{q_b}^{(-)}(\mathbf{r}_{bt})|V_{bc}|\Psi_i^{(+)}\rangle. \tag{15}$$

Various approximations to the exact wave function ($\Psi_i^{(+)}/\Psi_f^{(-)}$) will lead us to various reaction theories, which we shall investigate in the preceding subsections.

2.1 *The DWBA approximation*

Let us now introduce the distorted wave Born approximation (DWBA) [1] for the exact wave functions $\Psi_i^{(+)}$ and $\Psi_f^{(-)}$ in eqs (15) and (12).

If one assumes that the inelastic excitations of the projectile are small, then the wave function $\Psi_i^{(+)}$ can be approximated by

$$\Psi_i^{(+)} \approx \chi_{q_a}^{(+)}(\mathbf{r}_{at})\phi_a(\mathbf{r}_{bc}). \tag{16}$$

The post form DWBA T-matrix is then

$$T_{fi}^{(+)[\text{post}]}(\text{DWBA}) = \langle \chi_{q_b}^{(-)}(\mathbf{r}_{bt})\chi_{q_c}^{(-)}(\mathbf{r}_{ct})|V_{bc}|\chi_{q_a}^{(+)}(\mathbf{r}_{at})\phi_a(\mathbf{r}_{bc})\rangle. \tag{17}$$

If, on the other hand, one assumes that the final-state interaction between the breakup fragments, b and c, is not important, i.e. V_{bc} is weak in the final channel, then one can write the exact wave function $\Psi_f^{(-)}$ as

$$\Psi_f^{(-)} \approx \chi_{q_b}^{(-)}(\mathbf{r}_{bt})\chi_{q_c}^{(-)}(\mathbf{r}_{ct}). \tag{18}$$

This leads to the prior form DWBA T-matrix

$$T_{fi}^{(-)[\text{prior}]}(\text{DWBA}) = \langle \chi_{q_b}^{(-)}(\mathbf{r}_{bt})\chi_{q_c}^{(-)}(\mathbf{r}_{ct})|V_{ct} + V_{bt} - U_{at}|\chi_{q_a}^{(+)}(\mathbf{r}_{at})\phi_a(\mathbf{r}_{bc})\rangle. \tag{19}$$

It can be shown [5] that the DWBA T-matrices given by eqs (17) and (19) are equivalent to one another, i.e.,

$$T_{fi}^{(+)[\text{post}]}(\text{DWBA}) = T_{fi}^{(-)[\text{prior}]}(\text{DWBA}). \tag{20}$$

Thus, for actual calculations, one may use the T-matrix which seems more convenient. However, $T^{(-)[\text{prior}]}$ involves very complicated coordinate transformations as compared to the $T^{(+)[\text{post}]}$ form and hence it is very difficult to work with it in actual problems. Moreover, V_{bc} in eq. (17) is of a shorter range than $V_{ct}+V_{bt}-U_{at}$ in eq. (19), which would make the numerical evaluation of eq. (17) relatively easier. The post form DWBA has been extensively used to perform breakup calculations [6–9].

However, by introducing a different approximation for $\Psi_f^{(-)}$, an alternate prior form T-matrix can be obtained. If we assume that the final-state interaction V_{bc} between the fragments b and c is important, then one can approximate $\Psi_f^{(-)}$ as

$$\Psi_f^{(-)} \approx \chi_{\mathbf{Q}_f}^{(-)}(\mathbf{r}_{at})\phi_{a^*,\mathbf{q}_f}^{(-)}(\mathbf{r}_{bc}). \tag{21}$$

In eq. (21), the relative motion wave function of b and c (which could also be a resonant state) is described by $\phi_{a^*,\mathbf{q}_f}^{(-)}(\mathbf{r}_{bc})$, where $\mathbf{q}_f$ denotes the relative momentum between the fragments. The centre of mass (c.m.) motion of the unbound system ($a^* = b + c$) with respect to the target in the final state is given by $\chi_{\mathbf{Q}_f}^{(-)}(\mathbf{r}_{at})$ with momentum $\mathbf{Q}_f$. They are related to momenta $\mathbf{q}_b$ and $\mathbf{q}_c$ of fragments b and c by

$$\mathbf{Q}_f = \mathbf{q}_b + \mathbf{q}_c \tag{22}$$

and

$$\mathbf{q}_f = \frac{m_b}{m_a}\mathbf{q}_b - \frac{m_c}{m_a}\mathbf{q}_c, \tag{23}$$

respectively.

This approximation (eq. (21)) leads to an alternate prior form T-matrix

$$T_{fi}^{(-)[\mathrm{alt,prior}]}(\mathrm{DWBA}) = \langle \chi_{\mathbf{Q}_f}^{(-)}(\mathbf{r}_{at})\phi_{a^*,\mathbf{q}_f}^{(-)}(\mathbf{r}_{bc})|V_{ct}+V_{bt}-U_{at}|\chi_{q_a}^{(+)}(\mathbf{r}_{at})\phi_a(\mathbf{r}_{bc})\rangle \tag{24}$$

or

$$T_{fi}^{(-)[\mathrm{alt,prior}]}(\mathrm{DWBA}) = \langle \chi_{\mathbf{Q}_f}^{(-)}(\mathbf{r}_{at})\phi_{a^*,\mathbf{q}_f}^{(-)}(\mathbf{r}_{bc})|V_{ct}+V_{bt}|\chi_{q_a}^{(+)}(\mathbf{r}_{at})\phi_a(\mathbf{r}_{bc})\rangle. \tag{25}$$

In eq. (24), U_{at} depends on $\mathbf{r}_{at}$ while $\phi_{a^*,\mathbf{q}_f}^{(-)}(\mathbf{r}_{bc})$ and $\phi_a(\mathbf{r}_{bc})$ depend on $\mathbf{r}_{bc}$, and hence the explicit dependence on U_{at} in eq. (25) has dropped out because of the orthogonality of $\phi_{a^*,\mathbf{q}_f}^{(-)}(\mathbf{r}_{bc})$ and $\phi_a(\mathbf{r}_{bc})$. We note that eq. (25) contains only two distorted waves as against the three distorted waves in $T^{(+)[\mathrm{post}]}$. The distorted waves being oscillatory even at large distances, any reduction in their number will accelerate the convergence of the T-matrix. The alternate prior form has been used for performing breakup calculations by several authors [10–12]. It should be noted that $T_{fi}^{(-)[\mathrm{alt,prior}]}(\mathrm{DWBA})$ is no longer equivalent to $T_{fi}^{(+)[\mathrm{post}]}(\mathrm{DWBA})$.

The T-matrix (eq. (25)) describes a situation in which the projectile a is inelastically excited from the ground state to its continuum. If we ignore the nuclear interactions in the distorted waves in both incident and final channels and also ignore the nuclear parts of interactions V_{ct} and V_{bt}, then the alternate prior form T-matrix describes the Coulomb excitation of the projectile. The semiclassical counterpart of eq. (25) is the Alder–Winther theory of Coulomb excitation [13].

This is also the point of comparison between the DWBA and the other reaction theories, especially the CDCC and the eikonal model.

2.2 *The CDCC approximation*

As in the case of alternate prior form DWBA, in CDCC method too the breakup of the projectile, $a(=b+c)$, is considered as an inelastic excitation of the $b+c$ system from its ground state to excited states in the continuum for some momentum k and partial wave l. The CDCC method provides an approach for coupling ground and continuum states together [14,15].

The basic idea of the CDCC method is to expand exact wave function in terms of the eigenstates of the Hamiltonian describing the projectile $[u_{nl}(r_{bc})]$ having

eigenenergies ϵ_{nl}, with the coefficients of expansion as $f^J_{nlL}(r_{at})$, depending on the projectile–target distance. Thus,

$$\Psi^{(-)}_f \rightarrow \Psi^{\rm CDCC} = \frac{1}{r_{bc}r_{at}}\Sigma_{nlL}u_{nl}(r_{bc})f^J_{nlL}(r_{at})[Y_{lm}(\hat{\mathbf{r}}_{bc})\otimes Y_{LM}(\hat{\mathbf{r}}_{at})]^J. \tag{26}$$

In the above equation we define a particular channel $c = \{n, l, L\}$ with n, as the principal quantum number in the bound state or replaced by k if it is a continuum state. l and L are partial waves associated with the internal motion of the projectile and the projectile–target motion, respectively with the total angular momentum as J. The expansion coefficients $f^J_c(r_{at})$ are solutions of coupled set of equations

$$[T_{r_{at}} + V_{cc}(r_{at}) + \epsilon_{nl} - E]f^J_c(r_{at}) + \sum_{c\neq c'} V_{cc'}(r_{at})f^J_{c'}(r_{at}) = 0, \tag{27}$$

where $V_{cc}(r_{at})$ are coupling potentials (potential matrix elements) containing the interaction of the fragments with the target.

Replacing the continuum wave functions by a discretized continuum becomes inevitable at one stage and two different ways are generally used to construct the discretized continuum. In the first, states are constructed by expanding them in some square integrable basis [16] and in the other one constructs average scattering states over momentum bins [14,17].

The transition matrix is then given by

$$T^{(-)}_{fi}({\rm CDCC}) = \langle\Psi^{\rm CDCC}|V_{ct} + V_{bt}|\chi^{(+)}_{q_a}(\mathbf{r}_{at})\phi_a(\mathbf{r}_{bc})\rangle. \tag{28}$$

The alternate prior form DWBA can be regarded as the first iteration of CDCC equations (see, e.g. [17]). However, breakup studies of both stable isotopes [18,19] and halo nuclei [20–22] have shown that the alternate prior form DWBA is insufficient to describe the data and that higher-order coupling effects of the breakup channels are important in both the cases.

2.3 *The eikonal approximation*

This is a semiclassical theory originally introduced in quantum scattering theory by Moliere [23] and has been developed and generalized by Glauber [24]. The underlying physical assumption is that the projectile energy (E_a) is high enough so that it is not deviated much from its original trajectory after passing through the interaction zone. Thus, a condition for the validity of the eikonal approximation is that $|V|/E_a \ll 1$, where $|V|$ is the potential strength.

If we assume that the short-range potential (V) varies very slowly over the scale of the incident wavelength then one can factor out the plane-wave component from the total scattering wave function for the motion of the incident projectile and write

$$\Psi^{(-)}_f = {\rm e}^{ikz}\psi, \tag{29}$$

assuming that the incident wave vector, $\mathbf{k}$, is along the z-direction. In eq. (29), ψ is usually the slowly varying wave function for all trajectories at a given impact parameter. However, this (incoherent) assumption violates rotational symmetry about the z-axis (beam axis). Recently, it has been shown that one actually can use a coherent formulation, the dynamical eikonal approximation (DEA) [25], which preserves rotational symmetry along the beam axis. The wave functions at various azimuthal angles ϕ, which are generated by the operator $\exp(-i\phi j_z)$ can be written as

$$\Psi_f^{(-)} \to \Psi^{\mathrm{DEA}} = \mathrm{e}^{ikz}\mathrm{e}^{-i\phi j_z}\psi, \tag{30}$$

where j_z is the z component of the total angular momentum of the two-body projectile. For more details one is referred to ref. [25]. The transition matrix is then given by

$$T_{fi}^{(-)}(\mathrm{DEA}) = \langle \Psi^{\mathrm{DEA}} | V_{ct} + V_{bt} | \chi_{q_a}^{(+)}(\mathbf{r}_{at})\phi_a(\mathbf{r}_{bc}) \rangle. \tag{31}$$

2.4 *Applications, benchmark tests and future directions*

Application of these theories to the study of breakup reactions, especially those involving exotic projectiles, reveals an interesting problem. They all agree with each other, reasonably well, when inclusive observables like relative energy spectra or angular distributions of fragments are calculated. As examples two particular cases involving the breakup of ^{11}Be and ^{8}B are mentioned here.

In [25] and [26], the relative energy spectra are calculated in the DEA and the finite-range DWBA, respectively, in the breakup of ^{11}Be on a heavy ^{208}Pb target at 69 MeV/nucleon beam energy for two different angular ranges of the projectile centre of the mass scattering angle. The structure model for ^{11}Be in both cases were based on the potential model with slightly different Wood–Saxon parameters. Comparison with recently available experimental data [27] showed that both results explain the data quite well. Similarly, the angular distributions in the breakup of ^{8}B on ^{208}Pb target at 59 MeV/nucleon beam energy within the DEA [28] and the CDCC [29] method show reasonably good agreement.

However, this apparent agreement between different models will not be so obvious in the case of exclusive observables, like double and triple differential cross-sections, where interference effect between Coulomb and nuclear or between partial waves themselves will not be washed away by too many summations or integrations, unlike in inclusive observables.

This feature is apparent in the calculation [26] of neutron energy distributions in the breakup of ^{11}Be on ^{44}Ti at various beam energies and neutron emission angles. At 20 MeV/nucleon beam energy, the Coulomb breakup accounted for more of the cross-section than nuclear breakup and the Coulomb-nuclear interference (CNI) terms was constructive, in general. The importance of the CNI terms were again highlighted by the calculation at the beam energy of 30 MeV/nucleon, where the CNI terms at low neutron emission angles not only cancel out the nuclear terms, but also reduces the Coulomb terms so that the coherent total sum is less than

the Coulomb terms. At 40 MeV/nucleon beam energy, interference was generally constructive at smaller neutron angles, often being larger or almost equal to the individual nuclear terms. These results, thus, indicate that the CNI terms are not only dependent on energies and angles of the outgoing fragments, but also on the incident beam energy. It would indeed be quite interesting if in a future experiment exclusive cross-section measurements could be made at low beam energies where the effect of the CNI terms has been found to be substantial.

At this stage it is also worthwhile to mention that, in various limits of the parameter space, one can derive analytical results which can serve as benchmarks for more elaborate models [30–32]. This not only helps in the physical understanding of the problem, but also serves as arbitrators of theoretical models.

In ref. [33], two different theoretical models of breakup reactions (finite range DWBA and the Alder-Winther theory of Coulomb dissociation) were compared by calculating several reaction observables like relative energy spectra, angular distributions and breakup cross-sections, taking the neutron separation energy as a parameter. Theoretically, one thus simulated situations ranging from weakly bound isotopes to stable ones for the same angular momentum configuration of the system.

Investigation of the one-neutron breakup cross-section as a function of separation energy obtained by comparing the results of two theories revealed that, as one goes away from the valley of stability towards the drip lines (where one would encounter predominantly weakly bound isotopes) higher multipoles, other than the dipole, do not play any significant role in the breakup process. That the two results were almost identical also opens up an interesting opportunity. In the calculation of astrophysical S-factor via the Coulomb dissociation method, it is crucially important that only a particular multipolarity is almost solely responsible for the breakup process. Thus, calculating the same observable with the all-order finite range DWBA could provide an independent method to check whether a single multipolarity is indeed responsible for the breakup process.

To complement the progress in calculating the Coulomb breakup of neutron halo nuclei, efforts are also underway to construct a fully quantal theory of Coulomb breakup involving proton halo nuclei. The transition amplitude in this case splits into the structure and dynamics parts with the latter being related to the analytically solvable bremsstrahlung integral [34]. This theory could then be applied to calculate the breakup reaction $^8\mathrm{B} \rightarrow {}^7\mathrm{Be} + p$ on a heavy target and from thereon relate it to the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ radiative capture cross-section.

Nevertheless, several challenges remain in reaction theory which needs to be taken care of in the future. Sensitivity of the projectile structure is prime among them. It so appears that if the structure model reproduces the binding energy correctly, then the cross-section may not be too sensitive to the projectile structure. Closely related is the concept of 'spectroscopic factors'. In principle, they need to be derived from a more fundamental theory thereby predicting the probability of the cluster structure considered within the projectile. More often than not it is considered as a phenomenological factor for matching experimental cross-sections with theoretical predictions. Therefore, lot of scope exists to improve the projectile structure information by using *ab-initio* calculations and other microscopic models utilizing N–N forces and antisymmetrization.

3. Conclusion

Breakup reactions still remain one of the preferred ways to investigate nuclei both far and near the stability line. In this paper we have brought into focus the progress made in breakup reaction theories like the DWBA, CDCC and the dynamical eikonal approximation. Current experimental data cannot fully distinguish between the inclusive observables predicted by these theories, although they start with different approximations to the exact wave function. This calls for more calculations of exclusive observables, like double and triple differential cross-sections, where effects of interferences between Coulomb and nuclear terms as also between different partial waves will be more apparent. There is thus lot of scope to investigate these effects in future experiments. The utility of benchmark tests based on analytical results, which helps not only in the physical understanding of the problem, but also as arbitrators of theoretical models were emphasized.

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