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# Large-amplitude double layers in a dusty plasma with an arbitrary streaming ion beam

BRINDABAN DAS[1,*], DEBKUMAR GHOSH[1] and PRASANTA CHATTERJEE[1,2]
[1]Department of Mathematics, Siksha Bhavana, Visva Bharati University, Santiniketan 731 235, India
[2]Present address: Department of Physics, Plasma Physics Division, University of Malay, Malaysia
[*]Corresponding author. E-mail: brindabandas2006@rediffmail.com

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**Abstract.** Formation of large-amplitude double layers in a dusty plasma whose constituents are electrons, ions, warm dust grains and positive ion beam are studied using Sagdeev's pseudopotential technique. Existence of double layers is investigated. It is found that both the temperature of dust particles and ion beam temperature play significant roles in determining the region of the existence of double layers.

**Keywords.** Double layers; dusty plasma.

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## 1. Introduction

The study of different types of collective processes in dusty plasmas received much attention for the last two decades or so. This is because dusty plasmas occur in nature in various forms, such as planetary rings, cometary tails, interstellar clouds, etc. [1–3] Dusty plasmas play a vital role in understanding different types of new and interesting aspects in other fields like low-temperature physics, radiofrequency plasma discharge, coating and etching of thin films, plasma crystals etc. Such plasmas are also investigated in laboratory experiments [3,4].

Several authors have studied the non-linear wave phenomena in dusty plasmas. It began with the work of Bliokh and Yarashenko [5] who first theoretically observed the waves while dealing with waves in Saturn's ring. The discovery of dust-acoustic wave [6,7] and dust ion-acoustic wave [8,9] gave a new impetus to the study of waves in dusty plasmas. Later, it was found that the dust grain dynamics also introduced few new eigenmodes like Dust–Berstain–Greene–Kruskal (DBGK) mode, dust lattice (DL) mode [10], Shukla–Verma mode [11], dust-drift mode [12] etc. A number of theoretical studies of dust ion-acoustic (DIA) soliton [13,14], dust-acoustic soliton [15] and DL soliton have also been done with low-frequency dust electrostatic and electromagnetic waves. The DIA solitary and shock waves and DL solitary waves were investigated experimentally [16].

In the last few years, the formation of double layers has been a topic of great interest. Double layers are found in a variety of laboratory plasmas such as in constricted plasmas [17], mercury discharges [18], Q-machines [19], triple plasma devices [20] etc. The role of double layers in astrophysics is also considerable as they are thought to be present in the magnetosphere and responsible for the acceleration of electrons onto the upper atmosphere, creating the fantastic aurora [21]. Various theories on the formation of solar flares also involve double layers [22]. It was also proposed that double layers might play a significant role in supplying and accelerating plasma in magnetic coronal funnels [23]. Ion beam plasma system has been studied both theoretically and experimentally by several authors. Ion beams in laboratory dusty plasmas have become indispensable in materials processings such as etching, chemical vapour deposition and surface modification. Such circumstances in plasma applications and the case of realizing dusty plasmas on a laboratory scale have accelerated active studies on dust phenomena in plasmas. Nejoh [24] studied large-amplitude dust-acoustic waves in a plasma with an ion beam. He treated the ion beam as thermal without the inclusion of the particle streaming in his calculation of the beam current at the dust grain surface. Zank and McKenzie [25] studied the ion beam plasma system by adopting a multifluid model. Later, Zhang and Kuehl [26] modified their result by taking into account the effect of electron inertia. However, in both these works, the ion beam temperatures were neglected. El-Labany and El-Taibany [27] also studied dust-acoustic solitary waves and double layers in the presence of arbitrary streaming ion beam and found that there exists a critical ion beam velocity below which the ion beam is unable to generate solitons. Most of the authors used reduction perturbation technique (RPT) to study soliton solution and double layers. But, recently, Malfliet and Wieers [28] reviewed the studies on solitary waves and found that reduction perturbation technique (RPT) is based on the smallness of the amplitude. So their result is valid only for small-amplitude cases. But large-amplitude solitary waves and double layer also exist in nature and these were studied by several investigators [29,30] using Sagdeev's pseudopotential technique [31]. Sakanaka and Shukla [32] studied dust-acoustic double layers in the presence of charged dust grains.

In the present paper we consider the motion of dust particle in the presence of electrons, ions and an arbitrary streaming ion beam. The motivation of the paper is to study the existence of double layers without neglecting the dust temperature. The temperature of the dust particles is important, owing to thermalization with the ions or orbital effects. In fact, it is suggested that the dust in planetary rings may have a large value of dust temperature compared to the electron temperature. The organization of the paper is as follows. In §2 the basic equations are written and the Sagdeev's pseudopotential is derived. The conditions for the existence of double layers are given. Results and discussion is given in §3. Section 4 is kept for conclusions.

## 2. Basic equations and the existence of double layers

We consider an unmagnetized dusty plasma consisting of warm dust grains, electrons, ions and positive ion beam. Based on the continuity and momentum fluid equations, we have the following one-dimensional equations for warm dust grains and ion beam:

$$\frac{\partial n_{\mathrm{d}}}{\partial t} + \frac{\partial (n_{\mathrm{d}} u_{\mathrm{d}})}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_{\mathrm{d}}}{\partial t} + u_{\mathrm{d}} \frac{\partial u_{\mathrm{d}}}{\partial x} + 3\sigma_{\mathrm{d}} n_{\mathrm{d}} \frac{\partial n_{\mathrm{d}}}{\partial x} - z_{\mathrm{d}} \frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial n_{\mathrm{b}}}{\partial t} + \frac{\partial (n_{\mathrm{b}} v_{\mathrm{b}})}{\partial x} = 0, \tag{3}$$

$$\frac{\partial v_{\mathrm{b}}}{\partial t} + v_{\mathrm{b}} \frac{\partial v_{\mathrm{b}}}{\partial x} + 3\mu_{\mathrm{bd}} \sigma_{\mathrm{b}} n_{\mathrm{b}} \frac{\partial n_{\mathrm{b}}}{\partial x} + \mu_{\mathrm{bd}} \frac{\partial \phi}{\partial x} = 0, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = z_{\mathrm{d}} n_{\mathrm{d}} + n_{\mathrm{e}} - n_i - n_{\mathrm{b}}, \tag{5}$$

where $n_{\mathrm{d}}$ and $u_{\mathrm{d}}$ are the number density and fluid velocity of the dust grain respectively, and $n_{\mathrm{b}}$ and $v_{\mathrm{b}}$ are the corresponding beam parameters. Electrons and ions are assumed to have Boltzmann distribution. Thus we can express $n_{\mathrm{e}}$ and $n_{\mathrm{i}}$ as

$$n_{\mathrm{e}} = \frac{n_{\mathrm{e}0}}{z_{\mathrm{d}0} n_{\mathrm{d}0}} \exp(\beta_1 s \phi), \tag{6}$$

and

$$n_{\mathrm{i}} = \frac{n_{\mathrm{i}0}}{z_{\mathrm{d}0} n_{\mathrm{d}0}} \exp(-s \phi). \tag{7}$$

At equilibrium we have

$$n_{\mathrm{i}0} + n_{\mathrm{b}0} = n_{\mathrm{e}0} + z_{\mathrm{d}0} n_{\mathrm{d}0}, \tag{8}$$

where $n_{\mathrm{e}0}$, $n_{\mathrm{b}0}$, $n_{\mathrm{i}0}$ and $n_{\mathrm{d}0}$ are the unperturbed electron, beam, ion and dust number densities, respectively. $z_{\mathrm{d}}$ is the dust charge number and is normalized by $z_{\mathrm{d}0}$. $\phi$ is the electrostatic potential. The space coordinate $x$, time $t$, velocities and electrostatic potential $\phi$ are normalized by the Debye length $\lambda_{\mathrm{De}} = (T_{\mathrm{eff}}/4\pi n_{\mathrm{d}0} z_{\mathrm{d}0} e^2)^{1/2}$, the inverse of dust plasma frequency $\omega_{\mathrm{pd}}^{-1} = (m_{\mathrm{d}}/4\pi n_{\mathrm{d}0} z_{\mathrm{d}0}^2 e^2)^{1/2}$, the dust-acoustic speed $C_{\mathrm{d}} = (z_{\mathrm{d}0} T_{\mathrm{eff}}/m_{\mathrm{d}})^{1/2}$, and $T_{\mathrm{eff}}/e$, respectively.

Here $\mu_{\mathrm{bd}} = m_{\mathrm{d}}/m_{\mathrm{b}} z_{\mathrm{d}0}$, $\sigma_{\mathrm{b}} = (T_{\mathrm{b}}/T_{\mathrm{eff}}) = (1/s\beta_{\mathrm{b}})$, $\sigma_{\mathrm{d}} = T_{\mathrm{d}}/z_{\mathrm{d}0} T_{\mathrm{eff}}$, $\beta_1 \equiv T_{\mathrm{i}}/T_e$, $\beta_{\mathrm{b}} \equiv T_{\mathrm{i}}/T_{\mathrm{b}}$, $\delta_1 = n_{\mathrm{i}0}/n_{\mathrm{e}0}$, $\delta_2 = n_{\mathrm{b}0}/n_{\mathrm{e}0}$ and $s = T_{\mathrm{eff}}/T_{\mathrm{i}} = (\delta_1 + \delta_2 - 1)/(\delta_1 + \beta_{\mathrm{b}} \delta_2 + \beta_1)$, with temperature $T_{\mathrm{e}}$ for electrons, $T_{\mathrm{i}}$ for ions and $T_{\mathrm{b}}$ for beam, respectively.

$m_{\rm d}/m_{\rm b}$ is the mass of dust/beam particles. Thus, we have $n_{\rm e0}/z_{\rm d0}n_{\rm d0} = 1/(\delta_1 + \delta_2 - 1)$, $n_{\rm i0}/z_{\rm d0}n_{\rm d0} = \delta_1/(\delta_1 + \delta_2 - 1)$.

In order to find the Sagdeev's pseudopotential from eqs (1)–(5), we assume that all dependent variables depend on a single independent variable $\xi = x - vt$, where $v$ is the solitary wave velocity. The variable $\xi$ is the special coordinate in the coordinate system moving with the solitary wave velocity, i.e., the wave frame. Equations (1)–(5) now reduce to

$$-v\frac{{\rm d}n_{\rm d}}{{\rm d}\xi} + \frac{{\rm d}(n_{\rm d}u_{\rm d})}{{\rm d}\xi} = 0, \tag{9}$$

$$-v\frac{{\rm d}u_{\rm d}}{{\rm d}\xi} + u_{\rm d}\frac{{\rm d}u_{\rm d}}{{\rm d}\xi} + 3\sigma_{\rm d}n_{\rm d}\frac{{\rm d}n_{\rm d}}{{\rm d}\xi} - z_{\rm d}\frac{{\rm d}\phi}{{\rm d}\xi} = 0, \tag{10}$$

$$-v\frac{{\rm d}n_{\rm b}}{{\rm d}\xi} + \frac{{\rm d}(n_{\rm b}v_{\rm b})}{{\rm d}\xi} = 0, \tag{11}$$

$$-v\frac{{\rm d}v_{\rm b}}{{\rm d}\xi} + v_{\rm b}\frac{{\rm d}v_{\rm b}}{{\rm d}\xi} + 3\mu_{\rm bd}\sigma_{\rm b}n_{\rm b}\frac{{\rm d}n_{\rm b}}{{\rm d}\xi} + \mu_{\rm bd}\frac{{\rm d}\phi}{{\rm d}\xi} = 0, \tag{12}$$

$$\frac{{\rm d}^2\phi}{{\rm d}\xi^2} = z_{\rm d}n_{\rm d} + n_{\rm e} - n_{\rm i} - n_{\rm b}. \tag{13}$$

From eq. (9) we get

$$n_{\rm d} = \frac{v}{v - u_{\rm d}}. \tag{14}$$

From eq. (10) we get

$$\phi = -vu_{\rm d} + \frac{u_{\rm d}^2}{2} + \frac{3\sigma_{\rm d}}{2}\left(\frac{v}{v-u_{\rm d}}\right)^2 - \frac{3\sigma_{\rm d}}{2}. \tag{15}$$

Equation (15) can be solved to find $u_{\rm d}$ as an explicit function of $\phi$ and is given by

$$u_{\rm d} = v - \frac{1}{\sqrt{2}}[v^2 + 2\phi + 3\sigma_{\rm d} + \sqrt{(v^2 + 2\phi + 3\sigma_{\rm d})^2 - 12\sigma_{\rm d}v^2}]^{1/2}. \tag{16}$$

From eq. (11) we get

$$n_{\rm b} = \frac{\nu(v - v_0)}{v - v_{\rm b}}. \tag{17}$$

To derive the above results, we have used the boundary conditions $\phi \to 0$, $n_{\rm d} \to 1$, $u_{\rm d} \to 0$, $v_{\rm b} \to v_0$ and $n_{\rm b} \to \nu$, as $|\xi| \to \infty$.

From eqs (12) and (17) we can find $n_{\rm b}$ as an explicit function of $\phi$ and is given by

$$n_{\mathrm{b}} = \left[\frac{(A-2\mu_{\mathrm{bd}}\phi) \pm \sqrt{(A-2\mu_{\mathrm{bd}}\phi)^2 - 12\mu_{\mathrm{bd}}\sigma_{\mathrm{b}}B^2}}{6\mu_{\mathrm{bd}}\sigma_{\mathrm{b}}}\right]^{1/2}, \tag{18}$$

where $B = \nu(v - v_0)$, $A = (v - v_0)^2 + 3\mu_{\mathrm{bd}}\sigma_{\mathrm{b}}\nu^2$.

Integrating eq. (13) and using the boundary conditions given above we get

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\xi^2} = -\frac{\mathrm{d}\psi_{\mathrm{d}}}{\mathrm{d}\phi}, \tag{19}$$

where

$$\begin{aligned}\psi_{\mathrm{d}} = {} & \sigma_{\mathrm{d}} + vu_{\mathrm{d}} - \sigma_{\mathrm{d}}\left(\frac{v}{v-u_{\mathrm{d}}}\right)^3 \\ & + \frac{1}{s(\delta_1+\delta_2-1)}\left[\frac{1}{\beta_1}(1-\mathrm{e}^{\beta_1 s\phi}) + \delta_1(1-\mathrm{e}^{-s\phi})\right] \\ & + P\left[\frac{2}{3}(\mathrm{e}^{3\theta/2} - \mathrm{e}^{3\theta_0/2}) + 2(\mathrm{e}^{-\theta/2} - \mathrm{e}^{-\theta_0/2})\right].\end{aligned} \tag{20}$$

$P = \frac{1}{4\mu_{\mathrm{bd}}}(48\mu_{\mathrm{bd}}\sigma_{\mathrm{b}}B^6)^{1/4}$, $\theta = \cosh^{-1}[(12\mu_{\mathrm{bd}}\sigma_{\mathrm{b}}B^2)^{-1/2}(A - 2\mu_{\mathrm{bd}}\phi)]$ and as $\phi \to 0$, $\theta \to \theta_0$.

Note that eq. (19) is analogous to the standard potential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{\mathrm{d}\phi}{\mathrm{d}x}. \tag{21}$$

So in eq. (19) if $\phi$, the potential of the particle is termed as pseudoposition then obviously $\psi_{\mathrm{d}}(\phi)$ should be termed as pseudopotential. For the formation of double layers, the Sagdeev's pseudopotential $\psi_{\mathrm{d}}(\phi)$ must satisfy the following conditions:

(i) $\psi_{\mathrm{d}}(\phi) = 0$, $\psi'_{\mathrm{d}}(\phi) = 0$ and $\psi''_{\mathrm{d}}(\phi) < 0$ at $\phi = 0$.

(ii) $\psi_{\mathrm{d}}(\phi) = 0$, $\psi'_{\mathrm{d}}(\phi) = 0$ and $\psi''_{\mathrm{d}}(\phi) < 0$ at $\phi = \phi_m (\neq 0)$.

For the double layer solution, the Sagdeev's pseudopotential should be negative between $\phi = 0$ and $\phi_m$, where $\phi_m$ is some extremum value of the potential and is called the amplitude of the double layer. The condition $\partial\psi_{\mathrm{d}}/\partial\phi = 0$ at $\phi = \phi_m$ gives

$$\begin{aligned}& \frac{v}{v-u_{dm}} + \frac{1}{(\delta_1+\delta_2-1)}(\mathrm{e}^{\beta_1 s\phi_m} - \delta_1\mathrm{e}^{-s\phi_m}) \\ & \quad + P\left[\frac{2}{3}(\mathrm{e}^{3\theta_m/2} - \mathrm{e}^{3\theta_{0m}/2}) + 2(\mathrm{e}^{-\theta_m/2} - \mathrm{e}^{-\theta_{0m}/2})\right] = 0,\end{aligned} \tag{22}$$

where

$$u_{dm} = v - \frac{1}{\sqrt{2}}\left[v^2 + 2\phi_m + 3\sigma_{\mathrm{d}} + \sqrt{(v^2+2\phi_m+3\sigma_{\mathrm{d}})^2 - 12\sigma_{\mathrm{d}}v^2}\right]^{1/2}. \tag{23}$$

From the above equations we can determine $v$ and $\phi_m$ in terms of other parameters for the existence of double layers.

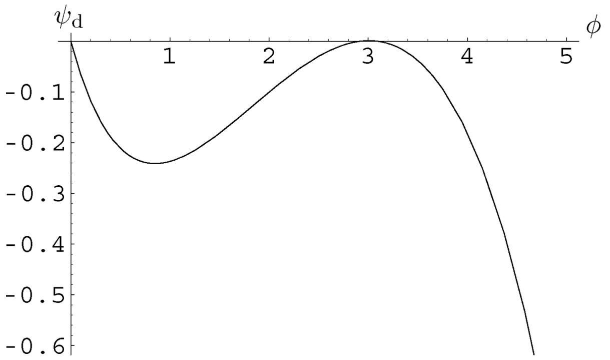


**Figure 1.** The plot of $\psi_{\rm d}(\phi)$ vs. $\phi$ for the set of parameters $v = 1.15$, $\sigma_{\rm d} = 0.06$, $\delta_1 = 10, \delta_2 = 5$, $\beta_{\rm b} = 0.1$, $\beta_1 = 0.5$, $v_0 = 0.2$, $\mu_{\rm bd} = 20$, $\nu = 1.05$.

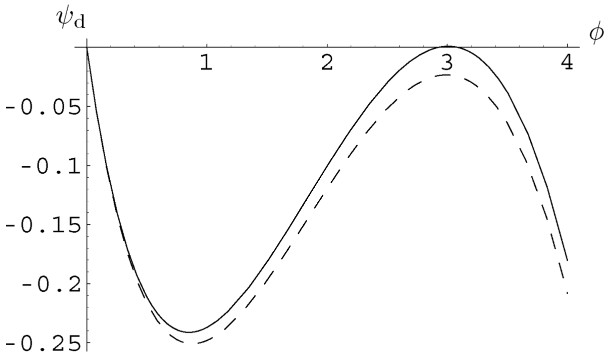


**Figure 2.** The plot of $\psi_{\rm d}(\phi)$ vs. $\phi$ for $\sigma_{\rm d} = 0.06$ and 0.00. Other parameters are $v = 1.15$, $\delta_1 = 10, \delta_2 = 5, \beta_{\rm b} = 0.1, \beta_1 = 0.5, v_0 = 0.2$, $\mu_{\rm bd} = 20, \nu = 1.05$. Here $\sigma = 0.06$ corresponds to the solid curve. The dashed curve is for $\sigma = 0.0$.

## 3. Results and discussions

To find the region of existence of double layer, one has to study the nature of the function $\psi_{\rm d}(\phi)$. Figure 1 shows the plot of $\psi_{\rm d}(\phi)$ vs. $\phi$, and it is seen that the double layers exist for the set of parameters $v = 1.15$, $\delta_1 = 10$, $\delta_2 = 5$, $v_0 = 0.2$, $\beta_{\rm b} = 0.1$, $\beta_1 = 0.5$, $\sigma_{\rm d} = 0.06$, $\nu = 1.05$ and $\mu_{\rm bd} = 20$. In figure 2, $\psi_{\rm d}(\phi)$ is plotted against $\phi$ for $\sigma_{\rm d} = 0.06$ and 0.0. Other parameters are $v = 1.15$, $\delta_1 = 10$, $\delta_2 = 5$, $v_0 = 0.2$, $\beta_{\rm b} = 0.1$, $\beta_1 = 0.5$, $\mu_{\rm bd} = 20$, $\nu = 1.05$. It is seen that for $\sigma_{\rm d} = 0.06$, both conditions (i) and (ii) are satisfied and hence a double layer exists. However, it is found that for $\sigma_{\rm d} = 0.0$, condition (i) is satisfied but $\psi_{\rm d}(\phi) \neq 0$ at $\phi = \phi_m$, i.e. $\psi_{\rm d}(\phi)$ does not cross the $\phi$-axis at any point other than 0. Hence double layer does not exist for $\sigma_{\rm d} = 0.0$. Here the solid curve corresponds to $\sigma = 0.06$ and $\sigma = 0.0$ corresponds to the dotted curve. So it is found that the finite dust temperature has a significant role in the formation of double layers. To see the effect of $\beta_{\rm b}$, in figure 3, $\psi_{\rm d}(\phi)$ is plotted against $\phi$ for $\beta_{\rm b} = 0.1$ and 0.15. It is seen from the solid curve that for $\beta = 0.1$, a double layer exists. But from the dotted curve it is seen

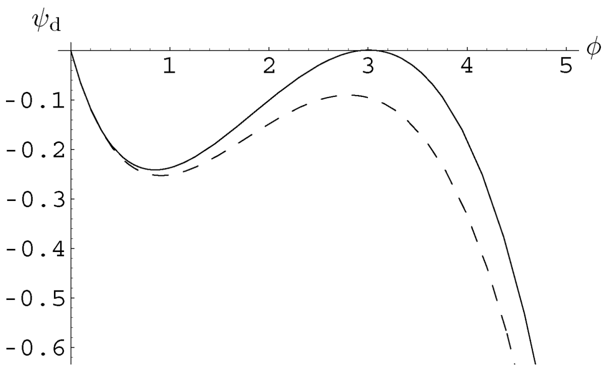


**Figure 3.** The plot of $\psi_d(\phi)$ vs. $\phi$ for $\beta_b = 0.1$ and 0.15. The other parameters are the same as those in figure 1. Here $\beta_b = 0.1$ corresponds to the solid curve. The dashed curve is for $\beta_b = 0.15$.

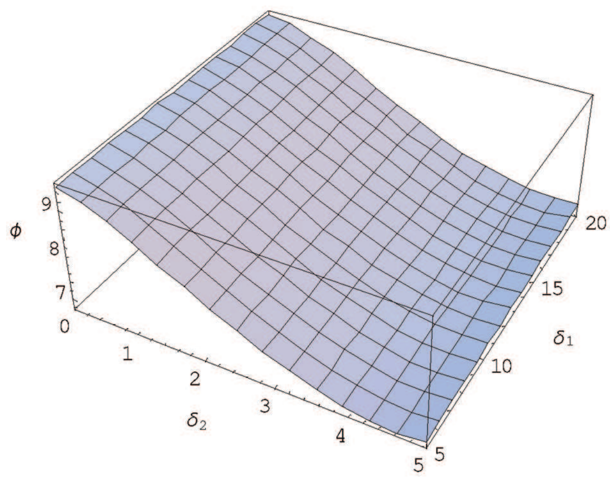


**Figure 4.** A three-dimensional plot of $\phi_m$ against $\delta_1$ and $\delta_2$ showing the region of existence for double layers. Here $v$ is kept fixed at 1.15.

that for a very small change in $\beta_b$, the double layer ceases to exist. Hence, both dust temperature and ion beam temperature play significant roles in the formation of double layers. Figure 4 shows the domain of the existence of double layers in a three-dimensional plot, where $\phi_m$ is plotted against $\delta_1$ and $\delta_2$. Here $v$ is kept fixed at $v = 1.15$. Amplitude of the double layers is plotted against $v$ in figure 5 and it is seen that as $v$ increases amplitude of the double layers decreases. It is to be noted that the model considered here is structurally unstable, in the sense that a small change in the parameters or inclusion of small additional effects will not produce just a small change in the solution, but completely change its nature.

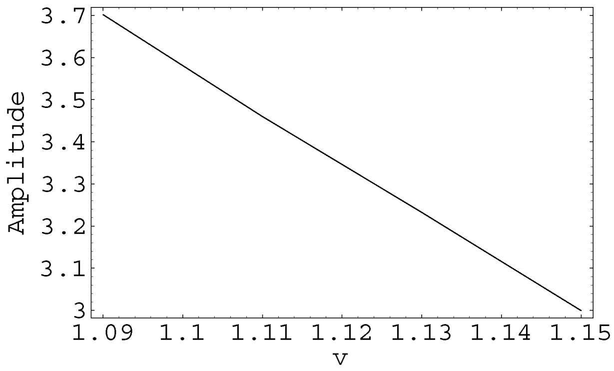


**Figure 5.** Amplitude of the double layers is plotted against $v$. All other parameters are same as those in figure 1.

## 4. Conclusion

We have used Sagdeev's approach to study large-amplitude double layers in dusty plasma in the presence of electrons, ions and arbitrary streaming ion beam. Existence of double layers is investigated. It is found that both the temperature of dust particles and ion beam temperature play significant roles in determining the region of the existence of double layers.

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